

Optimal Banking System for Private Money Creation

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Abstract

By taking deposits and making loans, banks create private money in an economy—the bank deposits. In this paper I study how the banking system should be designed such that banks can most efficiently create these money-like assets that are both *liquid* and *safe*. An endogenously created common pool of liquidity lies at the core of this private money creation process—loans made by one bank can generate a fresh supply of liquidity to the banking sector before they pay off, which helps to ensure the safety of bank deposits under the exposure to liquidity shocks. I show that under a setting of incomplete markets and incomplete contracts, three market failures potentially plague the private creation of safe assets. First, an *incentive* problem can lead to ex-ante under-production of the liquidity pool, when banking sector is subject to *idiosyncratic* liquidity risks. Second, a *commitment* problem can result in ex-post over-use of the liquidity pool, when banking sector is subject to *systemic* liquidity risks. Interestingly, more competitive banking markets ensure more efficient ex-ante incentive provision and hence alleviate the under-production problem; however, increased competition also introduces an ex-post commitment issue, which exacerbates the over-use problem. Finally, an ex-ante *coordination* problem potentially arises when banking markets become sufficiently competitive: no private safe assets can be created in the economy without proper liquidity regulations (e.g. uniform reserve requirements) being imposed. The analysis has implications on regulations of banking markets and bank liquidity for enhancing economy's resilience to negative shocks.

Keywords: Safe asset creation, Liquidity risks, Pecuniary externality, Banking market competition, Liquidity regulation.

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1. Introduction

A fundamental yet crucial role that banks play in an economy is as the creator of “money”. From its earliest form as record-keeping tokens to its modern form as demand deposits, these money-like assets created by banks lubricates transactions and activities in the economy by serving as a credible payment medium. Today 97% of the money circulating in the economy is created by banks, in the form of bank deposits, whilst just 3% is printed by the government.¹ The importance of bank-created safe assets as a store of value and medium of exchange has been well recognized in both the macro and the banking literature.² Yet less attention has been paid to the creation of these money-like assets. This paper studies the creation of private money by the banking sector.

A conventional wisdom of how banks create private money in an economy is the so called “fountain pen money” view. That is, banks can create money simply by making loans— loans made by one bank will immediately create matching deposits somewhere in the banking system and whichever banks receive them can keep making loans and so on so forth. While this notion of how banks create money seems logically appealing and is believed by many as an accurate depiction of the true picture, something fundamental is missing and flawed in it. First and most importantly, in the process of private money creation, banks maintain an illiquid capital structure and are thus subject to liquidity risks.³ Such an exposure to liquidity risks is problematic to banks’ money creation, as it poses a challenge to banks in guaranteeing the safety of the money they create. Second, loans do not generate deposits directly, but rather in an indirect way.⁴

My analysis starts with recognizing and fixing these two issues highlighted above in modeling banks’ private money creation. A crucial linkage immediately emerges once we combine the two— loans made by one bank can generate fresh supply of liquidity to the banking sector before they pay off, since whoever receives these credit would have a demand for a safe store to place the credit they receive. Importantly, the enlarged liquidity pool for the banking sector helps to ensure the safety of bank deposits under the exposure to liquidity shocks, during the period before their loans pay off. In a world with one

¹See *How Banks Create Money - Positive Money*, Bank of England quarterly bulletin 2014.

²See [Kiyotaki and Wright \(1989\)](#); [Gorton and Pennacchi \(1990\)](#); [Caballero \(2006\)](#); [Stein \(2012\)](#); [Hart and Zingales \(2014\)](#); [DeAngelo and Stulz \(2015\)](#); [Diamond \(2017\)](#) etc..

³The fact that banks, as the provider of liquidity insurance for individual households, are subject to risk of runs is pointed out by the canonical work by [Diamond and Dybvig \(1983\)](#). The necessity of maintaining a fragile capital structure in financing illiquid projects that require special payment collection skills is further made clear in [Diamond and Rajan \(2001\)](#).

⁴This point is first made clear by [Tobin \(1963\)](#) in his famous critique on the notion of bank money creation by saying “... It depends on whether somewhere in the chain of transactions initiated by the borrower’s outlays are found depositors who wish to hold new deposits equal in the amount of the new loan...”.

single bank, the bank will see that this liquidity pool, which is endogenously created by its loan making, be optimally produced ex-ante and properly utilized ex-post. However, in an environment with many banks, this endogenously created liquidity pool may not be optimally produced and utilized when markets and contracts are incomplete. The design of the banking system thus comes into the spotlight.

In this paper, I address the following question. In an environment where markets and contracts are incomplete, how should the private banking system be designed to most efficiently create money-like assets in the economy– the bank deposits, which need to be made both liquid and safe? Put differently, how should the banking system be designed to maximize the efficiency in its private safe assets creation under the exposure to liquidity risks?

In doing so, I first demonstrate that the combination of incomplete markets (and contracts) with a constraint imposed by safe asset creation under liquidity risks implies that several market failures of distinct natures could arise, in the process of private money creation in a decentralized economy. Having identified where the roots of inefficiencies are located, I then study the optimal design of the private banking system to alleviate the inefficiencies induced by these market failures and thus to maximize the efficiency of private money creation in a decentralized economy.

In this paper I consider two commonly used policy instruments in the design of banking system. The first policy tool that a policy maker (who is not a social planner) can deploy is the regulation on banking market competition. In particular, I focus on the (de)regulation of bank funding markets, such as imposing/removing restrictions on banks' branching and deposit taking operations. The second policy tool could be deployed to achieve efficient private money creation is bank liquidity regulation. In particular, the only form of liquidity regulation considered in this paper is simply imposing certain uniform reserve requirements.⁵ As the main results of the analysis, I show that banking market competition has a mixed effect on efficiency and liquidity regulation becomes necessary for the private safe asset creation once banking market becomes sufficiently competitive. Let us now elaborate.

The economy in the model, which consists of three groups of agents– banks, households and entrepreneurs– is located on a circle as in [Salop \(1979\)](#) and [Zentefis \(2019\)](#). Each bank possesses a natural territory, within which it enjoys certain monopoly power in both lending and funding markets. Two basic assumptions are made. First, there is a

⁵In other words, it is assumed that the policy maker in this economy does not have access to more complicate policy tools such as the liquidity regulations proposed in Basel III (e.g., minimum requirement on LCR, NSFR, etc.)

demand for money-like safe assets to support economic activities in the economy.⁶ Second, entrepreneurs' projects are illiquid when external financing is needed.⁷ In such an environment, banks perform a dual role on either sides of their balance sheets: they create a payment medium by issuing money-like deposits and they finance illiquid projects as delegated monitors or relationship lenders.

The money creation process is initiated at the moment banks take deposits from local households, at which point an exchange medium is created. Thanks to the ex-ante creation of such money-like deposits, profitable projects are generated in the economy. Banks then invest part of the received deposits into illiquid lending to finance local entrepreneurs' projects.⁸ In an important novelty of this model, banks' loan making triggers a whole chain of subsequent transactions in the economy, which generates a fresh supply of liquidity to the banking sector. In the model, this "liquidity pool creation" effect of loan making is captured by assuming a constant labor payment per unit of production, which is distributed to local households. This loan-created liquidity pool in the economy helps to guarantee that deposits issued by banks are indeed safe assets.

A key assumption maintained throughout the analysis in this paper is that for bank deposits to be money-like, they need to be made both *liquid* and *safe*. A tension thus arises in banks' private money creation— safety of banks deposits needs to be guaranteed with a fragile bank capital structure being maintained.⁹ This fragility-safety conflict is modeled by assuming that some banks will be hit by "liquidity shocks" before their loans pay off, in which case these banks need to (be able to) raise enough short-term liquidity to avoid bankruptcy.¹⁰ Banks play a sequential game in this economy— they play an ex-ante game in loan making and play an ex-post game in liquidity raising after liquidity shocks hit the banking sector. Thus, an ex-post solvency constraint under liquidity shocks is at the core of the safe asset creation: the safety of the ex-ante issued deposits is ensured only if the ex-post solvency constraint under liquidity shocks is guaranteed to hold.¹¹

⁶The demand for money in the economy is not micro founded in this paper. A rationale for the necessity of money can be found in [Kiyotaki and Moore \(2002, 2018\)](#). An alternative assumption that does same the job for our modeling purpose could be that consumers have idiosyncratic liquidity needs so that bank deposits must be made both redeemable and safe, as in [Diamond and Dybvig \(1983\)](#).

⁷See [Diamond \(1984\)](#), [Rajan \(1992\)](#), [Petersen and Rajan \(1994, 1995\)](#). The illiquidity problem here is essentially the same as the limited pledgeability problem that plagues the private creation of liquidity in [Holmström and Tirole \(1998\)](#).

⁸In the model, the illiquid feature of bank loans is captured by assuming that loans cannot be resold to their full value before maturity, but can only be borrowed against or liquidated to get a lower value.

⁹Bank capital structure being fragile is due to the fact that bank deposits needs to be liquid and redeemable on demand, while banks' assets are illiquid.

¹⁰This solvency constraint under liquidity shocks is essentially a panic-proof constraint. This constraint ensures each of the depositors to banks who get by "liquidity shocks" that running is not self-fulfilling.

¹¹The notion that only safe assets can perform the role as money in the economy is well recognized in the

The first main result of the paper is that the combination of this safe asset creation constraint under liquidity risks with certain markets being missing implies that the private money creation in a decentralized economy could be constrained inefficient. In particular, under a practically and theoretically reasonable environment of contracting and market space, I identify three market failures that can potentially plague the private money creation in a market economy. Two of them can cause inefficiencies by making the safe asset creation constraint inefficiently tight and the other can result in a complete collapse of private money creation.

Let us be more specific. Banks' safe asset creation relies crucially on a commonly shared liquidity pool, which is endogenously generated by each individual bank's ex-ante loan making and provides liquidity to banks hit by ex-post liquidity shocks. As such, tightness of banks' safe asset creation constraint depends both on the ex-ante creation of this liquidity pool, as well as on the ex-post usage of the liquidity pool. In a social planner's solution, both the ex-ante creation and the ex-post usage of the liquidity pool are optimized, which may not hold once the economy is decentralized.

The first potential inefficiency in private safe asset creation is due to (ex-ante) underproduction of the liquidity pool, when not all banks will be hit by liquidity shocks. Each bank's ex-ante loan making enlarges the size of the common liquidity pool in the system, which generates a positive externality by lowering other banks' ex-post liquidity raising cost when themselves are not hit by liquidity shocks.¹² In the constrained efficient benchmark, this positive externality can be fully internalized by the social planner. This is because the planner can guarantee that surplus liquidity is always traded at its marginal costs from banks surplus in liquidity to those in need of liquidity. This ability to frictionlessly reallocate surplus liquidity within the banking sector allows the social planner to fully account for the loosening effect on the safe asset creation constraint, of the liquidity generated by each individual bank's ex-ante loan making.

However, this is unlikely to be true in a decentralized economy. Although a positive externality is generated when the loan-making bank is not hit by liquidity shocks, the cost is an increase in the bank's own demand for liquidity when it gets hit. An incentive issue thus arises. Internalization of the positive externality requires sufficient incentives to be provided, which in turn depends on whether loan-making banks are properly compensated— in states where they do care about the compensation.¹³ As such, whenever

money and banking literature: see for instance [Gorton and Pennacchi \(1990\)](#)

¹²The nature of this positive externality is akin to that of the "collateral externality" discussed by [Davila and Korinek \(2016\)](#), in which individual decision maker's action can generate impact on certain aggregate state variables that affect the financial constraints faced by individual agents.

¹³As the creator of safe assets, banks care more about the price at which they can purchase liquidity when

banks are concerned that they cannot purchase surplus liquidity in the ex-post wholesale market at fair prices once in need of liquidity, the incentive provision for ex-ante creation of the liquidity pool would be inefficient. In such cases, a hold-up problem arises and the liquidity pool in the economy is under-produced, due to banks' concern of being held up in ex-post wholesale liquidity trading.

The second inefficiency that potentially arises in private money creation is due to (ex-post) over-use of the liquidity pool, when more than one bank will be hit by liquidity shocks. Behind this over-use problem is a negative externality one bank generates when dipping into the liquidity pool on others who are also in need of liquidity.¹⁴ In the constrained efficient benchmark, the planner can fully internalize this "distributive externality" with her ability to dictate each shocked banks' ex-post liquidity raising strategy—liquidating how much of their illiquid assets, as well as dipping how much into the liquidity pool. Importantly, the social planner can guarantee that the liquidity pool is always optimally used ex-post, contingent on any level of ex-ante production of the liquidity pool and any realization of liquidity shocks.

However, these conditions are likely to fail once the economy is decentralized. To achieve the time consistency in planner's constrained efficient solution—that is, to ensure the optimality of ex-post liquidity pool usage contingent on ex-ante liquidity pool production, certain futures markets are needed in a decentralized system. The futures contracts required to implement the constrained efficient benchmark should allow each bank to purchase pre-specified quantities of ex-post retail liquidity at pre-specified prices, contingent only on their ex-ante loan making choices while not affected by the ex-post behavior of other banks.¹⁵ Whenever this futures market is missing or is impaired, a commitment problem arises and banks hit by liquidity shocks end up being forced to over-use the liquidity pool in the ex-post equilibrium.¹⁶

As a brief summary of the analysis so far, two market failures can potentially lead to inefficiencies in private money creation by making the safe asset creation constraint inefficiently tight. The first is an *incentive* problem due to non-competitive pricing of ex-

they need it than about the price at which they can sell liquidity when it is in surplus.

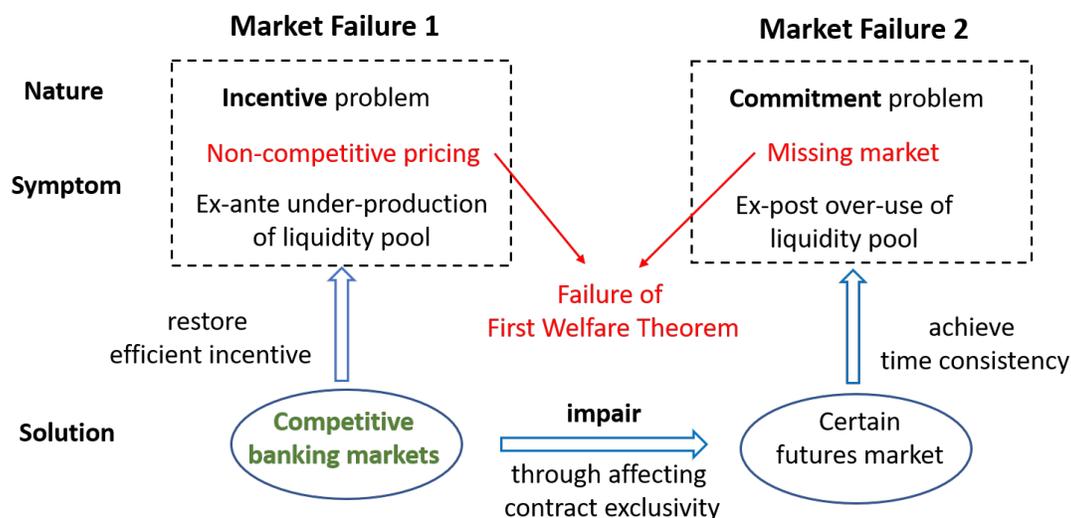
¹⁴The nature of this negative externality is akin to the "distributive externality" discussed by [Davila and Korinek \(2016\)](#), in which externalities are zero-sums across agents. Here one bank's usage of the ex-post liquidity pool results in less available retail liquidity at given prices for other banks that also need liquidity, when the size of liquidity pool is held fixed.

¹⁵Those banks who are not hit by liquidity shocks can then trade these ex-post retail liquidity to those in need of liquidity via the wholesale market.

¹⁶Existence of such a futures market is desirable both collectively and also individually. This is because as the money creator, banks care about the worst case scenario, as captured by the safe asset creation constraint under the worst realization of ex-post liquidity shocks. This makes individual banks' interest aligned with all banks' collective interests, which is guaranteeing the safety of all banks' deposits under liquidity risks.

post wholesale liquidity, which can result in ex-ante under-production of the liquidity pool. The second is a *commitment* problem due to certain futures market being missing or impaired, which can result in ex-post over-use of the liquidity pool. These two market failures are exactly the manifestations of the two scenarios in which the first welfare theorem could fail and Pareto inefficiencies could arise, as illustrated in Figure 1.

Fig. 1. Mixed effect of banking market competition on efficiency



Based on the above normative analysis, I derive the key positive results of this paper. I demonstrate an interesting role played by banking market competition in affecting the efficiency of private money creation. In particular, I show that increased competition in banking market has a mixed effect on efficiency– it alleviates the incentive problem and hence the ex-ante under-production of liquidity pool, but it also exacerbates the commitment problem and the ex-post over-use of liquidity pool.¹⁷

The intuitions are as follows. With a more competitive retail liquidity market, banks hit by ex-post liquidity shocks are able to purchase wholesale liquidity at a price closer to its marginal cost. This is because a competitive retail liquidity market effectively improves the threat point of liquidity-needy banks, in their bargaining with banks surplus in liquidity. Therefore, banks are less concerned about being extracted in their ex-post wholesale liquidity trading once hit by liquidity shocks and thus are more incentivized to account for the liquidity creation effect of their ex-ante loan making. As such, the hold-up problem is alleviated and efficiency is improved.

A more competitive banking market, however, would have an undesirable effect on

¹⁷The banking market competition here refers to that in bank funding markets– the retail deposit market in particular.

the futures markets that are necessary to get around the commitment problem in ex-post liquidity pool usage. With a highly competitive retail liquidity market, each individual bank's ability to write the future contracts on the ex-post retail liquidity usage and pricing would be impaired. This is because the quantity of retail liquidity a bank can obtain at certain price is likely to be affected by other banks' ex-post behavior, when a banking market is competitive. In other words, a competitive banking market impairs the contract exclusivity. This is problematic for the functioning of futures market because it means the liquidity pool usage specified in the futures contracts cannot be committed—because other banks' ex-post behavior could easily invalidate these ex-ante written contracts under a competitive market environment.¹⁸

In addition to the above two market failures that will lead to inefficient outcomes in private money creation, there is a third market failure that can potentially result in a complete collapse of the whole private system in its safe asset creation. When banking markets become sufficiently competitive, an ex-ante *coordination* issue may arise in banks' safe asset creation. In an environment of competitive banking markets, the perceived cost of an ex-ante deviation to make more loans is relatively low for each individual bank. This is because after making such a deviation, the competitive ex-post liquidity market allows the bank to cover the increased demand for external liquidity without having to post a much higher rate.¹⁹ As such, when the banking market is too competitive, the safe asset creation equilibrium in which all banks can just stay solvent under ex-post liquidity shocks cannot be coordinated ex-ante. In such cases, no private safe assets can be created in the economy.²⁰

The results of the analysis in this paper have some powerful policy implications. Our main results, that banking market competition has a mixed effect on the efficiency in private money creation, have a clear policy implication on the optimal level of banking market competition in an economy. If an economy is subject to idiosyncratic risks (e.g. has a diversified industry structure and hence diversified risk factor loading), then a competitive banking market helps. This is because for such an economy, alleviating the incentive problem would be more economically important than curing the commitment problem, since the former is relevant when the economy is subject to idiosyncratic shocks whereas the latter is relevant for systemic shocks. In such cases, competitive banking markets play

¹⁸The phenomenon described here shares some similar spirits to that illustrated in [Petersen and Rajan \(1995\)](#).

¹⁹This is more likely to be true when the number of banks is large so that the impact of each individual bank's ex-ante deviation on ex-post liquidity price is not large enough for self-disciplining.

²⁰This system-wise collapse in safe asset creation is reminiscent of results in [Jacklin \(1987\)](#) and [Farhi et al. \(2009\)](#), which demonstrates that the existence certain ex-post markets can impair the ex-ante liquidity creation.

the role as an incentivizing tool and can achieve better risk-sharing when idiosyncratic shocks hit.

On the contrary, if an economy is exposed to systemic risks (e.g. has a concentrated industry structure and hence concentrated risk factor loading), then competitive banking markets can be harmful. In this case, competitive banking markets can actually trap the economy in a high interest rate equilibrium once systemic shocks hit the economy, in which all shocked banks aggressively dip into the liquidity pool until the liquidity price gets bid up to an inefficiently high level. These predictions on how banking market competition would affect economy's resilience to unexpected shocks are supported by the empirical findings documented in [Xu \(2019\)](#). In that paper, through exploiting the 1986 oil price shock and the staggered intra-state banking deregulation in US during 1980s, I document the following facts: 1) for lightly hit areas, deregulation attenuates the shock; while 2) for heavily hit areas, deregulation amplifies the shock.

Finally, my analysis suggests that regulations on banks' liquid asset holding become essential for private safe asset creation when banking market becomes sufficiently competitive. In such cases, a properly designed ex-ante reserve requirement serves as a *coordinating* tool that commits all banks to choosing the (socially) desirable level of ex-ante loan making, which is the point that everyone can just stay solvent under ex-post liquidity shocks. The synchronized enforcement of nationally uniform reserve requirements with the banking deregulation in the "*Depository Institutions Deregulation And Monetary Control Act of 1980*" provides a real world example that is consistent with this implication.

Related literature This paper studies the creation by banks of private safe assets in the economy, with a threat of run being maintained on bank deposits. To the best of my knowledge, incorporating this fragility-safety conflict into private banks' money creation process is novel in the banking literature.

In my model, two components are crucial. First, the deposits that banks issue ex-ante must be guaranteed to be immune to panic under ex-post liquidity shocks. This safety requirement for bank deposits acknowledges the liquidity/ "money" creator role that banks play in the economy (e.g., [Diamond and Dybvig \(1983\)](#); [Gorton and Pennacchi \(1990\)](#); [Stein \(2012\)](#); [Hart and Zingales \(2014\)](#); [DeAngelo and Stulz \(2015\)](#); [Diamond \(2017\)](#); [Donaldson et al. \(2018\)](#), etc.), and the intrinsic illiquidity of banking operations (e.g., [Diamond and Dybvig \(1983\)](#); [Diamond and Rajan \(2000, 2001, 2005\)](#), etc.).

Second, in the spirit of the [Tobin \(1963\)](#) critique, the model features an "endogenous money creation" notion in which banks generate a fresh supply of deposits by extending loans. In my model, this deposits creation effect is reflected in the enlarged ex-post

endowments of households that receive labor income generated by the loan-making activities of banks. The enlarged ex-post liquidity pool lowers the ex-post retail liquidity price and helps to back up the safety of deposits issued ex-ante.

The sources of risks that present difficulties in banks' safe asset creation that we identify in this paper differ from those presented in the previous literature on bank safe asset creation (e.g. [Gorton and Pennacchi \(1990\)](#); [Stein \(2012\)](#); [DeAngelo and Stulz \(2015\)](#); [Diamond \(2017\)](#), etc.) While in these papers the safety of bank deposits is impaired by the risks inherent in the payoff of projects financed by banks, in my paper the liquidity risks induced by runnable deposits cause the problem. Furthermore, different from [DeAngelo and Stulz \(2015\)](#) and [Diamond \(2017\)](#) in which capital structure management is considered as the solution for safe assets manufacturing, an endogenously created ex-post liquidity pool is the key in our model.

My results— that private safe asset creation in a decentralized economy is constrained inefficient— is related to a large literature, initiated by [Geanakoplos and Polemarchakis \(1985\)](#) and [Greenwald and Stiglitz \(1986\)](#), that studies pecuniary externalities in incomplete markets. More specifically, my model features two pecuniary externalities of distinct natures, as distinguished in [Davila and Korinek \(2016\)](#).

First is a positive externality associated with ex-ante creation of the common liquidity pool in the economy. The economics of this externality are such that an individual bank's ex-post panic-proof constraint is affected by an aggregate state variable: the size of the liquidity pool in the economy. Although associated with the opposite sign, this externality has a nature similar to that of the firesale externalities that usually work through the impact from the market value of collaterals on individual agents' collateral constraints (e.g. [Kiyotaki and Moore \(1997\)](#); [Gromb and Vayanos \(2002\)](#); [Jeanne and Korinek \(2010\)](#); [Bianchi and Mendoza \(2011\)](#); [Stein \(2012\)](#); [He and Krishnamurthy \(2012\)](#); [Hart and Zingales \(2015\)](#), etc.). As a mirror image, while firesale externalities usually imply over-liquidation, in our model under-production of the liquidity pool is induced when this positive externality cannot be fully internalized due to banks' concerns of being held up.

Second is a negative externality associated with ex-post usage of the common liquidity pool in the economy. As distinguished by [Davila and Korinek \(2016\)](#), this externality is a "distributive externality" because it is zero-sum across banks when the size of the liquidity pool is fixed.²¹ In our model, inefficiencies associated with this externality arise because the extra incentive compatibility constraint on liquidity pool usage introduces wedges to the equilibrium marginal rates of substitution in banks' ex-post liquidity rais-

²¹One bank using more of the liquidity pool results in the liquidity supply curve for another bank being shifted upward.

ing, which distort them away from the optimal levels implied by the planner’s solution. As such, the problem induced by this externality is reminiscent of those discussed by [Allen and Gale \(1994, 2000, 2004, 2005\)](#); [Lorenzoni \(2008\)](#); [Farhi et al. \(2009\)](#); [Gale and Yorulmazer \(2011\)](#); [He and Kondor \(2016\)](#), etc., who note that in the decentralized economy, wedges in MRS drive inefficient outcomes.

In a key novel finding, I show that banking competitiveness plays a crucial role in determining the efficiency of private safe asset creation, by simultaneously affecting both externalities. In particular, a mixed effect is shown: a more competitive banking market alleviates the first inefficiency but exacerbates the second one.

Finally, I show that when the banking market is sufficiently competitive, regulation of banks’ liquid asset holding becomes essential. The economics of this result recall those proposed by [Jacklin \(1987\)](#) and [Farhi et al. \(2009\)](#), who show that restrictions on private trading among consumers are crucial for intermediaries’ liquidity creation.

The remainder of this paper is organized as follows. Section 2 sketches the basic setup of the model, wherein private safe assets are created through a constrained “money multiplier”. Section 3 characterizes the equilibrium of bank safe asset creation. In Section 4, I analyze the equilibrium first by characterizing the planner’s solution as a constrained efficient benchmark. Then I illustrate the problems that potentially arise when the economy is decentralized. Section 5 applies the model to study policy making by a regulator who faces the same problems found in a decentralized economy. Conclusions follow.

2. A Model of Private Safe Asset Creation

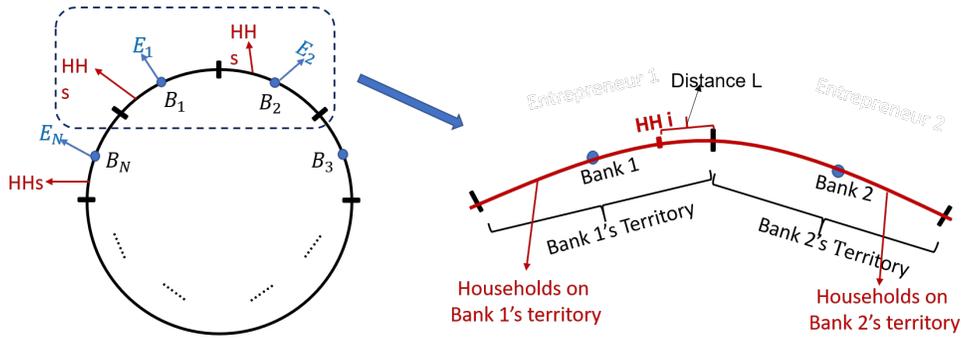
This section displays the model setup. I describe the model’s assumptions about the economy’s environment, endowments and technologies, as well as key frictions in the economy. In particular, a constrained “money multiplier” will be highlighted in banks’ private safe asset creation process.

A. The Economy

The economy considered in the model is located on a circle similar to those in [Salop \(1979\)](#) and [Zentefis \(2019\)](#); and it consists of three groups of agents: households, entrepreneurs, and banks, as displayed in Figure 2.²²

²²Unlike [Salop \(1979\)](#) and [Zentefis \(2019\)](#) in which each bank is a single point on the circle, in our model each bank possesses an entire band around its headquarter as its natural territory. The competition in banking market essentially captures how integrated the retail deposit market is, similar to how financial integration is modeled in [Gârleanu et al. \(2015\)](#)

Fig. 2. Structure of the Economy



The banking sector is comprised of N identical banks, whose headquarters are evenly scattered over the circle.²³ Each bank possesses a natural territory centered around its headquarter, within which it may enjoy certain monopoly power in both the lending and funding markets. The territories of two adjacent banks exactly touch each other without overlapping.

Infinitesimal and ex-ante identical households are uniformly scattered over the entire circle. Each individual household HH consists of two members: A (consumer) and B (worker). To simplify the algebra, I normalize the total mass of households located on each bank's territory to be unity.

For the corporate sector, there is one single entrepreneur located in each bank's territory and all N entrepreneurs are identical except for their various locations.²⁴

To define bank territory, I assume that entrepreneurs can only borrow from their local banks.²⁵ In the model, competition in the deposit market is crucial, and it is captured by a parameter κ . A household would incur a traveling cost $\kappa \cdot L$ if it deposits with a bank from which the distance is L . The distance L is zero if the household resides within the bank's territory; if not, L is the minimum distance to the boundaries of the bank's territory.

As such, banking market competition in our model is captured by parameter $\tau \equiv \frac{1}{\kappa}$ with larger τ suggesting higher level of competition among banks. Two extreme cases will be frequently referred to in the following analysis: 1) $\tau = \infty$, in which case the banking market is perfectly competitive; 2) $\tau = 0$, in which case the banking market is completely segmented or non-competitive.

B. Endowments and Technologies

²³The number N of banks in the economy is treated in this paper as exogenously given.

²⁴This single entrepreneur can be thought of as aggregating many infinitesimal entrepreneurs who reside in the same bank's territory.

²⁵This simplification of credit market competition can be rationalized by assuming that financing an entrepreneur's project requires frequent and intensive monitoring.

As in standard maturity mismatch models, there are three dates in the model: $t = 0, 1, 2$. In this subsection, I specify the model's assumptions regarding the endowments and technologies for each agents at relevant dates.

i) Households

At date 0, each household is endowed with one unit of (real) goods. I assume that these initial endowments are all owned by member A of each household, who has to rely on (local) banks for value storage.²⁶

Between date 0 and date 1, member B of households located in each bank i 's territory receives endowment $e_i = \bar{e} + n_i$. Here \bar{e} represents the exogenous component of the endowment, while the endogenous part n_i will be determined by the production scale of local entrepreneurs.

On date 1, each member B is equipped with a storage technology $h(y)$ that allows him to transfer value from date 1 to date 2, where function $h(y)$ satisfies: $h'(y) > 0$, $h''(y) < 0$.

ii) Entrepreneurs

Investment opportunities only arrive on date 0, at which point entrepreneurs are endowed with a productive project that requires inputs to be invested on date 0 and pays off with a constant return $R > 1$ on date 2. Entrepreneurs do not have internal funds and must rely on external financing from banks. Under the assumption that each entrepreneur can only be financed by the local bank, banks have local monopoly power in the lending market and entrepreneurs elastically produce with whatever is lent to them. Projects are riskless if kept to date 2.

As a crucial component of the model, a constant labor cost $w \in (0, 1)$ is required per unit of production and is paid before date 1. To make the analysis simpler, I assume that these labor payments are evenly distributed to the member B of local households who reside in the same bank territory. As such, if the production scale in bank i 's territory is α_i , then the labor income that each household in its territory receives before date 1 is $n_i = w \cdot \alpha_i$.

This labor payment made before date 1 is intended to capture the fresh supply of deposits generated in the whole chain of transactions triggered by entrepreneurs' initial outlay of their received lending. Importantly, these labor payments distributed to local households contribute to the creation of the ex-post liquidity pool in the economy.

iii) Banks

To focus on the banking sector's creation of money-like safe assets in the economy, I

²⁶Competition for ex-ante deposits is abbreviated in the model. The key in our analysis is banks' competition for ex-post liquidity.

assume that banks are all debt financed.²⁷ At date 0, each bank raises 1 unit of liability from member A of households that reside in its territory, in the form of deposits demandable on both dates 1 and 2. In the model, these date-0 deposits issued by banks are the private safe assets that can function as money in the economy. To simplify the algebra, the deposit rates required on these date-0 deposits are assumed to be one for both dates 1 and 2.²⁸

After receiving 1 unit of deposits, each bank i makes its asset portfolio choice $(\alpha_i, 1 - \alpha_i)$, in which it invests α_i to finance local entrepreneur's project while it invests $1 - \alpha_i$ in a liquid but low return asset (cash). The illiquidity of loans made to entrepreneurs, from which banks can get a return R if kept to date 2, is reflected in their relatively low proceeds if liquidated at date 1.²⁹

While the liquid cash holding has a return of 1 on both date 1 and date 2, a liquidation technology $g(t, \alpha)$ governs the proceeds that banks can get when liquidating part of their illiquid loans on date 1. Specifically, if a bank made α illiquid loan on date 0, then by liquidating $t \in [0, \alpha]$ of these illiquid assets on date 1, a proceeds $g(t, \alpha)$ can be generated. To reflect the illiquidity, I assume that $g(t, \alpha) < t$, for $t \in [0, \alpha]$.³⁰ Furthermore, the following regulatory conditions are assumed on function $g(t, \alpha)$: i) $\frac{\partial g(t, \alpha)}{\partial t} > 0$; ii) $\frac{\partial^2 g(t, \alpha)}{\partial t^2} < 0$; iii) $\frac{\partial g(t, \alpha)}{\partial \alpha} < 0$. While condition i) is trivial, condition ii) can be explained as a "pecking order" in liquidation and condition iii) can be micro-founded by relating the asset sale price (liquidation proceeds) to seller's bargaining position.

C. Sources of Friction

Two sources of friction in this economy drive our analysis. The first is related to a lack of double coincidence in consumer demand and limited pledgeability on consumers' future return from their specific human capital, as in [Kiyotaki and Wright \(1989\)](#) and [Hart and Zingales \(2014\)](#). Although not explicitly micro-founded in this paper, my model implicitly assumes (because of the issues described above) that there is a demand for safe

²⁷An implicit assumption made here is that equity issued by banks cannot function as money in the economy. However, while not explored in this paper, it is interesting to consider how bank equity can help make claims on bank debt more money-like. I leave this question for future research.

²⁸This simplifying assumption on date-0 deposit rates can be rationalized by the convenience yield on these money-like deposits, or by households' preference of holding safe assets (e.g. [Stein \(2012\)](#), [Diamond \(2017\)](#)).

²⁹Banks can get the full return from entrepreneurs' projects because credit market competition is shut down in the model—banks enjoy full monopoly power in local lending market.

³⁰The liquidation proceeds $g(t, \alpha)$ can be interpreted as the net revenue from liquidating t of the long-term assets, if a fixed cost in liquidation is required.

asset to facilitate transactions.³¹

The second source of friction stems from entrepreneurs' special human capital in generating a payoff from their projects, which is costly to acquire. While this special human capital is needed to generate value, it also makes entrepreneurs' projects illiquid when they need an external funding source to finance their operation. As in [Diamond and Rajan \(2001\)](#), this illiquidity is reflected as the entrepreneurs' inability to borrow from lenders who lack the skill to collect loan payments.

Banks arise to ameliorate these two fundamental frictions in the economy. Through deposit-taking activity in their liability-side operations, banks create a payment medium that facilitates otherwise restrained transactions. In their asset-side operations, by forming relationships (e.g., [Rajan \(1992\)](#); [Petersen and Rajan \(1994, 1995\)](#)) and behaving as delegated monitors (e.g., [Diamond \(1984\)](#)), banks enlarge the borrowing capacity of otherwise illiquid borrowers.

D. A Constrained “Money Multiplier”

In this economy, banks create money-like safe assets through the joint activities of deposit taking and loan making on either side of their balance sheets. As shown in [Figure 3](#), a “money multiplier” drives this safe asset creation, at the core of which is an endogenously generated liquidity pool in the economy. However, a fragility-safety conflict implied by banks' illiquid capital structure imposes a natural bound on the size of this multiplier in private money creation. We now elaborate this constrained “money multiplier”.

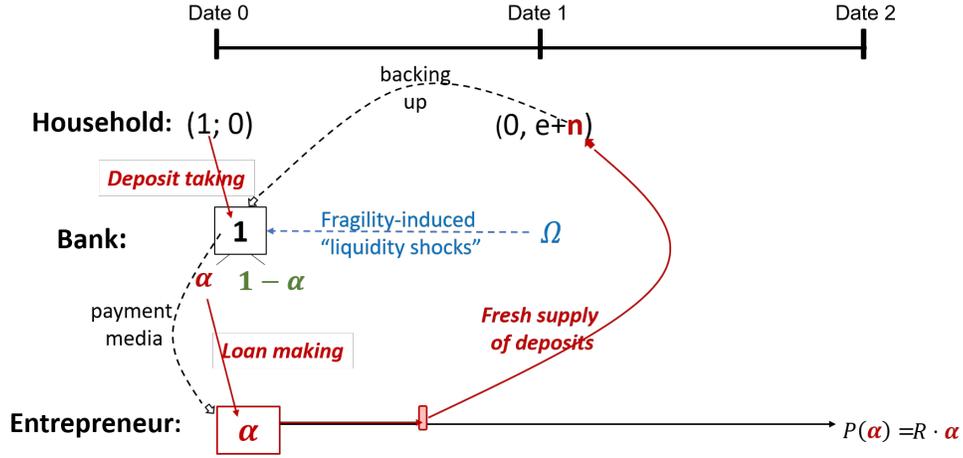
i) Deposits Create Loans

At date 0, through taking deposits on household member *A*'s endowments, banks create private safe assets in this economy that can be used as a payment medium in transactions between agents who otherwise have a limited ability to commit to paying and delivering. In the model, this demand of safe assets for value storing and credible wealth transferring (as in [Kiyotaki and Wright \(1989\)](#); [Caballero \(2006\)](#); [Hart and Zingales \(2014\)](#), etc.) is reflected in the requirement that these date-0 deposits must be made money-like. More specifically, only if deposits issued by banks on date 0 are guaranteed to be *liquid* and *safe* will investment opportunities with return R be generated in the economy.³²

³¹Alternatively, such a demand for liquid and safe deposits in the economy can be rationalized if risk-averse consumers are subject to idiosyncratic liquidity shocks, as in [Diamond and Dybvig \(1983\)](#).

³²This assumption on the economy's demand for perfectly safe assets does not affect the generality of the analysis that follows—the requirement for perfect safety is not essential for the results and is just a simplifying mathematical treatment.

Fig. 3. The constrained “money multiplier”



Related to the transactional demand that these date-0 deposits can fulfill is the low interest rates required on them. As in [Stein \(2012\)](#), [DeAngelo and Stulz \(2015\)](#) and [Diamond \(2017\)](#), a convenience yield is assumed on these date-0 deposits, as long as they can be made money-like.

Thanks to the creation of these payment media, profitable real investment opportunities arise in the economy. Banks thus engage in their asset-side operations, in which they allocate part of the received deposits to finance local entrepreneurs’ production. In this sense, lending opportunities (and hence loans) are endogenously generated by the ex-ante creation of these money-like date-0 deposits.

ii) Loans Create Deposits

The second component of this “money multiplier” is based on the “endogenous deposit creation” notion pointed out in the [Tobin \(1963\)](#) critique of banks as money creator. The essence of this “endogenous deposit creation” is that when a banker signs a loan approval, a chain of subsequent transactions is triggered by the borrowing entrepreneurs’ initial outlay. Through this process, a fresh supply of deposits is created for both the bank that extends the original loan and for the entire banking system.

In our model, this deposit creation effect of bank’s loan-making activities is captured by assuming that for each unit of loans that bank i ($1 \leq i \leq N$) makes, w units of new labor income will be earned by households that reside in bank i ’s territory. More specifically, the date-1 endowment e_i in the hands of member Bs of households that reside in bank i ’s territory will be $e_i = \bar{e} + w \cdot \alpha_i$, if bank i chooses an asset portfolio $(\alpha_i, 1 - \alpha_i)$ on date 0. The endogenous component $n_i = w \cdot \alpha_i$ in these date-1 endowments in the economy is generated by the loan-making activities of banks.

By enlarging the size of the (ex-post) liquidity pool in the economy, these loan-generated

endowments can help back up the money-like feature of the deposits that banks issue on date 0. In our model, an enlarged ex-post pool of liquidity can be beneficial because it lowers the marginal cost of liquidity raising for banks, which is governed by member B 's concave storage technology $h(y)$.

iii) A Fragility-Safety Conflict

To function as money in the economy, bank deposits must be made both *liquid* and *safe*. Being liquid requires deposits to be made runnable so that 1) depositors do not need to develop any specific payment collection skill to get paid (i.e., a threat of run needs to be maintained, as in [Diamond and Rajan \(2001\)](#)); 2) or alternatively, depositors can be insured against idiosyncratic liquidity risks (i.e., deposits need to be redeemable, as in [Diamond and Dybvig \(1983\)](#)).

Although it ensures that deposits are liquid, making deposits runnable could impair their safety. As pointed out in [Diamond and Dybvig \(1983\)](#), runnable deposits expose banks to a risk of coordination failure among their depositors, unless running can be guaranteed to be not self-fulfilling. Thus, a fragility-safety conflict arises in the banking sector's creation of deposits, which must be guaranteed to be both liquid and safe for reasons discussed before.

In the model, this fragility-safe conflict in banks' safe asset creation is modeled by subjecting banks to ex-post liquidity shocks on date 1 (see elaboration in Section 3). The deposits that banks issue at date 0 can be made money-like only if their safety can be ensured under the exposure to ex-post liquidity shocks on date 1. The presence of such a conflict effectively sets a natural bound on the "money multiplier" during the safe asset creation process because it constrains the illiquid lending α that each bank can make on date 0.

3. Equilibrium Characterization

In this section, I characterize the equilibrium in which private safe assets are created in this economy. In particular, I define the condition for safe asset creation and characterize the game played by banks during this process.

A. Ex-post Liquidity Shocks and Panic-Proof Constraint

The fragility-safety conflict in private safe asset creation is modeled as exposing the banking sector to ex-post liquidity shocks on date 1. More specifically, I assume that on date 1, all depositors of k ($1 \leq k \leq N$) randomly-selected banks see a sunspot while all

others do not. The scale of these ex-post liquidity shocks is parameterized by integer k , with larger k indicating that the liquidity shocks are more systemic. In the following analysis, I refer to shocks in which depositors of k banks seeing a sunspot as *level- k liquidity shocks*.

For notational convenience, let us denote the set of all level- k liquidity shocks as $\Omega_k = \{(i_1, \dots, i_k)\}$, where indices $1 \leq i_1 < \dots < i_k \leq N$. Also, denote each realization of an ex-post liquidity shock as $\omega \in \Omega_k$, where ω is a set that consists of the k banks that get hit by the shock. On date 0, all agents in the economy know that a level- k liquidity shock $\omega \in \Omega_k$ will be hitting the system on date 1, but they do not know the exact realization of ω .³³

On date 1, given a realization of ω , depositors at each bank $i \in \omega$ see a sunspot and decide whether or not to run. In the model, I assume that the date-0 deposits can be made money-like only if the coordination failure risk among the depositors at these banks in ω can be eliminated. The following assumption formally specifies the condition for safe asset creation.

Assumption 1 *When a date-1 liquidity shock ω hits, the deposits each bank $i \in \omega$ issued on date 0 will be guaranteed to be safe if and only running is not self-fulfilling for depositors at banks in ω .*

That is, on date 1, each bank $i \in \omega$ needs to assure each of its date-0 depositors that even if all other depositors (including those at other banks that get hit) immediately withdraw, choosing to stay is still a weakly dominating strategy. In this sense, this panic-proof constraint could be interpreted as a solvency constraint under *real* liquidity shocks, which force depositors (who get hit) to actually withdraw immediately on date 1.³⁴

Alternative Interpretation: Liquidity Stress Test with k Banks being Tested

While it seems somewhat obscure to imagine that depositors of k banks will miraculously see a sunspot at the same moment, this specification of ex-post liquidity shocks becomes more natural under an alternative interpretation. Stress tests are common practices that banking regulators adopt in designing policy tools that aim to enhance the stability of banking system and the whole economy. Prior to the 2008 financial crises, most of these stress tests run by policy makers are focusing attentions on bank *capital*. After the crises, stress tests on bank *liquidity* have gained more attentions and started to be considered in

³³That is, at date 0, everyone in the economy knows that k banks will be hit by the liquidity shock on date 1, but they do not know which k banks will get hit.

³⁴Rigorously speaking, the “sunspot shock” ω is not exactly a liquidity shock, because banks in ω do not actually suffer immediate withdrawal from their depositors. But for our modeling purpose, it works exactly the same as a real liquidity shock. So in our following analysis, we refer to ω simply as liquidity shocks, and being hit by one means that all date-0 deposits will withdraw on date 1.

central bankers' policy making.

These stress tests on bank liquidity normally work as follows: some banks are randomly picked and are asked to raise certain amount of short-term funding within a short duration of time. Such a liquidity stressful scenario for banks could be mapped onto real world events such as a sudden outflow (or withdrawal) of deposits, an increase in the short-term overdue ratio due to delayed loan payments, or a large drawdown in the credit line commitment, etc. Banks chosen to do the liquidity stress test can borrow against their long term assets to raise short-term liquidity, in order to pass the test.

In the concrete setting of this model, a liquidity stress test is run on date 1, in which k banks are randomly picked and each of these k banks needs to raise 1 unit of liquidity to pass the test. It's easy to see that for each individual bank under test, the condition of passing the liquidity stress test is mathematically equivalent to the run-proof condition elaborated above.

Under this alternative interpretation, the variable k – the number of banks that will be hit by liquidity shocks on date 1 has a very natural economic meaning. If an economy has a relatively concentrated industry structure and risk factor loading, then the relevant liquidity stress test for the policy maker in this economy should be associated with a larger value of k . On the contrary, for an economy with relatively diversified industry structure and risk factor loading, the relevant liquidity stress test to be run for such an economy should be associated with smaller value of k .

In the next subsection we ask: How can banks make themselves safe (immune to panic) under liquidity shocks at date 1, thus rendering their date-0 deposits money-like?

B. Coverage of Ex-post Liquidity Needs for Bank $i \in \omega$

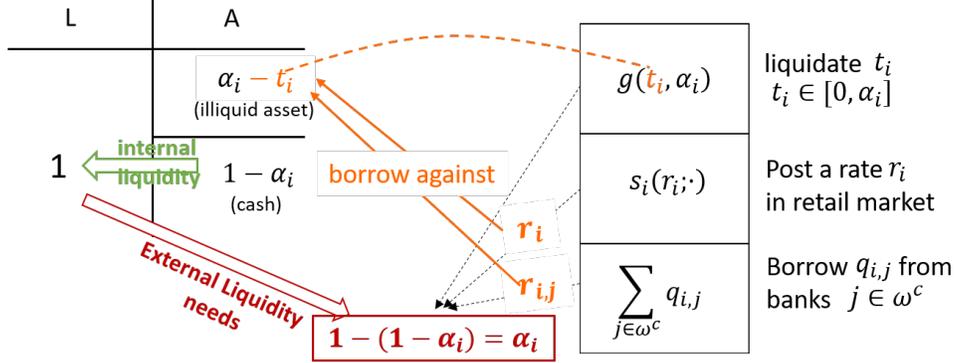
On date 1, after a liquidity shock ω hits, each bank $i \in \omega$ needs to be able to cover potential liquidity demand so that panic among its date-0 depositors can be deterred. In this subsection, I discuss the ex-post liquidity coverage for each bank gets hit by liquidity shocks on date 1.

i) Ex-post Solvency of Bank $i \in \omega$

As discussed above, the ex-post panic-proof constraint can be constructed as the solvency constraint on date 1 under *real* liquidity shocks that depositors of all banks get hit require immediate payment. Figure 4 illustrates the ex-post liquidity coverage for a bank $i \in \omega$ that chooses α_i in ex-ante loan making.

With α_i being invested in illiquid loans on date 0, the remaining $1 - \alpha_i$ liquid cash holding can be used as internal liquidity to cover bank i 's liquidity needs on date 1. To

Fig. 4. Ex-post liquidity coverage for bank $i \in \omega$



ensure that the liquidity shortage can be made up, the demand for liquidity to be raised externally on date 1 is $1 - (1 - \alpha_i) = \alpha_i$.

In the model, each bank $i \in \omega$ can employ three exhaustive methods to raise liquidity apart from its cash holding. First, it can deploy its liquidation technology $g(t, \alpha)$ and liquidate a certain amount $t_i \in [0, \alpha_i]$ of its long term assets (illiquid loans). In this manner, bank i can immediately raise ex-post liquidity $g(t_i, \alpha_i)$ on date 1, while the remaining long term assets kept on date 2 become $\alpha_i - t_i$.

Second, a bank $i \in \omega$ can post a rate on the retail market to raise liquidity from household member B 's date-1 endowments. I assume that banks cannot price discriminate in the retail market and thus that they can only post a single rate in date-1 liquidity raising.³⁵ The proceeds from posting a rate r_i are governed by a liquidity supply function $s_i(r_i, \cdot)$ for bank i , in which “ \cdot ” indicates that this supply function is parameterized by certain variables. The determination of this ex-post liquidity supply function $s_i(r_i, \cdot)$ will be elaborated in part ii) of this subsection.

Third, a bank $i \in \omega$ can raise liquidity by borrowing in the wholesale market from banks in surplus of liquidity. After a liquidity shock ω hits on date 1, each bank $i \in \omega$ that needs liquidity would borrow from each bank $j \in \omega^c$ that has a surplus of liquidity. In the model, these pairwise ex-post wholesale liquidity trades are determined as bilateral bargaining. The bargaining outcome specifies the quantity $q_{i,j}$ of date-1 liquidity to be delivered to the borrower as well as the price $r_{i,j}$ at which date-2 repayment is made. The determination of the bargaining outcome $(q_{i,j}, r_{i,j})$ between bank $i \in \omega$ and bank $j \in \omega^c$ will be elaborated in part iii) of this subsection.

On date 1, each bank $i \in \omega$ optimally determines its liquidity raising strategy (t_i, r_i) while the wholesale liquidity trading $\{(q_{i,j}, r_{i,j})\}_{j \in \omega^c}$ is determined as bargaining out-

³⁵No price discrimination in the retail liquidity market can be implied by assuming that banks cannot tell the identity of each household.

comes. Coverage of liquidity shortage on date 1 requires

$$g(t_i, \alpha_i) + s_i(r_i; \cdot) + \sum_{j \in \omega^c} q_{i,j} = \alpha_i$$

That is, the total proceeds of date-1 liquidity that come from adopting strategy (t_i, r_i) coupled with interbank trading $\{(q_{i,j}, r_{i,j})\}_{j \in \omega^c}$, should be just enough to cover bank i 's demand for external liquidity.

However, the ex-post liquidity raising strategy (t_i, r_i) is bounded by a feasibility constraint that is determined by bank i 's solvency on date 2. That is, the date-2 payoff from bank i 's unliquidated long-term assets must be sufficient to make the promised payments to the date-1 creditors in both retail and wholesale liquidity markets. The ex-post solvency condition for bank i can then be formulated as

$$S_i(\cdot, \omega) = R(\alpha_i - t_i) - r_i \cdot s_i(r_i; \cdot) - \sum_{j \in \omega^c} r_{i,j} \cdot q_{i,j}$$

where “.” indicates that the ex-post solvency of bank i is a function of certain variables. Bank i can stay solvent under liquidity shock ω with an ex-post liquidity raising profile $\{(t_i, r_i), \{(q_{i,j}, r_{i,j})\}_{j \in \omega^c}\}$ only if

$$S_i(\cdot, \omega) \geq 0$$

ii) Pricing of Retail Liquidity: Households' Problem

Household member B 's endowments on date 1 serve as the ex-post liquidity pool in the economy, and banks may dip into the pool once hit by liquidity shocks. Under the assumption that banks cannot price discriminate against households, a retail liquidity supply function $s_i(r_i; \cdot)$ governs the proceeds each bank i can raise at date 1 by posting a rate r_i .

This ex-post liquidity supply function is obtained by solving individual household's problem, in which each HH member B optimally determines the amount to deposit at the rate posted by banks on date 1 and the amount to deploy his storage technology $h(y)$. The following lemma describes the liquidity supply from each individual household.

Lemma 1 *On date 1, for a household member B with endowment $e + w \cdot \alpha$, its (individual) inverse supply of deposits is $r(d; \alpha) \equiv h'(e + w \cdot \alpha - d)$. Accordingly, its (individual) deposit supply function is $d(r; \alpha) = r^{-1}(d; \alpha)$.*

Depending on the competitiveness τ of the banking market, the ex-post liquidity sup-

ply function $s_i(r_i; \cdot)$ for each bank i can be formulated based on the individual liquidity supply. Two extreme cases are useful for illustration and will be frequently referred to in our following analysis.

When banking market is completely segmented (i.e., $\tau = 0$), each bank can only raise deposits from its own territory. In this case, for a bank i that chooses α_i on date 0, the ex-post liquidity supply curve it faces on date 1 is

$$s_i(r_i; \alpha_i) = d(r_i; \alpha_i)$$

which is parameterized by its own date-0 lending α_i .

In contrast, when the banking market is perfectly competitive (i.e., $\tau = \infty$), banks can freely compete for households that reside in the territories of other banks. In this case, if all other banks $n \neq i$ are posting a rate \bar{r} , the ex-post liquidity supply curve for an individual bank i on date 1 is

$$s_i(r_i; \bar{r}) = \begin{cases} 0 & r_i < \bar{r} \\ \text{anything} & r_i \geq \bar{r} \end{cases}$$

which is parameterized by other banks' choice \bar{r} .

For general cases, both banks' ex-ante choice of α and others banks' ex-post action \bar{r} will impact each individual bank's liquidity supply curve. The following lemma describes the ex-post liquidity supply for general cases.

Lemma 2 *For general cases in which $0 < \tau < \infty$, if all banks chose α on date 0 and all other banks $n \neq i$ are posting a rate \bar{r} on date 1, then the ex-post liquidity supply curve faced by bank i is*

$$s_i(r_i; \alpha, \bar{r}) = d(r_i; \alpha) \cdot [1 + 2\tau \cdot (V_H(r_i; \alpha) - V_H(\bar{r}; \alpha))]$$

where $V_H(r; \alpha) \equiv \max_d r \cdot d + h(e + s \cdot \alpha - d)$ is the optimized value for a HH with endowment $e + s \cdot \alpha$ and being offered a rate r .

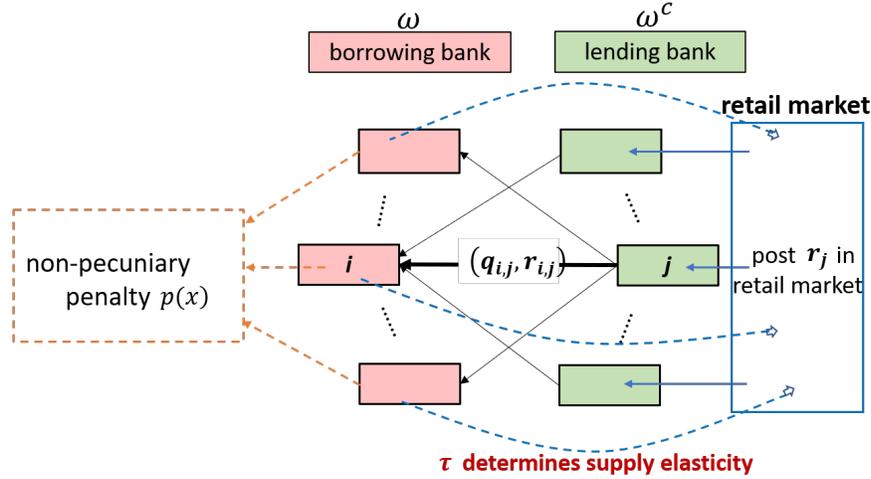
Parameter τ (i.e. $\frac{1}{\kappa}$) affects households' traveling cost, through which it determines the competitiveness of the banking market.³⁶ With higher values of τ indicating higher competitiveness, lemma 2 suggests that the ex-post liquidity supply is more elastic when the banking market is more competitive. Detailed analysis and algebra of retail liquidity supply are provided in Appendix [Section A1](#).

iii) Pricing of Wholesale Liquidity: Interbank Bargaining

³⁶A natural interpretation of the central banker's power to affect parameter τ is her ability to control the degree of competition in the retail markets by stipulating appropriate regulatory restrictions on the branching operations of banks.

In addition to raising liquidity by directly posting rates in the retail market, banks hit by liquidity shocks can also borrow in an ex-post wholesale liquidity market. Figure 5 illustrates ex-post interbank transactions in the wholesale liquidity market.

Fig. 5. Interbank bargaining



After the realization of a liquidity shock ω on date 1, each bank $i \in \omega$ establishes a trading deal $(q_{i,j}, r_{i,j})$ with each bank $j \in \omega^c$. The trading deal $(q_{i,j}, r_{i,j})$ specifies the quantity $q_{i,j}$ of liquidity to be delivered on date 1 by bank j to bank i and the rate $r_{i,j}$ at which repayment is made on date 2. To deliver the date-1 liquidity as specified in equilibrium trading deals, each bank $j \in \omega^c$ posts a rate r_j in the retail market and lends the proceeds to borrowing banks accordingly.³⁷

These interbank transactions on date 1 between liquidity-needy banks and liquidity-surplus banks are modeled as non-cooperative bilateral bargaining. The equilibrium trading deal $(q_{i,j}, r_{i,j})$ between bank $i \in \omega$ and bank $j \in \omega^c$ is determined as the Nash bargaining solution that maximizes the joint surplus from trading for the two banks. That is, given other equilibrium outcomes and the equilibrium strategies others, the ex-post trading deal between bank $i \in \omega$ and bank $j \in \omega^c$ is determined as

$$(q_{i,j}, r_{i,j}) = \arg \max_{(q,r)} [V_i(q,r) - V_i^R]^\beta \cdot [V_j(q,r) - V_j^R]^{1-\beta}$$

in which exogenous parameter β is the relative bargaining power, $V(q,r)$ is the valuation of an arbitrary deal (q,r) and V^R is the reservation value to each party.

³⁷To simplify analysis, I assume that banks in ω^c will only lend the proceeds from ex-post retail market, but not their ex-ante liquid cash holding. In the next section, I will argue that this simplifying assumption is innocuous for our efficiency analysis of private safe asset creation.

Reservation value V_j^R of the liquidity-surplus bank j is simply the extra profit bank j can make from date-1 trading, under the restriction that no deal with bank i is reached.³⁸ The reservation value V_i^R of the liquidity-needy bank i is determined by bank i 's alternative liquidity raising capacity and the penalty it will suffer once liquidity needs cannot be fully covered on date 1.

The elasticity of the retail liquidity supply curve that bank i faces plays a crucial role in determining its bargaining position in interbank trading: a more elastic retail liquidity supply allows bank i to raise more liquidity without having to offer a much higher rate once the deal breaks down. Finally, to pin down the reservation value of borrowing banks, we assume that a non-pecuniary penalty $p(x)$ will be imposed if a bank fails to meet the liquidity needs on date 1 with a margin $x > 0$, where function $p(\cdot)$ is exogenously given.³⁹ Detailed analysis and algebra for interbank bargaining in wholesale market are provided in Appendix [Section A2](#).

C. Equilibrium of Private Safe Asset Creation

Banks play a sequential game in the safe assets creation process, as shown in Figure 6. On date 0, banks play an *on equilibrium* game of money creation in which each bank i chooses its loan making α_i . On date 1, banks play an *off equilibrium* game of liquidity raising, in which a liquidity shock ω hits and banks in ω must meet the liquidity needs so that their ex-ante deposits can be made safe.⁴⁰ We now examine this game.

i) Subgame of Ex-post Liquidity Raising

We characterize this sequential game backward. Thus, we first describe the ex-post subgame played on date 1, given the ex-ante loan making profile $\{\alpha_n\}_{n=1}^N \in \mathcal{A}$ chosen on date 0.

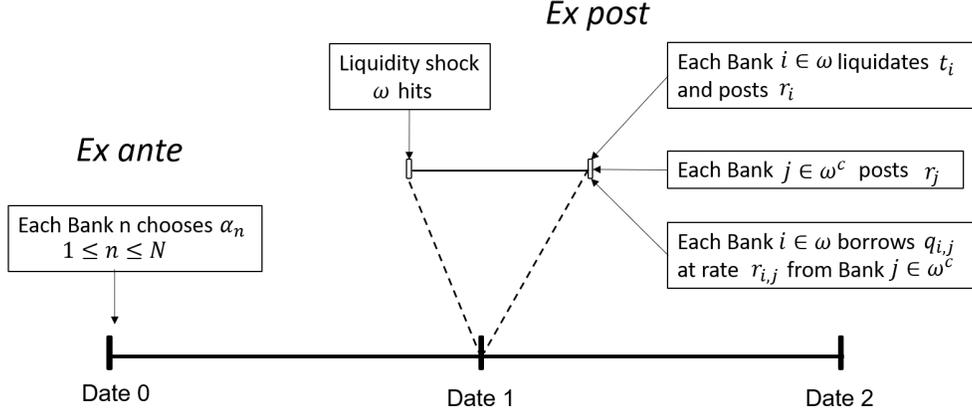
On date 1, after a liquidity shock $\omega \in \Omega_k$ hits, a subgame of liquidity raising is played in the economy. In this ex-post game, banks in ω have to raise sufficient date-1 liquidity to cover their liquidity needs, while banks in ω^c trade surplus liquidity to maximize their date-2 profits.

³⁸For instance, if bank i and j are the only banks in the economy, then $V_j^R = 0$.

³⁹This margin-contingent non-pecuniary penalty on a bank's failure to meet its liquidity needs can be interpreted as the reputation loss banks would incur in such events. It may also be viewed as a stigma associated with discount window facility usage, which a bank will have to resort to if exhausting all other liquidity sources is not sufficient to cover its liquidity needs. In this paper, I do not explicitly model the central bank's intervention policy. Similar treatments of central bank's intervention with an attached non-pecuniary penalty can be found in [Diamond and Rajan \(2012\)](#).

⁴⁰The game played on date 1 is *off equilibrium* because the liquidity shock ω is essentially a "sunspot shock" and running does not actually materialize on date 1, as long as banks can make their deposits panic-proof.

Fig. 6. Time line of gameplay



Upon a liquidity shock ω hits at date 1, an interbank trading deal $(q_{i,j}, r_{i,j})$ is established between each bank $i \in \omega$ and each bank $j \in \omega^c$. As discussed above, the equilibrium trading deals are determined as the non-cooperative bargaining solution that maximizes the joint surplus of both parties. Each party's reservation value and value gained from trading are perfectly known by the other. That is, for each pair $(i \in \omega, j \in \omega^c)$, the interbank deal $(q_{i,j}, r_{i,j})$ is determined as the Nash bargaining solution, given the equilibrium rates $\{r_n\}_{n \neq i,j}$ posted by others and all other interbank deals established in equilibrium.

Given the interbank trading deals established with banks in ω^c , each bank $i \in \omega$ optimally chooses its own liquidity raising strategy (t_i, r_i) to cover the remaining shortage of liquidity. That is, taking as given the rates $\{r_n\}_{-i}$ posted by others and the interbank deals $\{(q_{i,j}, r_{i,j})\}_{j \in \omega^c}$ established with banks in ω^c , each bank i chooses (t_i, r_i) to solve

$$\max_{t_i, r_i} R(\alpha_i - t_i) - r_i \cdot s_i(r_i; \cdot) - \sum_{j \in \omega^c} r_{i,j} \cdot q_{i,j}$$

subject to the liquidity coverage constraint

$$g(t_i, \alpha_i) + s_i(r_i; \cdot) + \sum_{j \in \omega^c} q_{i,j} = \alpha_i$$

That is, each bank $i \in \omega$ maximizes its solvency under the constraint that liquidity needs must be covered. If the optimized solvency can be made non-negative, then it means that bank i can stay solvent under liquidity shock ω .⁴¹

⁴¹The constrained optimization problem for each bank hit by liquidity shocks can be equivalently formulated as maximizing liquidity raising on date 1 under the constraint that the bank must stay solvent on date

The ex-post game playing of banks not hit by liquidity shocks is simpler. In equilibrium, each bank $j \in \omega^c$ posts rate r_j so as to deliver the quantity of date-1 liquidity specified in established interbank deals:

$$s_j(r_j; \cdot) = \sum_{i \in \omega} q_{i,j}$$

taking as given the equilibrium rates $\{r_n\}_{-j}$ posted by others.

For notational convenience, we define the following mappings Φ_N , Φ_S and Φ_I that map ex-ante loan making profile $\{\alpha_n\} \in \mathcal{A}$ and realization of ex-post liquidity shock $\omega \in \Omega_k$ into equilibrium outcomes of the ex-post subgame in liquidity raising. Specifically, for each pair of $\{\alpha_n\} \in \mathcal{A}$ and $\omega \in \Omega_k$,

- 1) a mapping $\Phi_N : \mathcal{A} \times \Omega_k \rightarrow \left\{ \{(t_i, r_i)\}_{i \in \omega} \right\}$ dictates the ex-post liquidity raising strategy (t_i, r_i) that each bank $i \in \omega$ adopts in equilibrium;
- 2) a mapping $\Phi_S : \mathcal{A} \times \Omega_k \rightarrow \left\{ \{r_j\}_{j \in \Omega/\omega} \right\}$ dictates the ex-post rate posting policy r_j for each bank $j \in \omega^c$;
- 3) a mapping $\Phi_I : \mathcal{A} \times \Omega_k \rightarrow \left\{ \{(q_{i,j}, r_{i,j})\}_{i \in \omega, j \in \Omega/\omega} \right\}$ specifies the ex-post interbank trading deal $(q_{i,j}, r_{i,j})$ between each bank $i \in \omega$ and each bank $j \in \omega^c$ in equilibrium.

ii) Game of Ex-ante “Money” Creation

Having characterized the ex-post subgame in liquidity raising, we are now ready to describe banks’ ex-ante decision in money creation/loan making. On date 0, banks play a “money creation” game in which each bank acts independently in choosing its loan making volume. On the premise that it is ensuring the money-like feature of its date-0 deposits, each bank i in this game chooses its asset portfolio $(\alpha_i, 1 - \alpha_i)$ to maximize the payoff on date 2.

To facilitate analysis, let us define the ex-post solvency function $S(i; \{\alpha_n\}, \omega)$ for each bank i , given any arbitrary ex-ante money creation profile $\{\alpha_n\} \in \mathcal{A}$ and ex-post realization of liquidity shock $\omega \in \Omega_k$. For banks $i \in \omega$, the ex-post solvency in the equilibrium of subgame played on date 1 is

$$S(i; \{\alpha_n\}, \omega) = R(\alpha_i - t_i) - r_i \cdot s_i(r_i; \cdot) - \sum_{j \in \Omega/\omega} r_{i,j} \cdot q_{i,j}$$

in which t_i , $\{r_n\}$ and $\{(q_{i,j}, r_{i,j})\}$ are evaluated as the subgame equilibrium outcomes implied by mappings Φ_N , Φ_S and Φ_I given $(\{\alpha_n\}, \omega)$. For banks $i' \notin \omega$, solvency under liquidity shock (or panic-proofness) is not a concern on date 1 and the solvency constraint

2.

would be slack. Consequently, given the choice of $\{\alpha_n\}$ on date 0, the ex-post solvency of bank i on date 1 under a liquidity shock ω can be formulated as

$$S(i; \{\alpha_n\}, \omega) \equiv \begin{cases} \epsilon > 0 & \text{if } i \notin \omega \\ R(\alpha_i - t_i) - r_i \cdot s_i(r_i; \cdot) - \sum_{j \in \omega^c} r_{i,j} \cdot q_{i,j} & \text{if } i \in \omega \end{cases}$$

Given the choice of $\{\alpha_n\}$ on date 0, bank i can stay solvent (or immune to panic) under liquidity shock ω if and only if

$$S(i; \{\alpha_n\}, \omega) \geq 0$$

In the loan-making game played on date 0, all banks simultaneously make their asset portfolio choice. In choosing its loan making volume α_i , each bank i takes the action profile $\{\alpha_n\}_{-i}$ of others as given and solves

$$\max_{\alpha_i \in [0,1]} R\alpha_i + (1 - \alpha_i)$$

subject to the **money/safe asset creation constraint**:

$$\min_{\omega \in \Omega_k} S(i; \{\alpha_i, \{\alpha_n\}_{-i}\}, \omega) \geq 0$$

That is, to ensure that the ex-ante deposits banks issued on date 0 are money-like, banks must guarantee that panic among their depositors is always prevented. This solvency constraint under the most unfavorable liquidity shocks guarantees that runnable deposits issued ex-ante will be safe assets.

iii) A Private Safe Asset Creation Equilibrium

The equilibrium of private safe asset creation in this economy takes the form of a subgame perfect Nash equilibrium, in which simultaneous games are sequentially played on date 0 and date 1. The private safe asset creation equilibrium associated with level- k liquidity shocks is defined as follows.

Equilibrium Definition

In this economy, a subgame perfect Nash equilibrium of bank money creation associated with level- k liquidity shocks consists of:

I) a loan-making strategy profile $\{\alpha_n^E\} \in \mathcal{A}$ and

II) mappings Φ_N , Φ_S and Φ_I as defined in part ii) of this subsection such that

I) On date 1, for any arbitrary $\{\alpha_n\} \in \mathcal{A}$ and $\omega \in \Omega_k$,

1. the strategy profile $\{(t_i, r_i)\}_{i \in \omega}$ for liquidity-needy banks implied by mapping Φ_N solves the constrained optimization problem in liquidity raising for each bank $i \in \omega$;
2. the strategy profile $\{r_j\}_{j \in \omega^c}$ for liquidity-surplus banks implied by mapping Φ_S guarantees

interbank liquidity delivery for each bank $j \in \omega^c$;

3. the interbank trading deals $\{(q_{i,j}, r_{i,j})\}_{i \in \omega, j \in \omega^c}$ implied by mapping Φ_I maximize the joint surplus from trading between each bank $i \in \omega$ and each bank $j \in \omega^c$;

given others' equilibrium actions.

II) On date 0, given the (perceived) ex-post solvency implied by mappings Φ_N , Φ_S and Φ_I , α_i^E solves the constrained optimization problem in money creation for each bank i , given others choosing $\{\alpha_n^E\}_{-i}$.

In the rest part of the paper, I analyze the properties of private safe asset creation in the economy, based on the subgame perfect Nash equilibrium $\{\alpha_n^E\}$ characterized above.

4. Analysis: Efficiency and Implementability

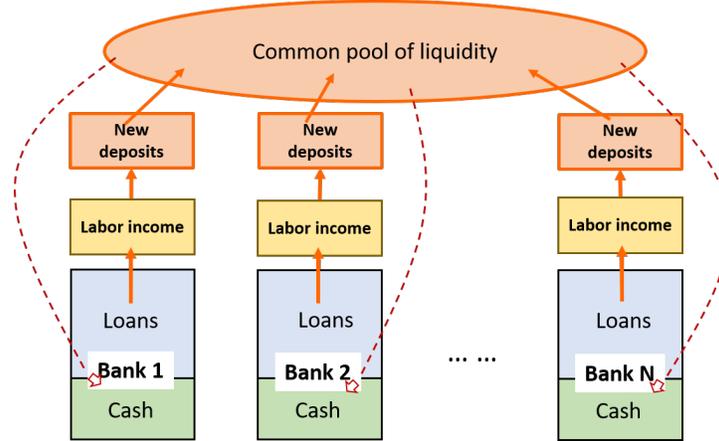
In this section I study properties of the private safe asset creation in the economy. As a benchmark, I first solve the constrained efficient solution in private safe asset creation that a planner can achieve. I then show that three commitment problems arise when the economy is decentralized: two of them can render the equilibrium outcomes inefficient while the third can cause the complete collapse of safe asset creation in the economy. Let us now elaborate.

A. The Constrained Efficient Benchmark: Planner's Solution

The creation of money-like liquid and safe deposits is performed not by any single bank but by the entire banking system. Interactions among banks during the process of safe asset creation can be crucial for equilibrium outcomes. The limited ability of individual banks to commit to not engaging in certain socially undesirable behavior can induce inefficiency or implementability issues in a decentralized economy. To highlight these potential problems, we first characterize the planner's solution as a constrained efficient benchmark that eliminates limited commitment problems.

The banking sector shares a common liquidity pool in its safe asset creation process. As illustrated in Figure 7, a liquidity pool is endogenously generated in the economy by banks' ex-ante loan making activities, and it provides liquidity to banks hit by ex-post liquidity shocks. As such, in the social planner's allocation for maximizing safe asset creation in the economy, both the ex-ante creation and the ex-post usage of this liquidity pool must be optimized.

Fig. 7. Liquidity pool in private safe asset creation



In the analysis that follows, we shut down the ex-post trading of the liquid cash banks hoarded ex-ante in order to focus on the ex-ante creation and ex-post usage of this commonly shared liquidity pool. That is, we assume that the liquid cash holding $1 - \alpha_i$ of each bank i will only be used to cover its own ex-post liquidity needs. The purpose of this assumption is to focus on the potential under-production of the commonly shared liquidity pool in the economy. In the Appendix [Section B1](#), we relax this assumption and show that all the results are qualitatively unaffected or even strengthened.⁴²

Facing the same panic-proof constraint induced by the fragility-safety conflict as in the decentralized economy, the planner solves the optimal allocation in private safe asset creation.⁴³ In making her allocation decisions, the planner has to respect each household's outside option implied by its storage technology $h(y)$ and the illiquidity of loans implied by liquidation technology $g(t, \alpha)$.

The planner's objective is to maximize the amount of economic activities that the banking sector can finance in the economy while ensuring that the deposits issued on date 0 are guaranteed safe under exposure to date-1 liquidity shocks. That is, on date 0 the planner chooses banks' ex-ante loan making profile $\{\alpha_n\}$ to maximize the total volume of loan making $\max_{\{\alpha_n\}} \sum_{n=1}^N \alpha_n$, subject to the constraint that all banks' date-0 deposits can

⁴²Each bank has natural monopoly power on their liquid asset holding, so the purpose of making this assumption is to tease out the contribution to the under-production of the commonly shared liquidity pool from this source. In fact, as we show in the Appendix [Section B1](#), the existence of the commonly shared liquidity pool can weaken the market power induced by privately held liquidity. Furthermore, we show that our efficiency analysis for both the ex-ante under-production and ex-post over-use of the commonly shared liquidity pool remains the same after the trading of ex-ante cash holding is introduced back to our model.

⁴³Here we are essentially assuming that the planner cannot resolve the limited commitment problem associated with banks' special loan collection skills, and so that the threat of run must also be maintained in planner's problem.

be (simultaneously) ensured safe under any ex-post liquidity shock $\omega \in \Omega_k$ that could hit on date 1.

Given the ex-ante loan-making profile $\{\alpha_n\}$, a liquidity pool is generated in the economy into which banks can dip for ex-post liquidity needs. Specifically, the date-1 endowments in the hands of households that reside in bank n 's territory will be $\bar{e} + w \cdot \alpha_n$ ($1 \leq n \leq N$). On date 1, with k banks getting hit by a liquidity shock $\omega \in \Omega_k$, the planner can optimally raise liquidity by dipping into the liquidity pool and liquidating the loans of banks in ω .⁴⁴ That is, on date 1 planner solves

$$L(\{\alpha_n\}, \omega) = \max_{\{t_i\}_{i \in \omega}, \{r_n\}} \sum_{i \in \omega} g(t_i, \alpha_i) + \sum_{n=1}^N d(r_n; \alpha_n)$$

subject to the feasibility constraint

$$\sum_{i \in \omega} R \cdot (\alpha_i - t_i) \geq \sum_{n=1}^N r_n \cdot d(r_n; \alpha_n)$$

That is, the date-2 payoff from the unliquidated loans of banks in ω must be sufficient to cover the promised payments to the date-1 creditors.

The ex-ante loan-making profile $\{\alpha_n^P\}$ that solves the planner's allocation problem on date 0 is then determined by

$$\{\alpha_n^P\} = \arg \max_{\{\alpha_n\}} \sum_{n=1}^N \alpha_n$$

subject to the **safe asset creation constraint**:

$$\min_{\omega \in \Omega_k} L(\{\alpha_n\}, \omega) - \sum_{i \in \omega} \alpha_i \geq 0$$

That is, the amount of liquidity the planner can raise by adopting the optimal feasible strategy must be sufficient to cover the total demand for external liquidity from banks in ω . For safe asset creation, this condition must hold even for the most unfavorable liquidity shock $\omega \in \Omega_k$.

In this paper I focus on symmetric equilibrium in which all banks adopt the same ex-ante loan-making strategy. The following proposition characterizes the symmetric constrained efficient equilibrium α_P that can be achieved in the planner's allocation.

⁴⁴It is assumed that the planner cannot liquidate the long term assets of banks that are not hit by liquidity shocks. Only the loans of banks in ω can be liquidated or borrow against on date 1.

Proposition 1 *The symmetric constrained efficient equilibrium α_P of safe asset creation associated with level- k liquidity shocks in a planner's solution is determined by*

$$k \cdot g(t^*, \alpha_P) + N \cdot d(r^*; \alpha_P) = k \cdot \alpha_P$$

where the optimal liquidity raising strategy (t^*, r^*) satisfies

$$\frac{d(r^*; \alpha_P)}{d'(r^*; \alpha_P)} + r^* = R / \frac{\partial g(t^*, \alpha_P)}{\partial t}$$

$$k \cdot R(\alpha_P - t^*) = N \cdot r^* \cdot d(r^*; \alpha_P)$$

The constrained efficient benchmark defined above is featured with two crucial trade-offs that are optimized in the planner's solution.

i) Trade-off 1: Optimal ex-ante creation of the liquidity pool

The first key trade-off in the planner's solution is related to the ex-ante creation of liquidity pool through banks' loan making activities. By increasing the ex-ante loan making α , the ex-post liquidity pool that the planner can deploy is enlarged. However, the demand for ex-post liquidity that the planner needs to cover on date 1 also increases.

The planner's solution α^P optimally balances these two opposing effects. For $\alpha < \alpha^P$, the ex-ante safe asset creation constraint is slack because the demand for ex-post liquidity is too small relative to what the planner is able to raise:

$$\min_{\omega \in \Omega_k} S^P(\{\alpha\}, \omega) > 0$$

where $S^P(\{\alpha_n\}, \omega) = L(\{\alpha_n\}, \omega) - \sum_{i \in \omega} \alpha_i$ is the planner's solvency function under liquidity shock ω . Similarly, for $\alpha > \alpha^P$, the ex-ante safe asset creation constraint cannot hold because the enlargement of the ex-post liquidity pool cannot keep up with the increased demand for liquidity:

$$\min_{\omega \in \Omega_k} S^P(\{\alpha\}, \omega) < 0$$

ii) Trade-off 2: Optimal ex-post usage of the liquidity pool

The second key trade-off regards the ex-post usage of the liquidity pool. The liquidity raised on date 1 by dipping into the liquidity pool needs to be repaid on date 2, which is accomplished by banks' payoff from their unliquidated loans. As such, an optimal marginal rate of substitution in the planner's ex-post liquidity raising (t, r) governs how much loans should be liquidated and what rate should be posted in the retail market. Specifically,

$$\frac{d(r^*; \alpha_P)}{d'(r^*; \alpha_P)} + r^* = R / \frac{\partial g(t^*, \alpha_P)}{\partial t}$$

pins down this optimal marginal substitution. Liquidating too many long term assets or

dipping too much into the ex-post liquidity pool would create wedges that distort the MRS in ex-post liquidity raising away from that characterized in the preceding equation.

Detailed analysis and calculations of planner's optimal allocation in safe asset creation are provided in Appendix [Section A3](#).

As a brief summary of our analysis of the planner's solution, the implementation of the constrained efficient benchmark hinges on three conditions that a social planner can guarantee to hold. In particular, the planner can ensure that

on date 1:

1) surplus liquidity in the economy can be frictionlessly reallocated within the banking system—that is, it can be guaranteed to be traded at a **competitive** price that equals its marginal cost;

2) the optimal liquidity raising strategy $\{t_i^{SP}, d_i^{SP}\}_{i \in \omega}$ and $\{d_j^{SP}\}_{j \in \omega^c}$ in planner's solution can be **committed**;

on date 0:

3) the lending profile $\{\alpha_i^{SP}\}_{i=1}^N$ can be **coordinated**.

While these conditions for implement the constrained efficient benchmark can guarantee to hold when there is a social planner, they are likely to fail in a decentralized economy where certain markets (and contracts) are missing. In the remainder of this section, I identify three market failures, which correspond to the three conditions outlined above, that could arise under a practically and theoretically reasonable environment of contracting and market space.

B. Inefficiency I: Ex-ante Under-production of Liquidity Pool

Once the economy is decentralized, the first inefficiency that can arise in the private safe asset creation is the ex-ante under-production of the liquidity pool in the economy. The key economics of this inefficiency is an incentive problem that originates from the non-competitive pricing of the liquidity traded in ex-post inter-bank markets.

Insurance Markets for Incentive Provision when $k < N$

On date 0, by increasing its loan making α_i , each bank i enlarges the size of the ex-post liquidity pool in the economy.⁴⁵ However, this makes each bank's own balance sheet more illiquid and increases its own demand for external liquidity on date 1, when it is hit by liquidity shocks. Proper compensation thus needs to be provided to incentivize banks to engage in ex-ante liquidity creation activities. Whenever not all banks will be hit by

⁴⁵This enlarging effect on the ex-post liquidity pool is reflected as the lowered marginal costs in retail liquidity raising in bank i 's territory on date 1.

ex-post liquidity shocks on date, i.e., $k < N$, the incentive provision for ex-ante liquidity pool creation would be relevant.

As the creator of “money” in the economy, banks’ valuation of the ex-post liquidity price will be state-contingent. Specifically, banks care more about the price of purchasing liquidity at when they need liquidity than the price for selling liquidity when there is a surplus.⁴⁶ This state-contingent valuation of the ex-post liquidity price implies that an insurance market generally would be needed for incentive provision.⁴⁷ An insurance market in which banks can insure against unfavorable liquidity states can restore potential incentive distortion because banks can use the profits they make in good states to purchase insurance to insure themselves against bad states.

However, such an insurance market usually does not exist in reality. Multiple reasons can contribute to this particular incompleteness in the space of marketable claims. For example, an exogenous reason could be that ex-post idiosyncratic liquidity shocks are observable but not verifiable. This would create a technical difficulty in writing contracts contingent on banks’ ex-post liquidity status. Endogenous reasons, too, can prevent such a market from being opened. Although not micro-founded in this paper, opening a market that allows banks to insure against ex-post liquidity shocks can induce ex-ante moral hazard issues that may render the entire system worse off.⁴⁸

Without such an insurance market, the incentive provision for individual bank’s ex-ante liquidity pool creation would then hinge on the pricing of ex-post liquidity.

Non-competitive Liquidity Pricing and a Hold-up Problem

The absence of the aforementioned insurance market itself does not cause the problem; it simply implies that ex-post rent distribution will distort ex-ante incentives. It must be certain socially inefficient behavior banks engage in that makes it problematic. In our model, this market incompleteness become relevant because of the rent extraction behavior by lending banks’ in ex-post interbank trading.

Whenever the ex-post interbank market is not perfectly competitive (as reflected in the lending banks’ ability to charging a positive markup in liquidity price), the ex-ante incentive provision for liquidity creation will be inefficient. This is so for the following reason: the incentive provided by the profit that a bank makes (by charging mark-ups) from in-

⁴⁶In the model, such a state-contingent valuation of ex-post liquidity price is reflected as the safe asset creation constraint being binding when hit by liquidity shocks and being slack when not hit.

⁴⁷In our concrete setting, the insurance market for incentive provision is a market in which contracts can be written such that banks can purchase ex-post liquidity at pre-committed prices. These pre-committed prices are set according to each banks’ ex-ante loan-making activities. In other words, the social value of liquidity creation is properly reflected in ex-post liquidity prices.

⁴⁸See [Diamond and Rajan \(2012\)](#) and [Farhi and Tirole \(2012\)](#) for detailed discussions of how ex-post intervention can cause ex-ante problems.

terbank trading when surplus in liquidity is not enough to offset the disincentivization of the high liquidity price (caused by the same mark-up) when itself is in need of liquidity.⁴⁹ As such, the ex-ante incentive for liquidity creation would be under-provided and, thus, in a decentralized system, the liquidity pool would be under-produced .

The following simple case illustrates this first inefficiency, the nature of which is a hold-up problem, in private safe asset creation in a decentralized economy.

Special Case Analysis: $k = 1, N = 2, \tau = 0$

This first market failure, which results in ex-ante under-production of the liquidity pool in the economy, is relevant whenever not all banks will be hit by ex-post liquidity shocks; i.e., $k \leq N - 1$. Here I analyze the simplest case that satisfies this condition, in which $k = 1, N = 2$. Furthermore, to highlight the potential hold-up in interbank trading, I assume $\tau = 0$. That is, the banking market is completely segmented.

Let us focus on symmetric equilibrium. That is, both banks choose α on date 0 in their ex-ante loan making. WLOG, suppose on date 1 bank 1 is hit by an ex-post liquidity shock while bank 2 is not. To cover its liquidity needs, bank 1 then

- i) liquidates $t_1 \in [0, \alpha]$ of its long term asset;
- ii) posts a rate r_1 in the retail liquidity market to raise liquidity from households;
- iii) borrows liquidity from bank 2 for quantity $q_{1,2}$ at rate $r_{1,2}$.

With the banking market being completely segmented (i.e. $\tau = 0$), the retail liquidity supply function on date 1 for each bank i ($i = 1, 2$) is

$$s(r_i; \alpha) = d(r_i; \alpha)$$

That is, each bank can only raise deposits from its own territory, regardless of the rate being offered by the other bank.

The interbank deal $(q_{1,2}, r_{1,2})$ is determined as the Nash bargaining solution that solves

$$(q_{1,2}, r_{1,2}) = \arg \max_{\hat{q}_{1,2}, \hat{r}_{1,2}} \left[V_1(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_1^R \right]^\beta \left[V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R \right]^{1-\beta}$$

in which V_i^R is each bank i 's reservation value and $V_i(\hat{q}_{1,2}, \hat{r}_{1,2})$ is each bank i 's valuation of any arbitrary deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$. The detailed expressions of V_i^R and $V_i(\hat{q}_{1,2}, \hat{r}_{1,2})$ are displayed in the Appendix [Section A4](#).

In this example with $\tau = 0$, we can show the following property of the equilibrium interbank deal $(q_{1,2}, r_{1,2})$.

Lemma 3 *With $\tau = 0$, the interbank deal $(q_{1,2}, r_{1,2})$ established in equilibrium satisfies $q_{1,2} < s(r_{1,2}; \alpha)$.*

⁴⁹Because the insurance market is absent, banks are not able to purchase insurance against unfavorable liquidity states with the profits they make in good states.

That is, a positive mark-up is being charged in the liquidity that bank 1 purchases from the wholesale market. By posting rate $r_{1,2}$ in bank 2's territory, the price at which bank 1 purchases liquidity from bank 2, an amount of $s(r_{1,2}; \alpha)$ retail liquidity could be raised. However, from its wholesale market trading with bank 2, bank 1 receives only $q_{1,2}$ liquidity, delivered at the same price $r_{1,2}$. In this sense, bank 1 is being held up in the ex-post interbank liquidity trading.

Coverage of ex-post liquidity shortage for bank 1 requires that in equilibrium

$$g(t_1, \alpha) + \underbrace{s(r_1; \alpha)}_{\text{retail liquidity}} + \underbrace{q_{1,2}}_{\text{wholesale liquidity}} = \alpha$$

while the solvency on date 2 (or panic-proofness at date 1) requires

$$R(\alpha - t_1) - \underbrace{r_1 \cdot s(r_1; \alpha)}_{\text{retail liquidity payment}} - \underbrace{r_{1,2} \cdot q_{1,2}}_{\text{wholesale liquidity payment}} \geq 0$$

As displayed in Table 1, the private safe asset creation outcome achieved in the decentralized economy α_E is compared with that achieved in the planner's solution α_P .

Table 1: Efficiency analysis: $k = 1, N = 2, \tau = 0$

	Equilibrium	Planner's problem
Liquidity coverage:	α_E is the maximum α such that $\exists t_1, r_1 :$ $g(t, \alpha) + s(r_1; \alpha) + q_{1,2} = \alpha$	α_P is the maximum α such that $\exists t_1, r_1, q_{1,2}, r_{1,2} :$ $g(t, \alpha) + s(r_1; \alpha) + q_{1,2} = \alpha$
Inter-bank liquidity pricing:	$(q_{1,2}, r_{1,2}) = \text{argmax}_{\hat{q}_{1,2}, \hat{r}_{1,2}} [V_1(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_1^R]^\beta [V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R]^{1-\beta}$ ← with mark-up	$q_{1,2} = s(r_{1,2}, \alpha)$ ← no mark-up
Solvency:	$R(\alpha - t_1) - [r_1 s(r_1; \alpha) + r_{1,2} q_{1,2}] \geq 0$	$R(\alpha - t_1) - [r_1 s(r_1; \alpha) + r_{1,2} q_{1,2}] \geq 0$

A banks' ex-ante incentive of liquidity creation is determined by the tightness of the ex-post solvency constraint under liquidity shocks. This tightness, in turn, depends on the price at which banks can purchase liquidity when hit by ex-post liquidity shocks. In the planner's solution, it is guaranteed that banks can purchase extra liquidity at prices that properly reflect the social value of their ex-ante loan making.⁵⁰

⁵⁰In the symmetric equilibrium that all banks choose α on date 0, the retail liquidity supply $s(r; \alpha)$ in the territory of banks not hit by liquidity shocks can be deployed by the planner to cover the liquidity needs

However, the wholesale liquidity price in the decentralized economy is determined by ex-post bargaining, during which lending banks are able to charge a positive mark-up above the marginal cost. Concerned that it could be held up in ex-post interbank trading if it is hit by liquidity shocks, each bank is thus less incentivized to engage in ex-ante liquidity creation activities. Consequently, in contrast to the planner's allocation, the ex-ante liquidity creation in the decentralized economy is not optimized; indeed, liquidity pool in the economy is under produced. As a result, the safe asset creation constraint in the decentralized economy would be inefficiently tight. More generally, we have the following proposition.

Proposition 2 *When the retail market competitiveness is sufficiently low, the laissez-faire outcomes of private safe asset creation associated with level- k ($k \leq N - 1$) liquidity shocks are inefficient. That is, there exists a threshold $\hat{\tau} \in [0, \infty]$ ($\hat{\tau}$ could be ∞), such that*

$$\alpha_E < \alpha_P$$

whenever $\tau < \hat{\tau}$ (or $\kappa > \frac{1}{\hat{\kappa}}$).

In our model, the panic-proof constraint faced by each individual bank is affected by an aggregate state variable– the size of the commonly shared liquidity pool in the economy– through its impact on ex-post retail liquidity price. Because this aggregate state variable is affected by each individual bank's ex-ante loan-making decision, inefficiencies arise when individual banks do not fully internalize such effects. Imperfect interbank market raises concerns of being extracted in ex-post interbank trading and thus results in the ex-ante under-production of the liquidity pool– a hold-up problem arises. The following corollary shows that a perfectly competitive banking market can restore efficiency.

Corollary 1 *The laissez-faire equilibrium of bank money creation associated with level-1 liquidity shock is efficient; i.e.,*

$$\alpha_E = \alpha_P$$

if $\tau = \infty$ or ($\kappa = 0$).

When $k = 1$, only this first efficiency will be present in the private safe asset creation.⁵¹ In this case, making the retail banking market perfectly competitive can dispel individual bank's concern of being extracted in ex-post interbank trading and thus eliminates the hold-up problem in banks' ex-ante liquidity pool creation. This helps restore constrained efficient safe asset creation in the decentralized economy. We discuss the impact of the

of those that are hit. The pricing of these extra liquidity reflects the social value of ex-ante loan making by those who get hit, as all banks choose the same α ex-ante.

⁵¹In the next subsection, it becomes clear why the second inefficiency is absent when $k = 1$.

banking market structure in more details in section 5.

C. Inefficiency II: Ex-post Over-use of Liquidity Pool

The second potential inefficiency in the decentralized economy's private safe asset creation is the ex-post over-use of the liquidity pool. The key economics of this inefficiency are reminiscent of the phenomenon demonstrated by [Hart \(1975\)](#)– that under certain market settings, adding certain spot markets can be counter-productive. As will be shown below, in our concrete setting the existence of a particular ex-post spot market can reduce welfare by impairing the functioning of certain futures markets, which are necessary for achieving the time consistency in banks' optimal sequential game play.

Necessity of Futures Markets for Time Consistency

In the process of private money creation, banks play a sequential game– they play an ex-ante game in loan making (or liquidity pool creation) on date 0 and play an ex-post game in liquidity raising after liquidity shocks on date 1. Implementing the constrained efficient benchmark requires both the incentive provision for ex-ante liquidity pool creation to be efficient, as well as a time consistency to be achieved in banks' ex-post usage of the liquidity pool created in the economy. That is, contingent on the level of ex-ante production of the liquidity pool, the ex-post utilization of the liquidity pool in the decentralized economy needs to be guaranteed optimized.

The second inefficiency that can potentially arise in private money creation when more than one bank will be hit by liquidity shocks, i.e., $k > 1$, is due to (ex-post) over-use of the liquidity pool. A negative externality, generated by one bank when it dip into the liquidity pool to others who are also in need of liquidity, is behind this over-use problem. The planner can fully internalize this "distributive externality" with her ability to dictate each bank's ex-post liquidity raising strategy and, thus, the liquidity pool usage can always guaranteed to be optimized contingent on any level of ex-ante production of the liquidity pool and any realization of liquidity shocks.

These conditions, however, are likely to fail once the economy is decentralized. To achieve the time consistency in planner's constrained efficient solution– that ex-post liquidity pool usage can be guaranteed optimized contingent on ex-ante liquidity pool creation, certain futures markets are needed in a decentralized system. The futures contracts required to implement the constrained efficient solution should allow each bank to purchase pre-specified quantities of ex-post retail liquidity at pre-specified prices, contingent only on their ex-ante loan making choices. Whenever such a futures market is absent or

impaired, the time consistency that is needed for achieving constrained efficient outcomes is likely to fail.

Ex-post Spot Market and Impaired Contract Exclusivity when $k > 1$

In this model, the spot market that can cause welfare loss is the ex-post retail liquidity market that allows (or makes it easier for) banks to raise liquidity in the territories of other banks, which is made possible by a competitive banking market. When such an ex-post spot market is introduced on date 1, an incentive compatibility constraint is placed on the ex-post liquidity raising strategy of banks that are hit by liquidity shocks. We now explain how.

Without the futures market as described above, an ex-post commitment problem could arise. Whenever a disciplining tool on ex-post competition is absent or not fully imposed, banks cannot commit to not competing for ex-post liquidity in each others' territories. Consequently, the equilibrium ex-post retail liquidity price in the territories of each bank hit by liquidity shocks must rise to a sufficiently high level. If it does not, other banks may be tempted to engage in liquidity stealing, which they achieve by offering slightly higher rates than those offered by native banks.

In this sense, a competitive banking market can generate an undesirable effect on the futures market that is needed to achieve the time consistency in efficient private money creation. With a highly competitive retail liquidity market, each individual bank's ability to write the futures contracts that specify ex-post retail liquidity usage and pricing would be impaired. This is because the quantity of retail liquidity a bank can obtain at certain price is likely to be affected by other banks' ex-post behavior, when a banking market is competitive. In other words, a competitive banking market impairs the contract exclusivity. This is problematic for the functioning of futures market because it means the liquidity pool usage specified in the futures contracts cannot be committed—because other banks' ex-post behavior could easily invalidate these ex-ante written contracts under a competitive market environment.

Due to the introduction of this ex-post spot market, banks are likely to become trapped in a dilemma: if they do not dip much enough into the liquidity pool they will not get anything from it. As a result, wedges are generated and distort banks' equilibrium marginal rates of substitution in ex-post liquidity raising away from the socially optimal levels. In this manner, banks hit by liquidity shocks would be forced to adopt ex-post liquidity raising strategies that are sub-optimal in the planner's solution. Specifically, in the decentralized economy, banks would use the liquidity pool too much yet liquidate their long-term assets too little.

Special Case Analysis: $k = 2, N = 2, \tau = \infty$

This second market failure, which results in an ex-post over-use of the liquidity pool, will be relevant whenever more than one bank is receiving ex-post liquidity shocks; i.e., $k \geq 2$. Here I analyze the simplest case that satisfies this condition, in which $k = 2, N = 2$. Furthermore, to highlight the welfare loss associated with introducing the ex-post spot market described above, I assume $\tau = \infty$. In other words, the banking market is perfectly competitive.

Again we focus on symmetric equilibrium, in which both banks choose α on date 0 in ex-ante loan making. Thus on date 1, the endowments of households in both banks' territories are $\bar{e} + w \cdot \alpha$. When the retail liquidity market is perfectly competitive, each bank can freely compete for liquidity in others' territory without having to offer extra higher rates. Therefore, on date 1, if the other bank posts a rate r_{-i} , then the retail liquidity supply for each bank i is

$$s(r; \alpha) = \begin{cases} 0 & \text{for } r < r_{-i} \\ \text{anything in certain range} & \text{for } r \geq r_{-i} \end{cases}$$

That is, bank i can raise nothing by posting any rate $r < r_{-i}$ or it can raise as much as it wants (bounded by certain limits) by posting a rate slightly higher than r_{-i} . Therefore, in the equilibrium on date 1, both banks must post the same rate \hat{r} in the retail liquidity market.

In equilibrium, when the ex-post retail liquidity price is \hat{r} , each bank i on date 1 finds optimal to also post the same rate in the retail market and optimally determines liquidation volume t by solving

$$\max_{t \in [0, \alpha]} R(\alpha - t) - \hat{r} \cdot [\alpha - g(t, \alpha)]$$

Here $\alpha - g(t, \alpha)$ is the liquidity shortage after liquidating t of the illiquid assets, which can be raised from the retail liquidity market at price \hat{r} .⁵²

The first order condition regarding t implies that in equilibrium

$$-R + \hat{r} \cdot \frac{\partial g(t, \alpha)}{\partial t} \geq 0$$

This is the incentive compatibility constraint on bank's ex-post usage of the liquidity pool that ensures no banks have incentive to deviate in equilibrium.⁵³

⁵²Since both banks choose the same α ex-ante, they would liquidate the same amount t in equilibrium. Hence the subscript in t is suppressed.

⁵³The deviation here is to steal liquidity in others' territory that are originally raised by those native

With the equilibrium ex-post retail liquidity price being \hat{r} , the aggregate supply of liquidity from the retail market is $2 \cdot d(\hat{r}; \alpha)$. The aggregate demand for retail liquidity from the banking sector is $2 \cdot [\alpha - g(t, \alpha)]$, in which t is the optimal liquidation volume implied by the above first order condition. The equilibrium rate \hat{r} equates supply with demand:

$$2 \cdot d(\hat{r}; \alpha) = 2 \cdot [\alpha - g(t, \alpha)]$$

Table 2 compares the private safe asset creation outcome achieved in the decentralized economy α_E and that achieved in the planner's solution α_P .

Table 2: Efficiency analysis: $k = 2, N = 2, \tau = \infty$

	Equilibrium	Planner's problem
Market clearing/ Liquidity coverage:	α_E is the maximum α such that $\exists t, \hat{r} :$ $d(\hat{r}; \alpha) = \alpha - g(t, \alpha)$	α_P is the maximum α such that $\exists t, \hat{r} :$ $g(t, \alpha) + d(\hat{r}; \alpha) = \alpha$
Incentive constraint:	$-R + \hat{r} \cdot \frac{\partial g(t, \alpha)}{\partial t} \geq 0$	N/A
Solvency:	$R(\alpha - t) - r \cdot d(\hat{r}; \alpha) \geq 0$	$R(\alpha - t) - r \cdot d(\hat{r}; \alpha) \geq 0$

Here the inefficiency in the decentralized economy's private safe asset creation is driven entirely by this extra incentive compatibility constraint on banks' ex-post usage of the liquidity pool. The existence of an ex-post spot market, implied by $\tau = \infty$, forces each bank hit by liquidity shocks to adopt a liquidity raising strategy that satisfies this extra IC constraint. However, the marginal rates of substitution in liquidity raising implied by this IC constraint generally differ from the optimal ones implied in the planner's solution.

As a result, the ex-post usage of liquidity pool in the decentralized economy is not optimized as it is in the planner's allocation. In particular, the liquidity pool in the economy is over-used. Because banks hit by ex-post liquidity shocks are forced to adopt strategies that are (socially) sub-optimal, the safe asset creation constraint that governs banks' ex-ante loan making will be inefficiently tight and it will result in inefficient safe asset creation. More generally, we have the following proposition.

Proposition 3 *The laissez-faire equilibrium of bank money creation associated with level- k ($k \geq 2$) banks, by posting a slightly higher rate.*

liquidity shocks is inefficient; i.e.,

$$\alpha_E < \alpha_P$$

whenever $\tau > 0$ (or $\kappa < \infty$).

In the model, the key component that makes this second inefficiency relevant is the assumption that banks' date-0 deposits must be made panic-proof when exposed to ex-post liquidity shocks on date 1. This panic-proof constraint implies an optimal marginal rate of substitution in ex-post liquidity raising that the planner would adopt. A competitive banking market introduces a commitment problem, as a consequence of which the socially optimal ex-post liquidity-raising policy cannot be implemented in the decentralized system. The following corollary shows that efficiency can be restored with complete segmentation in the retail liquidity market.

Corollary 2 *The laissez-faire equilibrium of bank money creation associated with level- N liquidity shocks is efficient; i.e.,*

$$\alpha_E = \alpha_P$$

if $\tau = 0$ (or $\kappa = \infty$).

When $k = N$, only this second inefficiency will be present in the private safe asset creation because there is no spare liquidity in the ex-post liquidity market. In this case, making the banking market completely segmented restores efficient safe asset creation in the decentralized economy. Detailed calculations for this inefficiency are provided in Appendix [Section A5](#).

D. Ex-ante Coordination in Safe Asset Creation

The two potential inefficiencies in private safe asset creation are essentially caused by ex-post commitment problems in the decentralized economy.⁵⁴ In addition to these two ex-post commitment issues, which can cause the laissez-faire solution of safe asset creation to be inefficient, an ex-ante commitment problem can also arise that impedes the private safe asset creation in the economy.

Equilibrium Implementability

In the ex-ante money creation/loan making on date 0, banks play a simultaneous game in which each bank chooses its own loan-making volume α and no banks know

⁵⁴The commitment problem relevant for the first inefficiency is that when the banking market is imperfect, lending banks cannot commit to not extracting rents from ex-post interbank trading. In the case of the second inefficiency, the problem is that once an ex-post spot market is opened, banks cannot commit to not stealing liquidity from each others' territories.

what choices the other banks have made. Consequently, banks cannot commit to an ex-ante loan-making strategy unless it is incentive compatible. Relatedly, we define the *automatic implementability* of an arbitrary ex-ante loan-making profile $\{\alpha_n\}$ as follows.

Definition *An ex-ante loan-making profile $\{\alpha_n\}$ is automatically implementable (AI) if and only if for each individual bank i , given the other banks choose $\{\alpha_n\}_{-i}$, it can commit to choosing α_i ; i.e., it optimizes by choosing α_i .*

The symmetric equilibrium α_E of private safe asset creation in this economy must satisfy $S(\alpha_E) = 0$, where function $S(\alpha)$ represents banks' ex-post solvency with everyone choosing α ex-ante. That is, when all banks choose α_E on date 0, everyone (who gets hit) can stay just solvent under ex-post liquidity shock at date 1.

To study whether or not the symmetric equilibrium $\{\alpha_E\}$ can be implemented, we define the following individual deviation function $S_D(\alpha; \alpha_E)$. This function $S_D(\alpha; \alpha_E)$ calculates the (perceived) ex-post solvency of an individual bank after it makes an arbitrary deviation α in its ex-ante loan making, while all other banks still choose α_E . Given this individual deviation function, the following lemma defines the conditions that ensure a safe asset creation equilibrium $\{\alpha_E\}$ to be *automatically implementable*.

Lemma 4 *A safe asset creation equilibrium $\{\alpha_E\}$ is automatically implementable if and only if*

$$\frac{\partial S_D(\alpha; \alpha_E)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+} \leq 0$$

That is, with all others choosing α_E at date 0, each individual bank can commit to choosing α_E in its ex-ante loan making.

The private safe asset creation equilibrium in which the banking sector can just stay solvent under ex-post liquidity shocks may not be automatically implementable. This is so because if the associated ex-ante loan making profile does not satisfy the necessary ex-ante incentive compatibility constraint, each individual bank cannot commit to choosing the loan making amount as specified in the profile. As illustrated in the special case analysis that follows, this ex-ante commitment problem will be acute when the banking market is competitive.

Special Case Analysis: $k = N = 10^6$

To highlight this ex-ante commitment problem in safe asset creation, we consider a case in which the number of banks in the economy is very large and the ex-post liquidity shock is systemic.

In this case with many banks in the economy, a perfectly competitive banking market ($\tau = \infty$) is likely to be problematic. Suppose in the equilibrium at date 1, the market

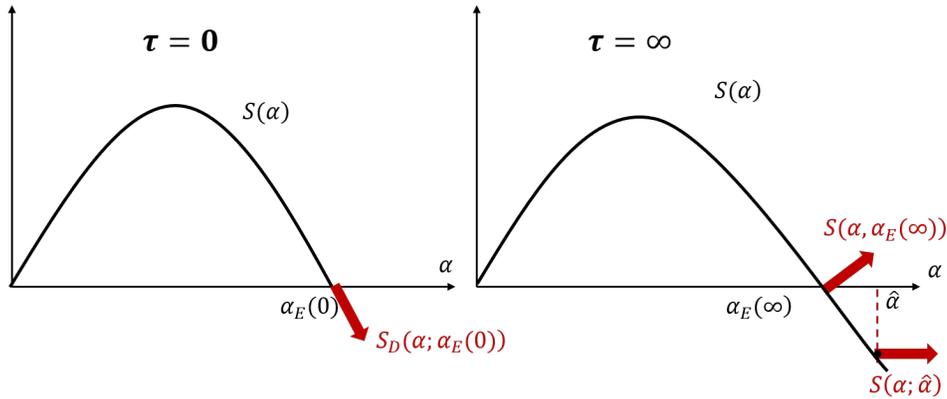
clearing retail liquidity price \hat{r} satisfies $\hat{r} < R$.⁵⁵ Then by making the following ex-ante deviation $\alpha = \alpha_E + \epsilon$ where $\epsilon > 0$ is infinitesimally small, an individual bank can be strictly better off. This is so because after making such an ex-ante upward deviation, this individual bank can adopt the following strategy in ex-post liquidity raising: it liquidates the same amount of long term assets as in old equilibrium and it raises additional ϵ liquidity from the retail market. As such, we have

$$S(\alpha_E + \epsilon; \alpha_E) \geq \epsilon \cdot R - \epsilon \cdot \hat{r} > 0$$

in which the retail liquidity price in the new ex-post equilibrium is still \hat{r} , thanks to the competitive retail market and the large number of banks in the economy.

Yet this commitment problem disappears when the banking market is completely segmented; i.e., $\tau = 0$. Now each individual bank only raises retail liquidity from its own territory, which makes the individual deviation function $S_D(\alpha; \alpha_E)$ perfectly aligned with the aggregate solvency function $S(\alpha_E)$. Figure 7 illustrates the comparison between $\tau = \infty$ and $\tau = 0$.

Fig. 8. Equilibrium implementability



In the left panel of Figure 8, where the banking market is completely segmented ($\tau = 0$), the safe asset creation equilibrium $\{\alpha_E(0)\}$ is both solvent and committed. In the right panel where the banking market is perfectly competitive ($\tau = \infty$), the equilibrium $\{\alpha_E(\infty)\}$ in which banks can just stay solvent ex-post cannot be committed ex-ante. In the equilibrium $\{\hat{\alpha}\}$ where ex-ante commitment can be guaranteed, everyone would however become insolvent once ex-post liquidity shocks hit.

In this sense, without a proper ex-ante commitment device, no safe asset can be created in the decentralized economy when the banking market is perfectly competitive! More generally, we have the following proposition.

⁵⁵Conditions of exogenous parameters that ensure this inequality are provided in the appendix.

Proposition 4 *For each (N, k) , there exists a threshold τ_{AI} in retail market competitiveness τ , such that the symmetric bank money creation equilibrium $\{\alpha_E\}$ that satisfies $S(\alpha_E; \tau) = 0$ is automatically implementable if and only if $\tau \leq \tau_{AI}$.*

The key intuition is that more competitive ex-post retail market makes ex-ante individual deviation less expensive. When the cost of making ex-ante deviation becomes low enough for individual banks such that everyone has incentive to make deviations at the equilibrium loan-making profile α_E that satisfies $S(\alpha_E) = 0$, an ex-ante commitment problem arises. This is because if every bank deviates ex-ante, no one bank can stay solvent ex-post. Detailed analysis and calculations are provided in Appendix [Section A6](#).

5. Policy Making for Safe Asset Creation

In this section, we conduct a policy analysis to study a regulator's optimal policy making for private safe asset creation in the economy.

A. The Regulator's Problem and Policy Instruments

The regulator seeks to maximize the safe asset creation in the economy. That is, the regulator's objective is to maximize the number of economic activities being financed by the banking sector while ensuring that the bank deposits are money-like.

However, unlike the planner, the regulator is constrained by several limited commitment problems. The following commitment issues impede the regulator's maximization of private safe asset creation:

- i) bank deposits cannot be made renegotiation-proof unless a threat of run is maintained;
- ii) banks cannot commit to not engaging in certain ex-post behavior (i.e., rent extraction, competition) unless proper commitment devices are imposed;
- iii) banks cannot commit to adopting an ex-ante loan making strategy α unless it is incentive compatible.

As discussed in Section 2, the commitment issue i) implies that the safety of deposits must be guaranteed under exposure to ex-post liquidity shocks that are endogenously implied by the fragile bank capital structure. Commitment issue ii) suggests that the two inefficiencies examined in Section 4 lie at the center of the regulator's optimization problem. Finally, commitment issue iii) reminds the regulator that the equilibrium in which the efficiency of private safe asset creation is maximized may not be automatically implementable unless proper regulatory policies is stipulated.

In this paper, I consider two policy instruments that the regulator can deploy in solving her problem:

- i) regulation of banking market competitiveness τ ;
- ii) regulation of banks' liquid asset holding.

Through banking market regulation (or deregulation), such as stipulating rules on the geographic scope of banks' branching operations, the regulator can manipulate competitiveness τ in the banking market. By stipulating a uniform requirement regarding banks' ex-ante liquid asset holding, the regulator is able to commit banks to adopting ex-ante loan-making strategies that might not be incentive compatible.⁵⁶

We now examine the effectiveness and efficacy of these two policy instruments.

B. Mixed Effects of Banking Market Competitiveness τ

In this subsection, we show that banking market competitiveness τ is a "double-edged sword" because it has mixed impacts on the two inefficiencies identified in Section 4.

High Competitiveness τ as (Ex-ante) Incentivizing Tool

In a highly competitive banking market, the ex-post retail liquidity supply for banks hit by liquidity shocks is relatively elastic. Given this elastic retail liquidity supply, when interbank deals in the wholesale market break down, banks in need of liquidity will not have to increase much the rates posted in the retail market to raise extra liquidity. As a result, the ability of lending banks to extract rents from ex-post interbank trading is weakened, due to the improved outside options of banks that need liquidity.

When banks are less concerned about getting held up in ex-post wholesale liquidity trading when liquidity shocks hit, they are more incentivized to engage in ex-ante liquidity creation activities. Through more efficient incentive provision, the first inefficiency in safe asset creation associated with ex-ante under-production of liquidity pool can be alleviated. In the extreme case illustrated in Corollary 1, when the banking market is perfectly competitive (i.e., $\tau = \infty$), inefficiencies caused by under-produced liquidity pool can be completely eliminated. More generally, we have the following proposition.

Proposition 5 *The laissez-faire equilibrium outcome of private safe asset creation associated with level-1 liquidity shocks $\alpha_E(\tau)$ is an increasing function in banking competitiveness τ .*

When $k = 1$, only the first inefficiency is present. As it dispels banks' concern about being held up in ex-post wholesale trading, more competitive banking market alleviates this ex-ante under-production inefficiency and monotonically improves welfare.

⁵⁶For instance, banks who are discovered failed to abide the liquid asset holding requirement will be penalized.

Low Competitiveness τ as (Ex-post) Disciplining Tool

As demonstrated in Section 4(C), the competitive retail liquidity market might be counter-productive when more than one bank is receiving ex-post liquidity shocks. In such cases, making the banking market more competitive works equivalently as introducing an ex-post spot market, which imposes an extra incentive compatibility constraint on banks' ex-post usage of the liquidity pool. Thus, unlike the optimal marginal rate of substitution guaranteed in planner's allocation, decentralized equilibrium forces banks to over-use the liquidity pool.

Making the banking market more segmented (less competitive)– i.e., reducing τ – can alleviate this second inefficiency associated with ex-post over-use of the liquidity pool. The economics here recall those of [Jacklin \(1987\)](#), who demonstrates that restricting ex-post trading among consumers is essential for financial intermediaries' ex-ante liquidity creation. In our problem, a less competitive banking market makes the extra IC constraint on banks' ex-post usage of liquidity pool more aligned with the optimal MRS in liquidity raising that the planner's solution implies. In effect, a lower τ functions as a disciplining tool on banks' ex-post competing behavior, and thus more effectively commits banks to using the ex-post liquidity pool at the socially desired level.

In the extreme case illustrated in Corollary 2, when the banking market is completely segmented ($\tau = 0$), inefficiency induced by the ex-post liquidity pool being over-used can be completely eliminated. More generally, we have the following proposition.

Proposition 6 *The laissez-faire equilibrium outcome of private safe asset creation associated with level- N liquidity shocks $\alpha_E(\tau)$ is a decreasing function in banking competitiveness τ .*

When $k = N$, only the second inefficiency is present. By better committing banks to using the liquidity pool at the socially desired level, a less competitive banking market alleviates this ex-post over-use inefficiency and strictly improves welfare.

Special Case Illustration: $k = 2, N = 3$

When the number of banks subject to ex-post liquidity shocks satisfies $2 \leq k \leq N - 1$, both inefficiencies will actively affect the private safe asset creation in the decentralized economy. In this part, we analyze a simplest case that satisfies this condition, in which $k = 2, N = 3$.

Consider an symmetric equilibrium in which all three banks choose α on date 0 and WLOG assume that bank 1 and bank 2 are hit by liquidity shocks on date 1 while bank 3 is not. In the equilibrium on date 1, both bank 1 and bank 2 will be:

- i) liquidating t_1 ($t_1 \in [0, \alpha]$) of their long term assets;
- ii) posting rate r_1 in the retail market to raise liquidity in the amount $s(r_1; \cdot, \tau)$;

iii) borrowing from bank 3 for quantity $q_{1,3}$ at rate $r_{1,3}$.

where the retail liquidity supply $s(r_i; \cdot, \tau)$ for each bank i is parameterized by banking competitiveness τ . The detailed expression of $s(r_i; \cdot, \tau)$ is presented in the appendix.

Combining the efficiency analysis carried out for the previous two special cases, Table 3 compares the decentralized equilibrium outcome α_E and planner's solution α_P of private safe asset creation. Detailed calculations are presented in the Appendix [Section A7](#).

Table 3: Efficiency analysis: $k = 2, N = 3$

	Equilibrium	Planner's problem
Market clearing/ Liquidity coverage:	α_E is the maximum α such that $\exists t_1, r_1 :$ $g(t_1, \alpha) + s(r_1; \cdot, \tau) + q_{1,3} = \alpha$	α_P is the maximum α such that $\exists t_1, r_1, q_{1,3}, r_{1,3} :$ $g(t_1, \alpha) + s(r_1; \cdot, \tau) + q_{1,3} = \alpha$
Inter-bank liquidity pricing:	$(q_{1,3}, r_{1,3}) =$ $\text{argmax}_{\hat{q}_{1,3}, \hat{r}_{1,3}} [V_1(\hat{q}_{1,3}, \hat{r}_{1,3}; \tau) -$ $V_1^R(\tau)]^\beta [V_2(\hat{q}_{1,3}, \hat{r}_{1,3}; \tau) - V_2^R(\tau)]^{1-\beta}$	$2q_{1,3} = s(r_{1,3}; \cdot, \tau)$
Incentive constraint:	$R \frac{\partial g(t_1, \alpha)}{\partial t} - r_1 = \frac{s(r_1; \cdot, \tau)}{s'(r_1; \cdot, \tau)}$	N/A
Solvency:	$R(\alpha - t_1) - [r_1 s(r_1; \alpha) + r_{1,3} q_{1,3}] \geq 0$	$R(\alpha - t_1) - [r_1 s(r_1; \alpha) + r_{1,3} q_{1,3}] \geq 0$

As shown in table 3, both inefficiencies are actively affecting the equilibrium tightness of the safe asset creation constraint in the decentralized economy. Higher banking market competitiveness improves the borrowing banks' bargaining position and, hence, it reduces the mark-up charged in the equilibrium interbank trading deal $(q_{1,3}, r_{1,3})$. As such, higher banking competitiveness τ can loosen the solvency constraint by making the wholesale liquidity price closer to the marginal cost.

In contrast, lower banking competitiveness brings the IC constraint on banks' ex-post usage of the liquidity pool more into alignment with the optimal MRS in liquidity raising that the planner's solution implies. Therefore, a lower banking market competitiveness τ can loosen the solvency constraint by reducing the sub-optimality of the equilibrium liquidity-raising strategy that banks adopt in the decentralized economy.

The level τ^* of the banking market competitiveness that maximizes private safe asset creation must optimally balance the two opposing effects, as described above.

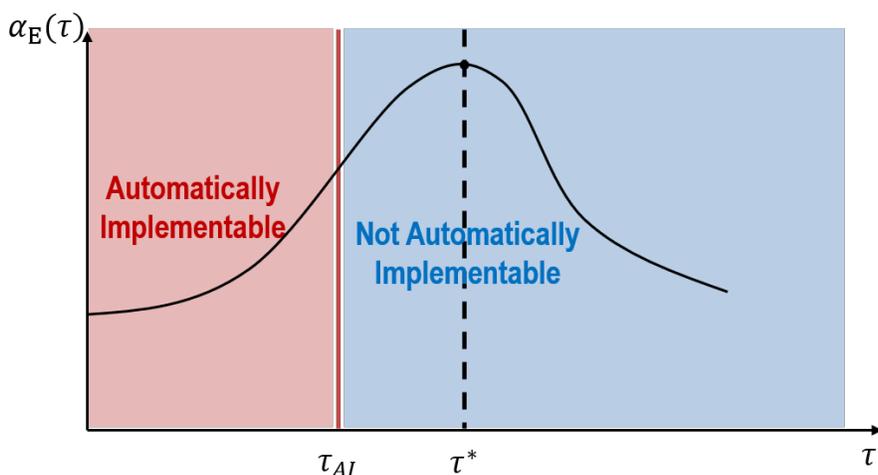
C. Liquidity Regulation for Equilibrium Implementation

Finally, we discuss the role played by bank liquidity regulation in maximizing the economy's private safe asset creation.

Reserve Requirement as (Ex-ante) Coordinating Tool

The private safe asset creation equilibrium associated with the optimal level of banking competitiveness τ^* may not be automatically implementable, as illustrated in Figure 9.

Fig. 9. Policy making for optimal safe asset creation



If the level of banking competitiveness that optimally balances the two ex-post commitment problems is higher than the threshold defined in proposition 4 that governs equilibrium implementability, i.e. $\tau^* > \tau_{AI}$, then the optimal equilibrium $\{\alpha_E(\tau^*)\}$ cannot be ex-ante committed. In such cases, to implement the desired private safe asset creation in the economy, the regulator needs to pair the banking market regulation that set $\tau = \tau^*$ with a liquidity regulation. Specifically, a reserve requirement on banks' ex-ante liquid asset holding $1 - \alpha$ needs to be imposed:

$$1 - \alpha \geq 1 - \alpha_E(\tau^*)$$

such that banks that fail to abide to this requirement will be penalized. Absent such a liquidity regulation, a coordination failure may arise in banks' safe asset creation because everyone has an incentive to make an upward deviation against $\alpha_E(\tau^*)$; thus, no one stay solvent ex-post. In this sense, the liquidity regulation on banks' liquid asset holding functions as a coordinating tool in the respect that it coordinates all banks in the economy to choose the right amount of illiquid lending ex-ante.

A Real World Example

The banking sector reform in the U.S. during the 1980s provides a concrete real world example for the theory developed in this paper. In the 1980s, the banking sector in the U.S. experienced significant regulatory reforms. The *Depository Institutions Deregulation and Monetary Control Act of 1980* is one of the most important pieces of legislation to affect the Federal Reserve during the last century.

This Act has two major titles. First, the Act deregulates financial institutions that accept deposits. Prior to 1980, commercial banks in the U.S. were subject to rigorous geographic restrictions on their branching operations and ceilings on the deposit rate they could offer. The *Depository Institutions Deregulation* removes these restrictions on branching operation and deposit raising. In our model, this banking market deregulation corresponds to an increase in banking market competitiveness τ .

However, the Act also strengthens the Fed's control on the liquidity of non-member banks; this is reflected in a set of reserve requirements that uniformly apply to all banks in the economy. Before 1980, the reserve requirements of nonmember institutions varied from state to state, and most state regulatory agencies imposed statutory requirements that were lower than Fed-mandated requirements.⁵⁷ The *Monetary Control* title imposes a set of requirements on the minimum liquid asset holding that all banks must follow.

The theory developed in this paper offers a novel rationale for the synchronized enforcement of a uniform reserve requirement with the banking deregulation as stipulated in the Act. The reserve requirement on banks' ex-ante liquid asset holding becomes essential for safe asset creation when the banking market is sufficiently competitive.

6. Conclusions

This paper develops a general equilibrium model of private safe asset creation in which banks create money-like deposits through a joint behavior of deposit taking and loan making. Liquid and safe deposits are manufactured through a "money multiplier", at the core of which is a common liquidity pool. This liquidity pool is endogenously generated by banks' loan-making activities, and it helps back up the money-like feature of the deposits that banks issued. In this process, a fragile bank capital structure imposes a constraint on the "money multiplier".

The first main result of the paper is that the combination of this safe asset creation constraint under liquidity risks with certain markets being missing implies that the private money creation in a decentralized economy could be constrained inefficient. In particular, under a practically and theoretically reasonable environment of contracting and market

⁵⁷See [Gilbert and Lovati \(1978\)](#).

space, I identify three market failures that can potentially plague the private money creation in a market economy. Two of them can cause inefficiencies by making the safe asset creation constraint inefficiently tight and the other can result in a complete collapse of private money creation.

Two market failures can potentially lead to inefficiencies in private money creation by making the safe asset creation constraint inefficiently tight. The first is an *incentive* problem induced by non-competitive pricing of ex-post wholesale liquidity, which can result in ex-ante under-production of the liquidity pool. The second is a commitment problem due to certain futures market being missing or impaired, which can result in ex-post over-use of the liquidity pool. These two market failures are exactly the manifestations of the two scenarios in which the first welfare theorem could fail and Pareto inefficiencies could arise.

Banking market competition plays an interesting role in affecting the efficiency of private money creation. In particular, I show that increased competition in banking market has a mixed effect on efficiency– it alleviates the incentive problem and hence the ex-ante under-production of liquidity pool, but it also exacerbates the commitment problem and the ex-post over-use of liquidity pool. These results have a clear policy implication on the optimal level of banking market competition in an economy. If an economy is subject to idiosyncratic risks, then a competitive banking market is beneficial– by playing the role as an incentivizing tool and help achieve better risk-sharing when idiosyncratic shocks hit. if an economy is exposed to systemic risks, then competitive banking markets can be harmful. In this case, competitive banking markets can trap the economy in a high interest rate equilibrium once systemic shocks hit the economy, in which all shocked banks aggressively dip into the liquidity pool until the liquidity price gets bid up to an inefficiently high level.

In addition to the above two market failures that will lead to inefficient outcomes in private money creation, a third market failure can potentially result in a complete collapse of the whole private system in its safe asset creation. When banking markets become sufficiently competitive, an ex-ante *coordination* problem may arise in banks' safe asset creation, at which point no private safe assets can be created in the economy without a proper reserve requirement being imposed. The analysis has implications for regulations of banking markets and bank liquidity.

References

- Allen, F. and Gale, D. (1994). "Limited Market Participation and Volatility of Asset Prices". *The American Economic Review* Vol. 84, No. 4 (Sep., 1994), pp. 933-955 (23 pages).
- Allen, F. and Gale, D. (2000). "Financial Contagion". *Journal of Political Economy* Vol. 108, No. 1 (February 2000), pp. 1-33.
- Allen, F. and Gale, D. (2004). "Financial Intermediaries and Markets". *Econometrica*, Volume 72, Issue 4 July 2004, Pages 1023-1061.
- Allen, F. and Gale, D. (2005). "From Cash-in-the-Market Pricing to Financial Fragility ". *Journal of the European Economic Association* Vol. 3, No. 2/3, Papers and Proceedings of the Nineteenth Annual Congress of the European Economic Association (Apr. - May, 2005).
- Caballero, R. J. (2006). "On the Macroeconomics of Asset Shortages". *NBER Working Papers* 12753.
- Davila, E. and Korinek, A. (2016). "Pecuniary Externalities in Economies with Financial Frictions". *Review of Economic Studies* 85(1), pp. 352-395.
- DeAngelo, H. and Stulz, R. (2015). "Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks". *Journal of Financial Economics*, 2015, vol. 116, issue 2, 219-236.
- Diamond, D. (1984). "Financial Intermediation and Delegated Monitoring". *The Review of Economic Studies* Vol. 51, No. 3 (Jul., 1984), pp. 393-414 (22 pages).
- Diamond, D. and Dybvig, P. (1983). "Bank Runs, Deposit Insurance, and Liquidity". *The Journal of Political Economy*, Vol. 91, No. 3. (Jun., 1983), pp. 401-419.
- Diamond, D. and Rajan, R. (2000). "A Theory of Bank Capital". *Journal of Finance* Volume 55, Issue 6 December 2000.
- Diamond, D. and Rajan, R. (2001). "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking". *Journal of Political Economy* Vol. 109, No. 2 (April 2001), pp. 287-327.
- Diamond, D. and Rajan, R. (2005). "Liquidity Shortages and Banking Crises". *Journal of Finance* Vol. 60(2), pages 615-647, 04.

- Diamond, D. and Rajan, R. (2012). "Illiquid Banks, Financial Stability, and Interest Rate Policy". *Journal of Political Economy*, 2012, vol. 120, issue 3, 552 - 591.
- Diamond, W. (2017). "Safety Transformation and the Structure of the Financial System". *Working Paper*.
- Donaldson, J. R., Piacentino, G., and Thakor, A. (2018). "Warehouse banking". *Volume 129, Issue 2, August 2018, Pages 250-267*.
- Farhi, E., Golosov, M., and Tsyvinski, A. (2009). "A Theory of Liquidity and Regulation of Financial Intermediation". *Review of Economic Studies* 76 (3): 973-992.
- Farhi, E. and Tirole, J. (2012). "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts". *American Economic Review* Vol. 102, No. 1, February 2012 (pp. 60-93).
- Gale, D. and Yorulmazer, T. (2011). "Liquidity Hoarding". *FRB of New York Staff Report No. 488*.
- Geanakoplos, J. and Polemarchakis, H. (1985). "Existence, Regularity, and Constrained Suboptimality of Competitive Allocations When the Asset Market Is Incomplete". *Cowles Foundation Discussion Papers* 764.
- Gilbert, A. and Lovati, J. (1978). "Bank reserve requirements and their enforcement: A comparison across states". *Federal Reserve Bank of St. Louis Review* (1978), pp. 22-32.
- Gorton, G. and Pennacchi, G. (1990). "Financial Intermediaries and Liquidity Creation". *Journal of Finance*, 1990, vol. 45, issue 1, 49-71.
- Greenwald, B. and Stiglitz, J. (1986). "Externalities in Economies with Imperfect Information and Incomplete Markets". *The Quarterly Journal of Economics*, Volume 101, Issue 2, 1 May 1986, Pages 229–264.
- Gromb, D. and Vayanos, D. (2002). "Equilibrium and welfare in markets with financially constrained arbitrageurs". *Journal of Financial Economics* Volume 66, Issues 2–3, Nov–Dec 2002, Pages 361-407.
- Gârleanu, N., Panageas, S., and Yu, J. (2015). Financial entanglement: A theory of incomplete integration, leverage, crashes, and contagion. *American Economic Review*, 105(7):1979–2010.
- Hart, O. and Zingales, L. (2014). "Banks Are Where The Liquidity Is". *Working Paper*.

- Hart, O. and Zingales, L. (2015). "Liquidity and Inefficient Investment". *Journal of European Economic Association*, Volume13, Issue5 October 2015.
- He, Z. and Kondor, P. (2016). "Inefficient Investment Waves". *Econometrica*, Volume84, Issue2 March 2016 Pages 735-780.
- He, Z. and Krishnamurthy, A. (2012). "Intermediary Asset Pricing". *AMERICAN ECONOMIC REVIEW VOL. 103, NO. 2, APRIL 2013 (pp. 732-70)*.
- Holmström, B. and Tirole, J. (1998). "Private and Public Supply of Liquidity". *Journal of Political Economy Vol. 106, No. 1 (February 1998), pp. 1-40*.
- Jacklin, C. (1987). "Demand Deposits, Trading Restrictions, and Risk Sharing". *E. Prescott and N. Wallace (eds.) Contractual Arrangements for Intertemporal Trade (Minneapolis: University of Minnesota Press) 26-47*.
- Jeanne, O. and Korinek, A. (2010). "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach". *American Economic Review*, 100 (2): 403-07.
- Kiyotaki, N. and Moore, J. (1997). "Credit Cycles". *Journal of Political Economy Vol. 105, No. 2 (April 1997), pp. 211-248 (38 pages)*.
- Kiyotaki, N. and Moore, J. (2002). Evil is the root of all money. *The American Economic Review*, 92(2):62-66.
- Kiyotaki, N. and Moore, J. (2018). Inside money and liquidity. *ESE Discussion Papers 115, Edinburgh School of Economics, University of Edinburgh*.
- Kiyotaki, N. and Wright, R. (1989). "On Money as a Medium of Exchange". *Journal of Political Economy Vol. 97, No. 4 (Aug., 1989), pp. 927-954*.
- Lorenzoni, G. (2008). "Inefficient Credit Booms". *The Review of Economic Studies Vol. 75, No. 3 (Jul., 2008), pp. 809-833 (25 pages)*.
- Petersen, M. and Rajan, R. (1994). "The Benefits of Lending Relationships: Evidence from Small Business Data". *The Journal of Finance Vol. 49, No. 1 (Mar., 1994), pp. 3-37*.
- Petersen, M. and Rajan, R. (1995). "The Effect of Credit Market Competition on Lending Relationships". *The Quarterly Journal of Economics*, 1995, vol. 110, issue 2, 407-443.
- Rajan, R. (1992). "Insiders and Outsiders: The Choice between Informed and Arm's-length Debt". *Journal of Finance*, Volume47, Issue4 September 1992.

- Salop, S. (1979). "Monopolistic Competition with Outside Goods". *The Bell Journal of Economics*.
- Stein, J. (2012). "Monetary Policy as Financial-Stability Regulation". *Quarterly Journal of Economics* 127 (1): 57-95.
- Tobin, J. (1963). "Commercial Banks as Creators of 'Money'". *TCowles Foundation Discussion Papers 159, Cowles Foundation for Research in Economics, Yale University*.
- Xu, X. (2019). "Bank Liquidity Price and Banking Market Competition". *Working Paper*.
- Zentefis, A. (2019). "Bank Net Worth and Frustrated Monetary Policy". *Journal of Financial Economics (Forthcoming)*.

Appendix

A. Proofs and Derivations

A1. Retail liquidity Pricing: Proofs of Lemma 1 and 2

In Section A1, we elaborate in detail the pricing of retail liquidity at date 1. The price of retail liquidity at date 1 is determined by household member B's problem.

Lemma 1 *On date 1, for a household member B with endowment $e + w \cdot \alpha$, its (individual) inverse supply of deposits is $r(d; \alpha) \equiv h'(e + w \cdot \alpha - d)$. Accordingly, its (individual) deposit supply function is $d(r; \alpha) = r^{-1}(d; \alpha)$.*

Proof For an individual HH member B with endowment $e + w \cdot \alpha$ (that is, his local bank makes α amount of lending at date 0), facing an interest rate r that pays off at date 2, this risk neutral decision maker solves

$$\max_d h(e + w \cdot \alpha - d) + r \cdot d$$

in which the HH member B deploys his storage technology $h(\cdot)$ to carry the remaining endowments into date 2. Under our regulatory assumptions on function $h(\cdot)$, the optimal choice of d to deposit at rate r is determined by the first order condition regarding d :

$$r = h'(e + w \cdot \alpha - d)$$

which immediately gives us the individual inverse supply of deposits $r(d; \alpha) \equiv h'(e + w \cdot \alpha - d)$ of this HH member B with endowment $e + w \cdot \alpha$, as a function of d . The inverse function of $r(d; \alpha)$ gives us the individual supply function of deposits from this HH member B: $d(r; \alpha) = r^{-1}(d; \alpha)$. ■

From the above individual deposit supply function, we can thus characterize the (aggregate) retail liquidity supply function $s_i(r; \cdot)$ faced by each individual bank i .

Lemma 2 *For general cases in which $0 < \tau < \infty$, if all banks chose α on date 0 and all other banks $n \neq i$ are posting a rate \bar{r} on date 1, then the ex-post liquidity supply curve faced by bank i is*

$$s_i(r_i; \alpha, \bar{r}) = d(r_i; \alpha) \cdot [1 + 2\tau \cdot (V_H(r_i; \alpha) - V_H(\bar{r}; \alpha))]$$

where $V_H(r; \alpha) \equiv \max_d r \cdot d + h(e + s \cdot \alpha - d)$ is the optimized value for a HH with endowment $e + s \cdot \alpha$ and being offered a rate r .

Proof To prove Lemma 2, we proceed in two steps.

Step 1

First, we prove that each individual HH member B would never deposit in more than one bank. Suppose not, let us assume that a HH member B simultaneously deposits with two banks, i and j . Suppose the rates offered by each bank are r_i and r_j , while this HH's distance to each bank are L_i and L_j respectively. Furthermore, suppose that the HH deposits d_i with bank i and deposits d_j with bank j , where both d_i and d_j are strictly positive.

WOLG, let us assume $0 \leq L_i \leq L_j$, then it must be that $r_i < r_j$, because otherwise this HH can strict be made better by deviating to just deposit with bank i . This is because when $r_i \geq r_j$, by making the proposed deviation, the households can get weakly increased payoff at date 2 (since $r_i \geq r_j$) while the traveling cost $L_j \cdot \tau > 0$ ($L_i \leq L_j$ implies $L_j > 0$) is saved.

However, if $r_i < r_j$, then the following profitable deviation exists. That is, the HH can choose the following depositing profile: $\hat{d}_i = d_i - \epsilon$ and $\hat{d}_j = d_j + \epsilon$, where $\epsilon > 0$ is an arbitrary small positive number such that $\hat{d}_i > 0$. Then following the proposed deviation strategy, the HH can be made strictly better off:

$$\begin{aligned} r_i \cdot \hat{d}_i + r_j \cdot \hat{d}_j &= r_i \cdot (d_i - \epsilon) + r_j \cdot (d_j + \epsilon) \\ &= r_i \cdot d_i + r_j \cdot d_j + (r_j - r_i) \cdot \epsilon \\ &> r_i \cdot d_i + r_j \cdot d_j \end{aligned}$$

where the inequality is because $(r_j - r_i) \cdot \epsilon > 0$. As such, each individual bank would only deposit in one single bank.

Step 2

We are now ready to characterize the retail liquidity supply function for each individual bank i . For a HH member B with endowments $\bar{e} + w \cdot \alpha$ who chooses to deposit with a bank offering rate r , define function

$$V_H(r; \alpha) \equiv \max_d r \cdot d + h(e + s \cdot \alpha - d)$$

This function $V_H(r; \alpha)$ gives us the optimal payoff this HH can gain from depositing with this bank (before subtracting traveling costs from doing so).

Now consider an individual bank i . If all banks choose α at date 0, then the date-1 endowments of all HH member B on the entire circle is $\bar{e} + w \cdot \alpha$. Suppose all other banks are posting rate \bar{r} in the retail market for liquidity raising. Then if bank i posts a rate $r < \bar{r}$, some HH that reside in bank i 's territory will be attracted by adjacent banks. Specifically,

the HH who resides within a distance L to the boundaries of bank $i - 1$ and bank $i + 1$ would be attracted away, where the distance L is given by

$$\begin{aligned} L &= \frac{1}{\kappa} \cdot (V_H(\bar{r}; \alpha) - V_H(r_i; \alpha)) \\ &\equiv \tau \cdot (V_H(\bar{r}; \alpha) - V_H(r_i; \alpha)) \end{aligned}$$

As such, by offer a rate $r_i < \bar{r}$, bank i will only be able to raise liquidity from $1 - 2 \cdot L$ HH, each of whom would deposit an amount of $d(r_i; \alpha)$. Here we are making use of the results we established in step 1, that each individual HH would choose to deposit in one single bank. Therefore, the total amount of retail liquidity bank i can raise by posting a rate $r_i < \bar{r}$ is

$$s_i(r_i; \alpha, \bar{r}) = d(r_i; \alpha) \cdot [1 - 2\tau \cdot (V_H(\bar{r}; \alpha) - V_H(r_i; \alpha))]$$

Similarly, we can show that by posting a rate $r_i \geq \bar{r}$, the amount of retail liquidity can be raised by bank i is

$$s_i(r_i; \alpha, \bar{r}) = d(r_i; \alpha) \cdot [1 + 2\tau \cdot (V_H(r_i; \alpha) - V_H(\bar{r}; \alpha))]$$

Combine both cases, Lemma 2 is proved. ■

A2. Wholesale Liquidity Pricing

In Section A2, we provide details for interbank trading in the wholesale liquidity market at date 1.

After a liquidity shock ω hits, each bank $i \in \omega$ trades with each bank $j \in \omega^c$ for date-1 liquidity. The trading deal $(q_{i,j}, r_{i,j})$ is determined as the bargaining outcome in the bilateral negotiation between bank i and bank j . Mathematically, $(q_{i,j}, r_{i,j})$ maximizes the joint surplus from trading for both parties:

$$(q_{i,j}, r_{i,j}) = \arg \max_{(q,r)} [V_i(q, r) - V_i^R]^\beta \cdot [V_j(q, r) - V_j^R]^{1-\beta}$$

In the above equation, reservation values V_i^R and V_j^R are determined as the outside options for each bank given that a deal between the two cannot be reached. Since the interbank trading in the wholesale market at date 1 is modeled as non-cooperative bargaining, in doing our calculations for V_i^R and V_j^R , the following are held as fixed: the equilibrium deposit rates posted by banks n , where $n \neq i, j$; and all other interbank deals established in the equilibrium at date 1.

Under this model specification of the interbank bargaining between each pair of bank $i \in \omega$ and bank $j \in \omega^c$, reservation values V_i^R and V_j^R are determined as follows. Under the restriction that no deal between the two banks could be established, these two banks play will play a “hypothetical game”, in which

- i) bank i tries to minimize the margin at which (could be zero) it fails to meet its liquidity obligation; and
- ii) bank j has to raise enough liquidity in order to deliver what are specified in its established interbank trading deals with other banks in ω .

Therefore, taking as given the old equilibrium rate posted by banks $n \neq i, j$ and the rate \hat{r}_j posted by bank j in the new hypothetical equilibrium, bank i chooses its new strategy (\hat{t}_i, \hat{r}_i) to solve

$$\min_{(\hat{t}_i, \hat{r}_i)} \alpha_i - g(\hat{t}_i, \alpha_i) - s_i(\hat{r}_i; \cdot, \hat{r}_j) - \sum_{j' \in \omega^c \setminus \{j\}} q_{i,j'}$$

subject to feasibility constraint

$$R \cdot (\alpha_i - \hat{t}_i) \geq r_i \cdot s_i(\hat{r}_i; \cdot, \hat{r}_j) + \sum_{j' \in \omega^c \setminus \{j\}} r_{i,j'} \cdot q_{i,j'}$$

Here the retail liquidity supply function $s_i(r_i; \cdot, \hat{r}_j)$ for bank i is affected by banks' date-0 loan making profile $\{\alpha_n\}$, the equilibrium date-1 rates r_n posted by banks $n \neq i, j$, and the new rate \hat{r}_j posted by bank j .

For bank j , taking as given the old equilibrium rate posted by banks $n \neq i, j$ and the rate \hat{r}_i posted by bank i in the new hypothetical equilibrium, it posts a rate \hat{r}_j to ensure wholesale liquidity delivery:

$$s_j(\hat{r}_j; \cdot, \hat{r}_i) = \sum_{i' \in \omega \setminus \{i\}} q_{i',j}$$

Likewise, the retail liquidity supply function $s_i(r_i; \cdot, \hat{r}_j)$ that bank j faces would be affected by rate \hat{r}_i posted by bank i in the new equilibrium.

Suppose the equilibrium outcome of this “hypothetical game” after interbank deal breaks is that bank i adopts $(\hat{t}_i^H, \hat{r}_i^H)$ while bank j posts rate \hat{r}_j^H . Then the reservation value of bank j can be calculated as

$$V_j^R = \sum_{i' \in \omega \setminus \{i\}} r_{i',j} \cdot q_{i',j} - \hat{r}_j^H \cdot s_j(\hat{r}_j^H; \cdot, \hat{r}_i^H)$$

which is the total profits bank j could make from its established trading deals with other banks in ω .

For bank i 's reservation value \hat{r}_i^H , we first calculate the margin m^H at which bank i will be failing to meet its liquidity obligation in the "hypothetic equilibrium":

$$m^H = \left\{ \alpha_i - g(\hat{t}_i^H, \alpha_i) - s_i(\hat{r}_i^H; \cdot, \hat{r}_j^H) - \sum_{j' \in \omega^c \setminus \{j\}} q_{i,j'} \right\}^+$$

Then the reservation value of bank i is

$$V_i^R = p(m^H)$$

where a non-pecuniary penalty of size $p(m^H)$ will be imposed on bank i for its failure in meeting the liquidity obligation.

The valuation of an arbitrary trading deal $V_i(q, r)$ and $V_j(q, r)$ to both parties can be obtained following the same procedure. Now with a deal (q, r) being established between the two banks, bank i chooses (\hat{t}_i, \hat{r}_i) to solve

$$\min_{(\hat{t}_i, \hat{r}_i)} \alpha_i - g(\hat{t}_i, \alpha_i) - s_i(\hat{r}_i; \cdot, \hat{r}_j) - \sum_{j' \in \omega^c \setminus \{j\}} q_{i,j'} - q$$

subject to feasibility constraint

$$R \cdot (\alpha_i - \hat{t}_i) \geq r_i \cdot s_i(\hat{r}_i; \cdot, \hat{r}_j) + \sum_{j' \in \omega^c \setminus \{j\}} r_{i,j'} \cdot q_{i,j'} + r \cdot q$$

and bank j posts a rate \hat{r}_j to ensure wholesale liquidity delivery:

$$s_j(\hat{r}_j; \cdot, \hat{r}_i) = \sum_{i' \in \omega \setminus \{i\}} q_{i',j} + q$$

A3. Constrained Efficient Benchmark: Proof of Proposition 1

In Section A3, we provide details for the characterization of the planner's solution in maximizing private safe asset creation in the economy. Planner's authority in liquidity raising at date 1 is what ensures that the optimal private safe creation can be achieved. In particular, the planner's ability of credibly committing to implementing the following two behavior is crucial:

1) In the planner's allocation, each individual bank i only raises deposits from its own

territory at date 1, at a rate r_i dictated by the planner;

2) The planner can ensure that the wholesale liquidity traded in the interbank market is always priced at lending banks' marginal cost in deposit raising.

We now prove Proposition 1, which characterizes the symmetric constrained efficient equilibrium of safe asset creation that can be achieved in the planner's solution.

Proposition 1 *The symmetric constrained efficient equilibrium α_P of safe asset creation associated with level- k liquidity shocks in a planner's solution is determined by*

$$k \cdot g(t^*, \alpha_P) + N \cdot d(r^*; \alpha_P) = k \cdot \alpha_P$$

where the optimal liquidity raising strategy (t^*, r^*) satisfies

$$\frac{d(r^*; \alpha_P)}{d'(r^*; \alpha_P)} + r^* = R / \frac{\partial g(t^*, \alpha_P)}{\partial t}$$

$$k \cdot R (\alpha_P - t^*) = N \cdot r^* \cdot d(r^*; \alpha_P)$$

Proof We proceed in three steps.

Step 1

First we show that with all banks choosing the same α on date 0 (since we are focusing on symmetric equilibrium), in the planner's optimal allocation on date 1, all banks are posting the same rate r in the retail market.

Suppose all banks choose α on date 0, then the date-1 endowments of all households over the circle are $\bar{e} + w \cdot \alpha$. After a liquidity shock ω hits on date 1, which makes k banks in the economy in need of liquidity, the planner optimally raises liquidity in the economy to cover such needs. Specifically, the planner dictates the rate r_n each bank n to post in the retail market and the liquidation amount t_i for each bank $i \in \omega$ to solve

$$L(\alpha, \omega) \equiv \max_{\{t_i\}_{i \in \omega}, \{r_n\}} \sum_{i \in \omega} g(t_i, \alpha) + \sum_{n=1}^N d(r_n; \alpha)$$

subject to the feasibility constraint:

$$\sum_{i \in \omega} R \cdot (\alpha - t_i) \geq \sum_{n=1}^N r_n \cdot d(r_n; \alpha)$$

Form the Lagrangian

$$\mathcal{L} = \sum_{i \in \omega} g(t_i, \alpha) + \sum_{n=1}^N d(r_n; \alpha) + \lambda \left[\sum_{i \in \omega} R \cdot (\alpha - t_i) - \sum_{n=1}^N r_n \cdot d(r_n; \alpha) \right]$$

Take the first order conditions:

$$\begin{aligned} [r_n] : d'(r_n; \alpha) - \lambda \cdot [d(r_n; \alpha) + r_n \cdot d'(r_n; \alpha)] &= 0 \\ [t_i] : \frac{\partial g(t_i, \alpha)}{\partial t} - \lambda R &= 0 \end{aligned}$$

The FOC regarding t_i ($i \in \omega$) immediately implies that in planner's allocation, all banks in ω will liquidate the same amount of their long term asset. The FOC regarding r_n ($1 \leq n \leq N$) implies that

$$\begin{aligned} \lambda &= \left[\frac{d(r_n; \alpha)}{d'(r_n; \alpha)} + r_n \right]^{-1} \\ &\equiv [F(r_n)]^{-1} \end{aligned}$$

Given our assumption that HH's date-1 storage technology $h(\cdot)$ is concave, we can easily show that $d''(r_n; \alpha) < 0$. Therefore, we have

$$\begin{aligned} F'(r_n) &= 1 + \frac{d'(r_n; \alpha)}{d'(r_n; \alpha)} + \frac{-d(r_n; \alpha) \cdot d''(r_n; \alpha)}{[d'(r_n; \alpha)]^2} \\ &> 0 \end{aligned}$$

As such, all banks must be posting the same retail rate for liquidity raising in the planner's optimal allocation on date 1.

Finally, it's easy to observe that in the planner's allocation problem, the exact realization of $\omega \in \Omega_k$ does not matter. Given this fact, we can write function $L(\alpha, \omega)$ simply as $L(\alpha)$, which is a function only of the date-0 loan making choice α .

Step 2

In the next step of our proof, we show that the date-0 loan making α^P in the planner's optimal allocation must satisfy

$$L(\alpha_P) = k \cdot \alpha^P$$

Obviously we must have $L(\alpha_P) \geq k \cdot \alpha_P$. Otherwise if the planner chooses an α such that $L(\alpha) < k \cdot \alpha_P$, then after a liquidity shock hits the banking sector on date 1, even with the optimal liquidity raising strategy the planner will not be able to meet the aggregate demand for liquidity from banks that get hit. As a result, the deposits issued by banks in the economy on date 0 cannot be guaranteed to be safe assets.

On the other hand, if the date-0 loan making α_P in the planner's allocation satisfies

$L(\alpha_P) > k \cdot \alpha_P$, the safe asset creation is then not optimized. This is because the continuity of function $L(\alpha)$ implies that there exists a positive $\epsilon > 0$ such that $L(\alpha_P + \epsilon) > k \cdot \alpha_P$. Therefore, by increasing the date-0 loan making from α_P to $\alpha_P + \epsilon$, the planner can increase the amount of economic activities financed by banks in the economy while ensuring that the safety of deposits issued by banks on date 0 are still maintained. In other words, the planner's allocation of safe asset creation cannot be optimized by choosing α_P at which $L(\alpha_P) > k \cdot \alpha_P$

Therefore, in the planner's optimal allocation for safe asset creation, we must have $L(\alpha_P) = k \cdot \alpha_P$.

Step 3

Based on the results established in Step 1 and Step 2, we are now ready to prove this proposition. In planner's optimal allocation, with the k banks receiving liquidity shocks adopting the same strategy (t^*, r^*) and all other banks posting the same rate r^* , we have

$$L(\alpha_P) = k \cdot g(t^*, \alpha_P) + N \cdot d(r^*; \alpha_P)$$

Combined with the results from Step 2, we immediately have

$$k \cdot g(t^*, \alpha_P) + N \cdot d(r^*; \alpha_P) = k \cdot \alpha_P$$

To derive the conditions that t^* and r^* satisfy, combine the FOCs regarding r_n and t_i derived in Step 1, we get

$$\frac{d(r^*; \alpha_P)}{d'(r^*; \alpha_P)} + r^* = R / \frac{\partial g(t^*, \alpha_P)}{\partial t}$$

which gives us the optimal marginal rate of substitution between liquidation and retail liquidity raising implied by planner's solution.

Finally, at retail deposit rate r^* , the total supply of retail liquidity must equal the demand for them, when banks in ω are liquidating t^* of their long-term assets. That is,

$$k \cdot R(\alpha_P - t^*) = N \cdot r^* \cdot d(r^*; \alpha_P)$$

which finishes the proof. ■

A4. Inefficiency I: Proofs of Lemma 3, Proposition 2 and Corollary 1

In Section A4, we provide details of calculations in our analysis for efficiency I: the ex-ante under-production of the liquidity pool.

In our special case where $k = 1, N = 2, \tau = 0$, bank 1 is hit by the liquidity shock on date 1 while bank 2 is not. In the wholesale market for date-1 liquidity, bank 1 reaches a deal $(q_{1,2}, r_{1,2})$ with bank 2, which is determined by

$$(q_{1,2}, r_{1,2}) = \arg \max_{\hat{q}_{1,2}, \hat{r}_{1,2}} \left[V_1(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_1^R \right]^\beta \left[V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R \right]^{1-\beta}$$

In this Nash bargaining, bank 2's reservation value from outside option is simply

$$V_2^R = 0$$

This is because when no deals could be made with bank 1, the liquidity-surplus bank 2 cannot make any extra profits from date-1 liquidity trading: no one else would be purchasing wholesale liquidity from it. We now calculate the other terms in the above equation.

Suppose both banks make α illiquid lending on date 0. With an arbitrary deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$ established with bank 1, the extra profit from wholesale liquidity trading that bank 2 can make on date 1 is

$$V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) = \hat{q}_{1,2} \cdot (\hat{r}_{1,2} - r_2)$$

where r_2 is the rate bank 2 posts in the retail market in order to deliver the wholesale liquidity trading specified in the interbank deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$.

With the banking market being completely segmented (i.e. $\tau = 0$), each bank can only raise deposits from its own territory. The retail liquidity supply function on date 1 for each bank i ($i = 1, 2$) is $s(r_i; \alpha) = d(r_i; \alpha)$, regardless of the rate being offered by the other bank. Therefore, the rate r_2 that bank 2 posts in the retail liquidity market satisfies

$$d(r_2; \alpha) = \hat{q}_{1,2}$$

For bank 1, its reservation value is determined by the outcomes of its re-optimization after the deal with bank 2 breaks down. That is, facing the new feasibility constraint after the deal with bank 1 breaks down, bank 2 re-optimizes its liquidity raising to solve

$$\max_{(t_1, r_1)} g(t_1, \alpha) + s(r_1; \alpha)$$

subject to the new feasibility constraint

$$R \cdot (\alpha - t_1) \geq r_1 \cdot s(r_1; \alpha)$$

Denote the solution by (t_1^R, r_1^R) and the optimized objective by $L^R(\alpha) \equiv g(t_1^R, \alpha) + s(r_1^R; \alpha)$. The reservation value of bank 1 is then

$$V_1^R = -p(m^R(\alpha))$$

where $m^R(\alpha) \equiv (\alpha - L^R(\alpha))^+$ is the margin at which bank 1 fails to meet its liquidity obligation after adopting strategy (t_1^R, r_1^R) .

Similarly, for any arbitrary trading deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$, bank 1's valuation $V_1(\hat{q}_{1,2}, \hat{r}_{1,2})$ can be calculated as

$$V_1(\hat{q}_{1,2}, \hat{r}_{1,2}) = -p(m(\alpha; \hat{q}_{1,2}, \hat{r}_{1,2}))$$

The failure margin $m(\alpha; \hat{q}_{1,2}, \hat{r}_{1,2})$ is formulated as $m(\alpha; \hat{q}_{1,2}, \hat{r}_{1,2}) = (\alpha - L^R(\alpha; \hat{q}_{1,2}, \hat{r}_{1,2}))^+$, in which

$$L^R(\alpha; \hat{q}_{1,2}, \hat{r}_{1,2}) = g(\hat{t}_1, \alpha) + s(\hat{r}_1; \alpha)$$

where (\hat{t}_1, \hat{r}_1) is the optimal liquidity raising strategy bank 1 should adopt given a wholesale liquidity trading deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$ being established.

Having provided the details for interbank bargaining, we are now ready to prove the results regarding the first inefficiency in private safe asset creation.

Lemma 3 *With $\tau = 0$, the interbank deal $(q_{1,2}, r_{1,2})$ established in equilibrium satisfies $q_{1,2} < s(r_{1,2}; \alpha)$.*

Proof Suppose instead $q_{1,2} \geq s(r_{1,2}; \alpha)$. Then by the strict monotonicity of function $s(r; \alpha) \equiv d(r; \alpha)$, we know that

$$r_{1,2} \leq s^{-1}(q_{1,2}; \alpha) = r_2$$

Therefore, this implies that in equilibrium $V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) = \hat{q}_{1,2} \cdot (\hat{r}_{1,2} - r_2) \leq 0$. But $V_2^R = 0$, thus the value gained from trading for bank 2 is non-positive, i.e., $V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R \leq 0$.

However, when $\tau = 0$, it's easy to show that the set of trading deals $(\hat{q}_{1,2}, \hat{r}_{1,2})$ such that $V_1(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_1^R > 0$ and $q_{1,2} = s(r_{1,2}; \alpha)$ is non-empty. Therefore, whenever the bargaining parameter $\beta < 1$, the Nash bargaining solution must satisfy that $V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) -$

$V_2^R > 0$. Hence $q_{1,2} \geq s(r_{1,2}; \alpha)$ is impossible to hold in equilibrium. ■

Proposition 2 *When the retail market competitiveness is sufficiently low, the laissez-faire outcomes of private safe asset creation associated with level- k ($k \leq N - 1$) liquidity shocks are inefficient. That is, there exists a threshold $\hat{\tau} \in [0, \infty]$ ($\hat{\tau}$ could be ∞), such that*

$$\alpha_E < \alpha_P$$

whenever $\tau < \hat{\tau}$ (or $\kappa > \frac{1}{\hat{\kappa}}$).

Proof We proceed in two steps.

Step 1

First we show that when the retail market competitiveness τ is sufficiently, a positive mark-up will be charged in the liquidity that banks hit by liquidity shocks purchase from the wholesale market. For simplicity we show this result for the case $k = 1, N = 2$, proofs for the general cases follow the same arguments.

The trading deal established in equilibrium will be featuring a positive mark-up in liquidity price whenever the following is true: there exists a trading deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$ such that

$$\begin{aligned} V_1(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_1^R &> 0 \\ V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R &\geq 0 \end{aligned}$$

This is because if such a trading deal exists, then $\beta < 1$ implies that at the $(q_{1,2}, r_{1,2})$ that maximizes the joint surplus, we must have $V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R > 0$.

We now show that when the retail market competitiveness τ is sufficiently low, such a trading deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$ would always exist. Denote by (t_1^*, r_1^*) the optimal solution to

$$\max_{(t_1, r_1)} g(t_1, \alpha) + 2s(r_1; \alpha)$$

subject to the feasibility constraint

$$R \cdot (\alpha - t_1) \geq 2r_1 \cdot s(r_1; \alpha)$$

Then trading deal $(q_{1,2}^*, r_{1,2}^*)$ constructed as follows satisfies the above conditions: $r_{1,2}^* = r_1^*$ and $q_{1,2}^* = d(r_1^*; \alpha)$. Let us now verify.

With the established interbank trading deal being $(q_{1,2}^*, r_{1,2}^*)$, by construction, in equilibrium bank 1's optimal liquidity raising strategy will be (t_1^*, r_1^*) . Therefore, the equilibrium rate bank 2 needs to post in order to deliver $q_{1,2}^*$ wholesale liquidity is $r_2 = r_1^*$. As

such,

$$\begin{aligned} V_2(q_{1,2}^*, r_{1,2}^*) - V_2^R &= q_{1,2}^* \cdot (r_{1,2}^* - r_2) - 0 \\ &= 0 \end{aligned}$$

Finally, we need to show that when retail market competitiveness τ is sufficiently low, $V_1(q_{1,2}^*, r_{1,2}^*) - V_1^R > 0$. Suppose now the deal between the two banks breaks down and bank 2 will not be posting rates in the retail market anymore. In this case, if bank 1 can freely raise retail liquidity over the entire circle, then by construction the optimal strategy it should adopt is (t_1^*, r_1^*) . This gives us the upper bound of bank 1's reservation value from its outside option.

However, when retail market competitiveness τ becomes low enough, bank 1 will not be able to raise $q_{1,2}^* = d(r_1^*; \alpha)$ amount of retail liquidity from bank 2's territory by posting rate r_1^* . This is because the traveling cost (implied by the low τ) could be large enough to discourage some households that reside in bank 2's territory to deposit with bank 1 at rate r_1^* . When this happens, the reservation value of bank 1 would be strictly worse than its valuation of a deal $(q_{1,2}^*, r_{1,2}^*)$ with bank 2, i.e.,

$$V_1(q_{1,2}^*, r_{1,2}^*) - V_1^R > 0$$

Therefore, whenever the bargaining parameter $\beta < 1$, the wholesale liquidity would be traded with a positive mark-up if the retail market competitiveness is sufficiently low.

Step 2

This part of the proof is simple. When the retail market competitiveness is low so that in the decentralized economy wholesale liquidity trading are featured with positive mark-up, the panic-proof constraint that binds banks' ex-ante choice of α is inefficiently tight. This is because if such positive mark-up in the liquidity that banks purchase from the wholesale market can be eliminated, as can be guaranteed in the planner's solution, the constraint could be loosened and banks are able to make more illiquid lending on date 0 while staying immune to panic on date 1. In other words, when private safe asset creation in the decentralized economy is constrained inefficient when the retail market competitiveness τ is sufficiently low. ■

Corollary 1 *The laissez-faire equilibrium of bank money creation associated with level-1 liquidity shock is efficient; i.e.,*

$$\alpha_E = \alpha_P$$

if $\tau = \infty$ or $(\kappa = 0)$.

Proof When $k = 1$, only one bank will be receiving liquidity shocks on date 1. In this case, only the first inefficiency is active in affecting the private safe asset creation. Therefore, constrained efficiency (second best) can be achieved if this first inefficiency can be eliminated.

Again, we prove this result for case $k = 1, N = 2$ and the argument for general cases follows similarly. WLOG, suppose bank 1 is hit while bank 2 is not. From the analysis in our proof of Proposition 2, it suffices to show that when $\tau = \infty$, the liquidity traded in the wholesale market will always be priced at its marginal cost. To show this, we need to show that the following is true: there does not exist any $(\hat{q}_{1,2}, \hat{r}_{1,2})$, such that

$$\begin{aligned} V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R &> 0 \\ V_1(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_1^R &\geq 0 \end{aligned}$$

By way of contradiction, suppose there exists a $(\hat{q}_{1,2}, \hat{r}_{1,2})$ satisfying the above conditions. Because $\tau = \infty$, the rates \hat{r}_i both banks post in the retail market must be equalized, i.e., $\hat{r}_1 = \hat{r}_2$. Suppose in equilibrium the mass of households depositing with bank 2 is \hat{n}_2 , then $\hat{q}_{1,2} = \hat{n}_2 \cdot d(\hat{r}_2; \alpha)$. Bank 2's valuation of deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$ is $V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) = \hat{q}_{1,2} \cdot (\hat{r}_{1,2} - \hat{r}_2)$, therefore $V_2(\hat{q}_{1,2}, \hat{r}_{1,2}) - V_2^R > 0$ implies that $\hat{r}_{1,2} > \hat{r}_2$.

To calculate bank 1's reservation value from its outside option, suppose the deal between the two banks breaks down. In this case, a feasible option for bank 1 is to post the same rate \hat{r}_1 as it did in the original equilibrium. By doing so, bank 1 will be able to raise the same amount of retail liquidity from those households that deposited with them in the original equilibrium. Furthermore, it can raise $d(\hat{r}_2; \alpha)$ from each of the \hat{n}_2 households that used to deposit with bank 2.

Therefore, following this strategy bank 1 can raise the same amount of retail liquidity as it did in the original equilibrium, but the required date-2 payment is reduced. This is because $\hat{r}_{1,2} > \hat{r}_2 = \hat{r}_1$. As such, bank 1's reservation value from its outside option must be strictly higher than its valuation of the so-constructed trading deal $(\hat{q}_{1,2}, \hat{r}_{1,2})$, which is a contradiction. ■

A5. Inefficiency II: Proofs of Proposition 3 and Corollary 2

In Section A5, we provide detailed calculations for our analysis of the second efficiency in private safe asset creation: the ex-post over-use of the liquidity pool.

Proposition 3 *The laissez-faire equilibrium of bank money creation associated with level- k ($k \geq 2$) liquidity shocks is inefficient; i.e.,*

$$\alpha_E < \alpha_P$$

whenever $\tau > 0$ (or $\kappa < \infty$).

Proof We prove this result for case $k = 2, N = 2$, the argument for general cases follows similarly. Again, we proceed in two steps.

Step 1

The first step of our proof is to show that for any loan-making profile α chosen on date 0, the date-1 liquidity raising strategy (t^*, r^*) that solves the planner's constrained optimization problem cannot be committed in the decentralized economy, whenever retail market competitiveness $\tau > 0$.

From Proposition 1, we know that the optimal date-1 liquidity raising (t^*, r^*) in the planner's allocation satisfies

$$\frac{d(r^*; \alpha)}{d'(r^*; \alpha)} + r^* = R / \frac{\partial g(t^*, \alpha)}{\partial t}$$

Let us now check whether (t^*, r^*) can be committed when the economy is decentralized. WOLG, we consider the incentive compatibility problem for bank 1. With bank 2 posting rate r^* , the retail liquidity supply function faced by bank 1 is

$$\begin{aligned} s_1(r; \alpha, r^*) &= d(r; \alpha) \cdot [1 + 2\tau \cdot (V_H(r; \alpha) - V_H(r^*; \alpha))] \\ &\equiv d(r; \alpha) \cdot [1 + 2\tau \cdot F(r; \alpha)] \end{aligned}$$

where function $F(r; \alpha) \equiv V_H(r; \alpha) - V_H(r^*; \alpha)$. It's easy to show that this function F satisfies $F(r^*; \alpha) = 0$ and $\frac{\partial F(r^*; \alpha)}{\partial r} > 0$.

Facing this retail liquidity supply function, bank 1 solves

$$\max_{(t_1, r_1)} R(\alpha - t_1) - r_1 \cdot s_1(r_1; \alpha, r^*)$$

subject to the liquidity coverage constraint

$$g(t_1, \alpha) + s_1(r_1; \alpha, r^*) = \alpha$$

Form the Lagrangian:

$$\mathcal{L}(t_1, r_1) = R(\alpha - t_1) - r_1 \cdot s_1(r_1; \alpha, r^*) + \lambda \cdot [g(t_1, \alpha) + s_1(r_1; \alpha, r^*) - \alpha]$$

From the first order condition regarding t ,

$$-R + \lambda \cdot \frac{\partial g(t^*, \alpha)}{\partial t} = 0$$

we know that at (t^*, r^*) , the shadow value of relaxing the liquidity coverage constraint is

$$\lambda = R / \frac{\partial g(t^*, \alpha)}{\partial t}$$

For (t^*, r^*) to be committed, bank 1 must have no incentive to make the following deviation: posting a slightly higher rate $r^* + \epsilon$ and reduces the liquidation volume accordingly. For this to be true, the following needs to be true:

$$\frac{\partial \mathcal{L}(t^*, r^*)}{\partial r_1} \leq 0$$

However, whenever $\tau > 0$ this cannot be true. This is because

$$\begin{aligned} \frac{\partial \mathcal{L}(t^*, r^*)}{\partial r_1} &= -s_1(r^*; \alpha, r^*) - r^* \cdot s'_1(r^*; \alpha, r^*) + \lambda \cdot s'_1(r^*; \alpha, r^*) \\ &= s'_1(r^*; \alpha, r^*) \cdot \left[\lambda - r^* - \frac{s_1(r^*; \alpha, r^*)}{s'_1(r^*; \alpha, r^*)} \right] \\ &= s'_1(r^*; \alpha, r^*) \cdot \left[R / \frac{\partial g(t^*, \alpha)}{\partial t} - r^* - \frac{s_1(r^*; \alpha, r^*)}{s'_1(r^*; \alpha, r^*)} \right] \\ &= s'_1(r^*; \alpha, r^*) \cdot \left[\frac{d(r^*; \alpha)}{d'(r^*; \alpha)} - \frac{s_1(r^*; \alpha, r^*)}{s'_1(r^*; \alpha, r^*)} \right] \end{aligned}$$

where the last line of derivation makes use of the property that planner's optimal allocation (t^*, r^*) satisfies. When the retail market competitiveness $\tau > 0$,

$$\begin{aligned} s'_1(r^*; \alpha, r^*) &= d'(r^*; \alpha) \cdot [1 + 2\tau F(r^*; \alpha)] + d(r^*; \alpha) \cdot 2\tau \cdot \frac{\partial F(r^*; \alpha)}{\partial r} \\ &= d'(r^*; \alpha) + d(r^*; \alpha) \cdot 2\tau \cdot \frac{\partial F(r^*; \alpha)}{\partial r} \end{aligned}$$

where the second line of derivation makes use of the fact that $F(r^*; \alpha) = 0$. Therefore,

$$\begin{aligned} \frac{s_1(r^*; \alpha, r^*)}{s'_1(r^*; \alpha, r^*)} &= \frac{d(r^*; \alpha)}{d'(r^*; \alpha) + d(r^*; \alpha) \cdot 2\tau \cdot \frac{\partial F(r^*; \alpha)}{\partial r}} \\ &< \frac{d(r^*; \alpha)}{d'(r^*; \alpha)} \end{aligned}$$

where the second line of derivation is based on the fact that $\frac{\partial F(r^*, \alpha)}{\partial r} > 0$.

As such, whenever the retail market competitiveness $\tau > 0$, $\frac{\partial \mathcal{L}(t^*, r^*)}{\partial r_1} > 0$. This implies that the optimal liquidity raising strategy (t^*, r^*) implied in the planner's allocation cannot be committed in the decentralized economy. Specifically, at (t^*, r^*) each bank has incentive to reduce the liquidation of its long-term assets by posting a slightly higher rate in the retail market.

Step 2

Having established the result that banks in the decentralized economy cannot commit to adopting the socially optimal liquidity raising strategy on date 1, we now argue that the safe asset creation on date 0 would be inefficient when the economy is decentralized.

Suppose $\alpha_E = \alpha_P$ and denote by (t_E, r_E) the strategy each bank adopts at date 1. Since both banks must stay immune to panic under liquidity shocks on date 1, we have

$$\begin{aligned} g(t_E, \alpha_E) + d(r_E; \alpha_E) &\geq \alpha_E \\ R \cdot (\alpha_E - t_E) &\geq r_E \cdot d(r_E; \alpha_E) \end{aligned}$$

As such the date-1 liquidity raising profile $\{(t_E, r_E)\}$ is in the feasibility set of planner's allocation. Furthermore, following this feasible liquidity raising profile, the total amount of date-1 liquidity that the planner can raise satisfies

$$\sum_{i=1,2} g(t_E, \alpha_E) + d(r_E; \alpha_E) \geq 2\alpha_E$$

However, based on the results from Step 1, when the retail market competitiveness $\tau > 0$, the strategy (t_E, r_E) each individual bank adopts is strictly sub-optimal as solution to the planner's allocation problem. Therefore, the optimal amount of liquidity the planner can raise must satisfy

$$L(\alpha_E) > \sum_{i=1,2} g(t_E, \alpha_E) + d(r_E; \alpha_E) \geq 2\alpha_E$$

If $\alpha_E = \alpha_P$, then it implies that $L(\alpha_P) > 2\alpha_P$. This is a contradiction to Proposition 1. Therefore, whenever $\tau > 0$, the private safe asset creation in the decentralized economy must be inefficient; i.e., $\alpha_E < \alpha_P$. ■

Corollary 2 *The laissez-faire equilibrium of bank money creation associated with level-N liquidity shocks is efficient; i.e.,*

$$\alpha_E = \alpha_P$$

if $\tau = 0$ (or $\kappa = \infty$).

Proof From our proof of Proposition 3, we have

$$\frac{s_1(r^*; \alpha, r^*)}{s'_1(r^*; \alpha, r^*)} = \frac{d(r^*; \alpha)}{d'(r^*; \alpha)}$$

when the retail liquidity market is completely segmented; i.e., $\tau = 0$. Therefore, the socially optimal liquidity raising strategy (t^*, r^*) can be fully committed by each individual bank when the economy is decentralized, since $\frac{\partial \mathcal{L}(t^*, r^*)}{\partial r_1} = 0$. As such, the inefficiency induced by banks adopting sub-optimal liquidity raising strategy in the decentralized economy is completely eliminated.

In this case where the ex-post liquidity shock is systemic (i.e., $k = N$), only this second inefficiency will be relevant for private safe asset creation. Therefore, the equilibrium of safe asset creation achieved in the decentralized economy will be efficient if the retail market is completely segmented; i.e., $\tau = 0$. ■

A6. Implementability: Proofs of Lemma 4 and Proposition 4

In Section A6, we provide detailed calculations for our analysis of the equilibrium implementability.

Lemma 4 *A safe asset creation equilibrium $\{\alpha_E\}$ is automatically implementable if and only if*

$$\frac{\partial S_D(\alpha; \alpha_E)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+} \leq 0$$

That is, with all others choosing α_E at date 0, each individual bank can commit to choosing α_E in its ex-ante loan making.

Proof By definition, the symmetric equilibrium α_E of private safe asset creation must satisfy $S(\alpha_E) = 0$. In other words, if all banks choose α_E on date 0, then following the game played on date 1 after the liquidity shocks hit, banks receiving the liquidity shocks can just stay solvent.

On date 0, with all other banks choosing α_E , an individual bank i will not have incentive to make downward deviations; i.e., choosing any level of α that is below α_E . This is because doing so would result in strictly lowered date-2 profits.

On the other hand, an individual bank may have incentives to make upward deviations. This is because if it can still guarantee its safety under liquidity shocks on date 1 after making such upward deviations on date 0, strictly increased profits can be earned on date 2.

Therefore, a safe asset creation equilibrium $\{\alpha_E\}$ can be implemented in the decentralized economy if and only if the following is true: given all other banks choosing α_E on

date 0, an individual bank would become insolvent under liquidity shocks on date 1 if it makes any arbitrary upward deviation $\alpha_E + \epsilon$. As such, this is equivalent to the following condition

$$\frac{\partial S_D(\alpha; \alpha_E)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+} \leq 0$$

That is, at equilibrium α_E , any upward deviation made on date 0 would render the bank's solvency function $S_D(\alpha; \alpha_E)$ liquidity shocks on date 1 negative. ■

Proposition 4 *For each (N, k) , there exists a threshold τ_{AI} in retail market competitiveness τ , such that the symmetric safe asset creation equilibrium $\{\alpha_E\}$ that satisfies $S(\alpha_E; \tau) = 0$ is automatically implementable if and only if $\tau \leq \tau_{AI}$.*

Proof We prove this result for the case where $k = N = \infty$. The argument for general cases follows similarly. Let us construct the solvency function after deviation $S_D(\alpha; \alpha_E, \tau)$ first.

Given all other banks choosing α_E on date 0, for an individual bank that makes a deviation to choose any arbitrarily higher α , it solves the following problem on date 1:

$$S_D(\alpha; \alpha_E, \tau) = \max_{t, r} R(\alpha - t) - r \cdot s(r; \cdot)$$

subject to the feasibility constraint

$$g(t, \alpha) + s(r; \cdot) = \alpha$$

After making such a deviation in its date-0 loan making, the retail liquidity supply function $s(r; \cdot)$ for this bank is

$$s(r; \cdot) = d(r; \alpha) + d(r; \alpha_E) \cdot 2\tau \cdot F(r; \alpha_E)$$

where function $F(r; \alpha_E) \equiv V_H(r; \alpha_E) - V_H(r_E; \alpha_E)$ satisfies $F(r_E; \alpha_E) = 0$ and $F'(r_E; \alpha_E) > 0$. Here r_E is the rate each bank will be posting on date 1 in (old) equilibrium. In this case where $k = N = \infty$, after one bank makes a deviation in choosing α on date 0, the rate posted by other banks in the new equilibrium on date 1 would still be r_E .

Let us now form the Lagrangian

$$\mathcal{L}(\alpha; \alpha_E, \tau) = R(\alpha - t) - r \cdot s(r; \cdot) + \lambda \cdot [g(t, \alpha) + s(r; \cdot) - \alpha]$$

Following the same procedure as in our analysis of Proposition 4, we can show that at the

optimal solution, the following must hold

$$\lambda^* = r^* + \frac{s(r^*; \cdot)}{s'(r^*; \cdot)}$$

To study how the solvency on date 1 changes after an individual bank makes a arbitrarily small rightward deviation, let us examine the local property of function $S_D(\alpha; \alpha_E, \tau)$ at point $\alpha = \alpha_E$. Specifically, take the right derivative and by the envelope theorem we have

$$\frac{\partial S_D(\alpha; \alpha_E, \tau)}{\partial \alpha} = \frac{\partial \mathcal{L}(\alpha; \alpha_E, \tau)}{\partial \alpha}$$

and furthermore,

$$\frac{\partial \mathcal{L}(\alpha; \alpha_E, \tau)}{\partial \alpha} = R - r^* \cdot \frac{\partial d(r^*; \alpha)}{\partial \alpha} + \lambda^* \cdot \left[\frac{\partial g(t^*, \alpha)}{\partial \alpha} + \frac{\partial d(r^*; \alpha)}{\partial \alpha} - 1 \right]$$

in which we use the fact that $\frac{\partial s(r; \cdot)}{\partial \alpha} = \frac{\partial d(r; \alpha)}{\partial \alpha}$.

Based on Lemma 1, we can show that $\frac{\partial d(r; \alpha)}{\partial \alpha} = w$. We can thus further simplify the above equation to

$$\frac{\partial \mathcal{L}(\alpha; \alpha_E, \tau)}{\partial \alpha} = R - w \cdot r^* + \lambda^* \cdot \left[\frac{\partial g(t^*, \alpha)}{\partial \alpha} + w - 1 \right]$$

Evaluating the above partial derivative at $\alpha \rightarrow \alpha_E^+$, we have

$$\begin{aligned} \frac{\partial S_D(\alpha; \alpha_E, \tau)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+} &= R - w \cdot r_E + \lambda^* \cdot \left[\frac{\partial g(t_E, \alpha_E)}{\partial \alpha_E} + w - 1 \right] \\ &= R - w \cdot r_E + \left(r_E + \frac{s(r_E; \cdot)}{s'(r_E; \cdot)} \right) \cdot \left[\frac{\partial g(t_E, \alpha_E)}{\partial \alpha_E} + w - 1 \right] \end{aligned}$$

where we make use the fact that when $\alpha \rightarrow \alpha_E$, the optimal liquidity raising strategy (t^*, r^*) that the deviating bank will adopt on date 1 converges to (t_E, r_E) .

Similar to our analysis for Proposition 4, we can show that

$$\frac{s(r_E; \cdot)}{s'(r_E; \cdot)} = \frac{d(r_E; \alpha_E)}{d'(r_E; \alpha_E) + d(r_E; \alpha_E) \cdot 2\tau \cdot \frac{\partial F(r_E; \alpha_E)}{\partial r}}$$

Therefore, as long as $\frac{\partial g(t^*, \alpha)}{\partial \alpha} + w - 1 < 0$, which can be guaranteed by regulatory conditions on model parameters, $\frac{\partial S_D(\alpha; \alpha_E, \tau)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+}$ will be increasing in the retail market competitiveness τ for any given level of α_E . This monotonicity of the local deviation at equi-

librium level of loan making $\frac{\partial S_D(\alpha; \alpha_E, \tau)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+}$ regarding retail market competitiveness implies that there would be a threshold τ_{AI} that governs whether or not the ex-ante loan making α_E satisfying $S(\alpha_E)$ can be implemented in the decentralized economy.

It would be interesting to examine the two extreme cases. When the retail liquidity market is perfectly competitive, i.e., $\tau = \infty$, we have $\frac{s(r_E; \cdot)}{s'(r_E; \cdot)} = 0$. Therefore in this case

$$\begin{aligned} \frac{\partial S_D(\alpha; \alpha_E, \tau)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+} &= R - w \cdot r_E + r_E \cdot \left[\frac{\partial g(t_E, \alpha_E)}{\partial \alpha_E} + w - 1 \right] \\ &= R - r_E \cdot \left[1 - \frac{\partial g(t_E, \alpha_E)}{\partial \alpha_E} \right] \end{aligned}$$

Therefore, whenever in the date-1 equilibrium $R > r_E \cdot \left[1 - \frac{\partial g(t_E, \alpha_E)}{\partial \alpha_E} \right]$, which can be guaranteed by certain conditions on model parameters, the safe asset creation equilibrium associated with perfectly competitive retail market cannot be automatically implemented in the decentralized economy.

On the other hand, when the retail market is completely segmented, i.e., $\tau = 0$, we have $\frac{s(r_E; \cdot)}{s'(r_E; \cdot)} = \frac{d(r_E; \alpha_E)}{d'(r_E; \alpha_E)}$. Applying the envelope theorem to the planner's allocation problem on date 1 as in Section A3, it's easy to see that the individual deviation $\frac{\partial S_D(\alpha; \alpha_E, \tau)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+}$ when $\tau = 0$ is perfectly aligned with the local deviation in planner's problem $\frac{\partial \mathcal{L}^P(\alpha)}{\partial \alpha} \Big|_{\alpha \rightarrow \alpha_E^+}$. As such, completely segmented retail market $\tau = 0$ guarantees that ex-ante loan making α_E satisfying $S(\alpha_E)$ can be fully committed in the decentralized economy. ■

A7. Mixed Effect of Competitiveness τ : Proofs of Proposition 5 and 6

In Section A7, we provide detailed calculations for our analysis of how the banking market competitiveness τ affects the efficiency of private safe asset creation in the decentralized economy.

Proposition 5 *The laissez-faire equilibrium outcome of private safe asset creation associated with level-1 liquidity shocks $\alpha_E(\tau)$ is an increasing function in banking competitiveness τ .*

Proof We prove this result for the case $k = 1, N = 2$. The argument for general cases follows similarly. WOLOG, suppose bank 1 is hit by the liquidity shock while bank 2 is not. For any level of α both banks choose on date 0, in the equilibrium on date 1, the interbank trading deal $(q_{1,2}, r_{1,2})$ is determined as

$$(q_{1,2}^*, r_{1,2}^*) = \arg \max_{(q_{1,2}, r_{1,2})} \left[V_1(q_{1,2}, r_{1,2}; \alpha, \tau) - V_1^R(\alpha, \tau) \right]^\beta \cdot \left[V_2(q_{1,2}, r_{1,2}; \alpha, \tau) - V_2^R(\alpha, \tau) \right]$$

Functions $V_i(q_{1,2}, r_{1,2}; \alpha, \tau)$ and $V_i^R(\alpha, \tau)$ are formulated as in our analysis in Section A2.

The retail market competitiveness τ affects the elasticity of the retail liquidity supply, through which it affects the valuation functions in bargaining. Importantly, for any given α , we can show the following properties of functions $V_i(q_{1,2}, r_{1,2}; \alpha, \tau)$ and $V_i^R(\alpha, \tau)$:

- 1) higher τ improves bank 1's reservation value, i.e., $\frac{\partial V_1^R(\alpha, \tau)}{\partial \tau} \geq 0$;
- 2) for any arbitrary $(q_{1,2}, r_{1,2})$ (in the equilibrium-relevant set), higher τ improves payoff for bank 1, i.e., $\frac{\partial V_1(q_{1,2}, r_{1,2}; \alpha, \tau)}{\partial \tau} > 0$;

In other words, increased retail market competitiveness τ improves bank 1's reservation value from its outside option when no deal could be made, as well as improves bank 1's valuation from any trading deal that may arise in equilibrium. Following the analysis in [Thomson \(1987\)](#) about monotonicity of bargaining solutions regarding disagreement point, these properties we established above implies that holding fixed the α chosen at date 0, increased τ improves the equilibrium payoff of bank 1. That is, for any $\tau > \tau'$, we have

$$V_1(q_{1,2}^*(\tau), r_{1,2}^*(\tau); \alpha, \tau) \geq V_1(q_{1,2}^*(\tau'), r_{1,2}^*(\tau'); \alpha, \tau')$$

Finally, since in equilibrium banks' ex-ante choice of α must satisfy $V_1(q_{1,2}^*(\tau), r_{1,2}^*(\tau); \alpha_E, \tau) = 0$, it follows that $\alpha_E(\tau)$ is increasing in τ . This is because holding τ fixed, $V_1(q_{1,2}^*(\tau), r_{1,2}^*(\tau); \alpha, \tau)$ as a function of α is downward sloping at point $\alpha_E(\tau)$. The above inequality suggests that for any $\tau > \tau'$, function $V_1(q_{1,2}^*(\tau), r_{1,2}^*(\tau); \alpha, \tau)$ is above function $V_1(q_{1,2}^*(\tau'), r_{1,2}^*(\tau'); \alpha, \tau')$. Therefore, the intercept of the former must locate to the right of the latter. ■

Proposition 6 *The laissez-faire equilibrium outcome of private safe asset creation associated with level- N liquidity shocks $\alpha_E(\tau)$ is a decreasing function in banking competitiveness τ .*

Proof We prove this result for the case $k = 2, N = 2$. The argument for general cases follows similarly.

Let us first hold fixed the choice of α made by both banks on date 0. On date 1, taking as given the rate r_E posted by the other bank, each bank chooses its liquidity raising strategy (t, r) to maximize the date-2 solvency

$$S(\alpha, \tau) \equiv \max_{(t, r)} R(\alpha - t) - r \cdot s(r; \alpha, r_E)$$

subject to liquidity coverage constraint

$$g(t, \alpha) + s(r; \alpha, r_E) = \alpha$$

Here the retail liquidity supply function each bank faces is

$$s(r; \alpha, r_E) = d(r; \alpha) [1 + 2\tau \cdot F(r; \alpha, r_E)]$$

where $F(r; \alpha, r_E) \equiv V_H(r; \alpha) - V_H(r_E; \alpha)$ satisfies $F(r_E; \alpha, r_E) = 0$ and $\frac{\partial F(r_E; \alpha, r_E)}{\partial r} > 0$.

Form the Lagrangian

$$\mathcal{L}(\alpha, \tau) = R(\alpha - t) - r \cdot s(r; \cdot) + \lambda [g(t, \alpha) + s(r; \cdot) - \alpha]$$

Then we have

$$\lambda^* = r^* + \frac{s(r^*; \cdot)}{s'(r^*; \cdot)}$$

where

$$\frac{s(r; \cdot)}{s'(r; \cdot)} = \frac{d(r; \alpha)}{d'(r; \alpha) + d(r; \alpha) \cdot 2\tau \cdot \frac{\partial F(r; \alpha, r_E)}{\partial r}}$$

Apply the envelope theorem, we have

$$\frac{\partial \mathcal{L}(\alpha, \tau)}{\partial \tau} = -r^* \cdot \frac{\partial s(r^*; \cdot)}{\partial \tau} + \frac{\partial}{\partial \tau} \left[\left(r^* + \frac{s(r^*; \alpha)}{s'(r^*; \alpha)} \right) \cdot s(r^*; \cdot) \right]$$

In the above equation, we have

$$\frac{\partial s(r^*; \cdot)}{\partial \tau} = 2d(r^*; \alpha) \cdot F(r^*; \alpha, r_E)$$

But since in equilibrium $r^* = r_E$, thus we have

$$\frac{\partial s(r^*; \cdot)}{\partial \tau} = 0$$

where we use the fact that $F(r_E; \alpha, r_E) = 0$. Based on this result, we can simplify the above equation as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}(\alpha, \tau)}{\partial \tau} &= s(r^*; \alpha, r_E) \cdot \frac{\partial}{\partial \tau} \left(r^* + \frac{s(r^*; \alpha)}{s'(r^*; \alpha)} \right) \\ &= s(r^*; \alpha, r_E) \cdot \frac{\partial}{\partial \tau} \left(\frac{d(r^*; \alpha)}{d'(r^*; \alpha) + d(r^*; \alpha) \cdot 2\tau \cdot \frac{\partial F(r^*; \alpha, r_E)}{\partial r}} \right) \end{aligned}$$

Furthermore, we have

$$\frac{\partial}{\partial \tau} \left(\frac{d}{d' + d \cdot 2\tau \cdot \frac{\partial F(r^*; \alpha, r_E)}{\partial r}} \right) = \frac{-d^2 \cdot \frac{\partial F(r_E; \alpha, r_E)}{\partial r}}{\left(d' + d \cdot 2\tau \cdot \frac{\partial F(r^*; \alpha, r_E)}{\partial r} \right)^2} < 0$$

where we use the fact that $\frac{\partial F(r_E; \alpha, r_E)}{\partial r} > 0$. As such, we prove the following important result: for any fixed α chosen on date 0,

$$\frac{\partial S(\alpha, \tau)}{\partial \tau} < 0$$

Based on this result, it immediately follows that the private safe asset creation equilibrium $\alpha_E(\tau)$ in the decentralized economy is decreasing in the retail market competitiveness τ .

B. Model Extensions and Alternative Specifications

B1. Relaxing Assumptions on Wholesale Liquidity Trading

In our model, to focus on the ex-ante creation and the associated under-production problem of the commonly shared liquidity pool in the economy, we make a simplifying assumption that each bank i 's date-0 liquid asset hoarding $1 - \alpha_i$ can only be used to cover its own liquidity needs on date 1. In other words, the ex-post wholesale liquidity trading on date 1 does not involve these liquid cash banks hoarded on date 0.

In this section, we show that this simplifying assumption on ex-post interbank trading is innocuous to our analysis of the efficiency of private safe asset creation. Specifically, we relax this assumption by allowing banks' ex-ante liquid cash hoarding to be traded in ex-post wholesale market and show that the main results of the paper still hold (or even get strengthened). Let us first start with the planner's problem.

The Planner's Problem

In the planner's optimal allocation for safe asset creation in the economy, because more liquidity resources can be deployed on date 1, it's natural to expect that more ex-ante loan making on date 0 could be supported. Specifically, for any loan-making profile $\{\alpha_n\}$ chosen on date 0, after a liquidity shock ω hits on date 1, the planner solves the following problem

$$L(\{\alpha_n\}, \omega) \equiv \max_{\{t_i\}_{i \in \omega^c}, \{r_n\}} \sum_{i \in \omega} g(t_i, \alpha_i) + \sum_{n=1}^N d(r_n; \alpha_n) + \sum_{j \in \omega^c} (1 - \alpha_j)$$

subject to the feasibility constraint:

$$\sum_{i \in \omega} R \cdot (\alpha_i - t_i) \geq \sum_{n=1}^N r_n \cdot d(r_n; \alpha_n) + \sum_{j \in \omega^c} 1 \cdot (1 - \alpha_j)$$

In solving this date-1 liquidity raising problem, the planner can optimally choose the liquidation profile $\{t_i\}_{i \in \omega}$ of banks in ω and the the profile of retail liquidity rates $\{r_n\}$ posted by all banks in the economy. Furthermore, the liquid cash holding by each bank $j \in \omega^c$ can also be deployed to cover the liquidity shortage of banks in ω . On date 2, these liquid cash lent on date 1 needs to be paid back at the price according to the outside option of bank $j \in \omega^c$, which is 1.

The characterization of the planner's optimal allocation in liquidity raising on date 1 proceeds exactly the same as in our analysis in Proposition 1. Specifically, we can show that the optimal safe asset creation α_P in the planner's allocation is determined by

$$k \cdot g(t^*, \alpha_P) + N \cdot d(r^*; \alpha^P) = N \cdot \alpha^P - (N - k)$$

where the optimal liquidity raising strategy (t^*, r^*) is determined by

$$\begin{aligned} \frac{d(r^*; \alpha^P)}{d'(r^*; \alpha^P)} + r^* &= R / \frac{\partial g(t^*, \alpha^P)}{\partial t} \\ k \cdot R (\alpha^P - t^*) &= N \cdot r^* \cdot d(r^*; \alpha^P) + (N - k) \cdot (1 - \alpha_P) \end{aligned}$$

Safe Asset Creation in the Decentralized Economy

When the economy is decentralized, the liquid cash held by banks not by liquidity shocks on date 1 would in general not be at the price that reflects lending banks' outside option. This is because when borrowing banks' reservation value from their outside options is low, lending banks will be able to charge a positive mark-up in the trading price of the liquid cash they lend.

As such, by serving as the outside option for borrowing banks in their bargaining with liquidity-surplus banks on date 1, the pool of liquidity that is commonly shared in the economy becomes even more importantly. This is because the existence of such common liquidity pool in the economy improves borrowing banks' bargaining position and thus allows them to purchase the privately owned liquidity (the date-0 cash holding) from liquidity-surplus banks at more fair prices.

We elaborate the argument above by analyzing a special case in which $k = 1, N = 2$. WOLOG, assume that bank 1 is hit by the liquidity shock on date 1 while bank 2 is not.

Suppose both banks choose α on date 0. Then after a liquidity shock hits bank 1 on date 1, the trading deal $(q_{1,2}^*, r_{1,2}^*)$ it establishes with bank 2 is determined as the bargaining solution that solves

$$(q_{1,2}^*, r_{1,2}^*) = \arg \max_{(q_{1,2}, r_{1,2})} \left[V_1(q_{1,2}, r_{1,2}; \alpha, \tau) - V_1^R(\alpha, \tau) \right]^\beta \cdot \left[V_2(q_{1,2}, r_{1,2}; \alpha, \tau) - V_2^R(\alpha, \tau) \right]$$

Now being able to lend out its liquid cash holding, bank 2's valuation from an arbitrary trading deal $(q_{1,2}, r_{1,2})$ (such that $q_{1,2} > 1 - \alpha$) is

$$V_2(q_{1,2}, r_{1,2}; \alpha, \tau) = q_{1,2} \cdot r_{1,2} - (1 - \alpha) - r_2 \cdot [q_{1,2} - (1 - \alpha)]$$

where the rate r_2 it posts satisfies $s(r_2; \cdot) = q_{1,2} - (1 - \alpha)$.

The wholesale liquidity bank 2 delivers to bank 1, the total volume of which is $q_{1,2}$, consists of two parts. The first part is bank 2's own liquid cash holding, $1 - \alpha$. The marginal cost of lending out this part of liquidity is the return of these liquid cash to bank 2, which is unity. The second part of the wholesale liquidity is the same as in our original model, which are raised by bank 2 from the retail market by posting a rate r_2 . The marginal cost of this part of wholesale liquidity, the quantity of which is $q_{1,2} - (1 - \alpha)$, is the rate r_2 that bank 2 needs to post in the retail market in order to raise this quantity of liquidity.

When the retail market competitiveness τ is low, bank 1's reservation from its outside option, which is to reach out to the retail market when no deals could be made with bank 2, would be low due to the inelastic retail liquidity supply. This weakened outside option allows bank 2 to extract rents from the bargaining and thus drives up the wholesale liquidity price $r_{1,2}$ above the marginal costs of its two components.

Accordingly, more competitive retail liquidity market (higher τ) improves borrowing banks' outside option and thus reduces the mark-ups in wholesale liquidity price. With the privately owned liquidity (date-0 cash holding) being able to be traded in the wholesale market, this improvement in the borrowing banks' outside option becomes even more valuable. This is so because a competitive retail liquidity market not only brings down the mark-up in the wholesale liquidity that raised from the retail market, it also brings down the mark-up lending banks charged on their privately owned liquidity.

In this sense, when banks' privately owned liquidity can also be traded in the wholesale market, the under-production problem becomes even more acute. This is because a natural hold-up problem exists on banks' privately owned liquidity and the alleviation of such natural hold-up relies crucially on the creation of a commonly shared liquidity

pool in the economy. As such, it becomes even more important to ensure that sufficient incentives can be provided for the creation of such common liquidity pool in the economy.