Game Theory 2: Extensive-Form Games and Subgame Perfection

Dynamics in Games

How should we think of strategic interactions that occur in sequence?

Who moves when?

And what can they do at different points in time?

How do people react to different histories?

Modeling Games with Dynamics

Players

Player function

- Who moves when

Terminal histories

- Possible paths through the game

Preferences over terminal histories

Strategies

A strategy is a complete contingent plan

Player i's strategy specifies her action choice at each point at which she could be called on to make a choice

An Example: International Crises

Two countries (A and B) are competing over a piece of land that B occupies

Country A decides whether to make a demand

If Country A makes a demand, B can either acquiesce or fight a war

If A does not make a demand, B keeps land (game ends)

A's best outcome is Demand followed by Acquiesce, worst outcome is Demand and War

B's best outcome is No Demand and worst outcome is Demand and War

An Example: International Crises

A can choose: Demand (D) or No Demand (ND)

B can choose: Fight a war (W) or Acquiesce (A)

Preferences

$$u_A(D, A) = 3 > u_A(ND, A) = u_A(ND, W) = 2 > u_A(D, W) = 1$$

$$u_B(ND, A) = u_B(ND, W) = 3 > u_B(D, A) = 2 > u_B(D, W) = 1$$

How can we represent this scenario as a game (in strategic form)?

International Crisis Game: NE

Country B

Country A

	W	A
D	$1, 1$	$3^{\checkmark}, 2^{\checkmark}$
ND	$2^{\checkmark}, 3^{\checkmark}$	$2, 3^{\checkmark}$

- Is there something funny here?
- Is there something funny here?
- Specifically, (ND, W)?
- Is there something funny here?
- Specifically, (ND, W)?

Non-Credible Threats

The equilibrium (ND, W) depends on a "non-credible threat"

Once A makes a demand, B does not want to fight a war

But to rule out such behavior, we need a stronger solution concept

One that incorporates the fact that actions are taken in sequence

Why Rule out Non-credible Threats

Equilibrium as a steady state

War is only a best-response for B because when no demand is made, B is indifferent

If A accidentally made a demand, war is not a sequential best-response for B. B should acquiesce instead

- Read the strategy W as "if A makes a demand, I will go to war"

Subgame Perfect Nash Equilibrium

A strategy specifies what a player will do at every decision point

- Complete contingent plan

Strategy in a SPNE must be a best-response at each node, given the strategies of other players

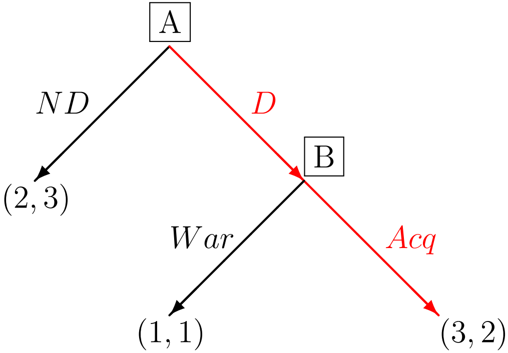
Backward Induction

But First!

Let's introduce a way of incorporating the timing of actions into the game explicitly

Use a game tree to represent the sequential aspect of choices

An Example: International Crises

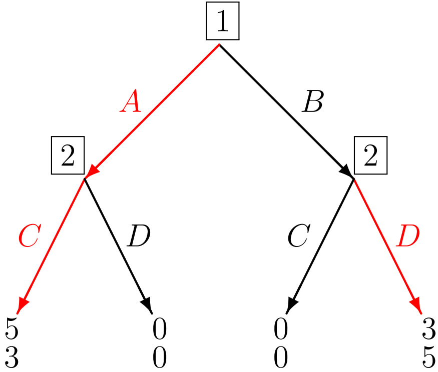


Another Example

		Player 2	
		C	D
Player 1	A	$5^{\checkmark}, 3^{\checkmark}$	$0, 0$
	B	$0, 0$	$3^{\checkmark}, 5^{\checkmark}$

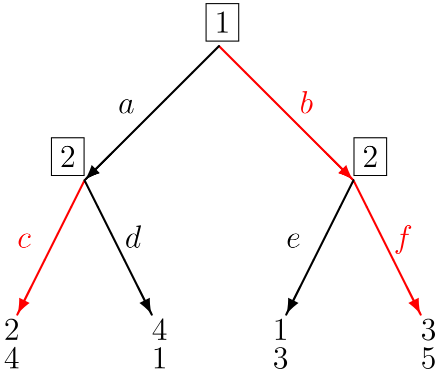
Suppose that player 1 moves first and player 2 moves second.

Another Example



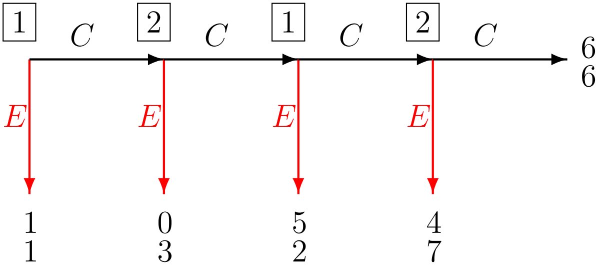
SPNE: $(A, (C, D))$

Practice Game



SPNE: $(b, (c, f))$

The Centipede Game

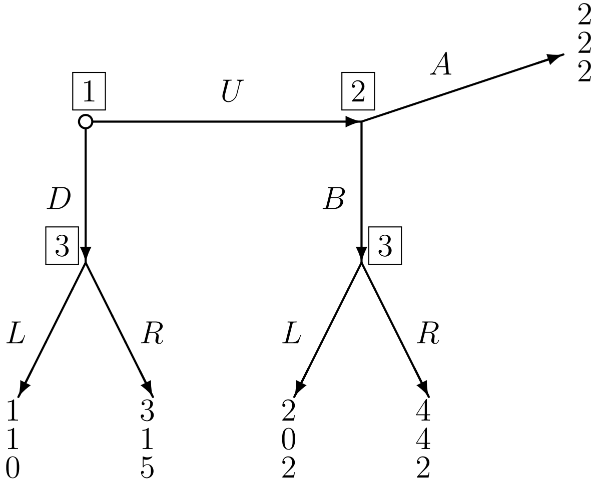


Unique SPNE: $((E, E), (E, E))$

Equilibrium payoffs $(1, 1)$

Pareto dominated by 3 outcomes!

Multiple Equilibria



SPNE 1: (D, A, (R,L))
SPNE 2: (U,B,(R,R))

Buying Votes (Osborne)

Legislature with k members (k odd) votes on two bills, X vs. Y

Interest groups I_X and I_Y try to buy votes in favor of preferred bill

First I_X makes payments (could be 0) to each legislator

- $(x_1, \ldots, x_k)$

Then I_Y makes payments (could be 0) to each legislator

- $(y_1, \ldots, y_k)$

Legislators vote for whomever paid them more (ties go to Y)

Payoffs

Group I_X values at $V_X > 0$ and Y at 0

Group I_Y values Y at $V_Y > 0$ and X at 0

Payoff is value from bill minus sum of payments made

Example 1: $k = 3$ and $V_X = V_Y = 300$

Start by finding I_Y's best response to (x_1, x_2, x_3)

Suppose x_1 is the smallest and x_2 is the second smallest

If $x_1 + x_2 < 300$, then want to buy the two cheapest legislators

- $y_1 = x_1$, $y_2 = x_2$, $y_3 = 0$

If $x_i + x_j \geq 300$, not worth it ot buy the vote

- $y_1 = y_2 = y_3 = 0$

Example 1: I_X's best response

In order to win, I_X must spend at least 300 on each pair of two legislators

The cheapest way to do this is $x_1 = x_2 = x_3 = 150$

But then I_X's payoff is

$$300 - 450 < 0$$

So I_X will instead choose $x_1 = x_2 = x_3 = 0$

SPNE of Example 1

$x_1 = x_2 = x_3 = 0$

If the two smallest payments by I_X sum to less than 300, I_Y matches them and offers 0 to the third legislator

If the two smallest payments by I_X sum to 300 or more, I_Y offers 0 to all legislators

Example 2: $k = 3$, $V_X = 300$, and $V_Y = 100$

Start by finding I_Y's best response to (x_1, x_2, x_3)

Suppose x_1 is the smallest and x_2 is the second smallest

If $x_1 + x_2 < 100$, then want to buy the two cheapest legislators

- $y_1 = x_1$, $y_2 = x_2$, $y_3 = 0$

If $x_i + x_j \geq 100$, not worth it ot buy the vote

- $y_1 = y_2 = y_3 = 0$

Example 2: I_X's best response

In order to win, I_X must spend at least 100 on each pair of two legislators

The cheapest way to do this is $x_1 = x_2 = x_3 = 50$

I_X's payoff from this is

$$300 - 150 > 0$$

SPNE of Example 2

$x_1 = x_2 = x_3 = 50$

If the two smallest payments by I_X sum to less than 100, I_Y matches them and offers 0 to the third legislator

If the two smallest payments by I_X sum to 100 or more, I_Y offers 0 to all legislators

Second move advantage

- When values are equal, second move wins
- For first mover to win, has to value winning a lot more than first move

The Ultimatum Game

Players 1 and 2 are bargaining over a dollar

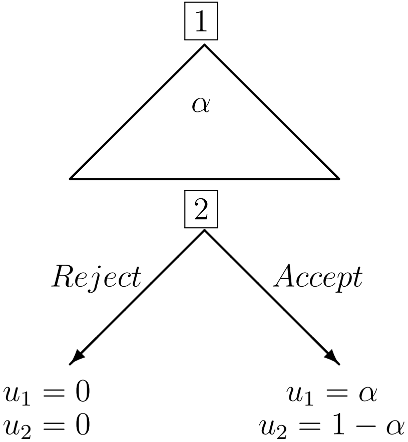
Player 1 makes a take it or leave it offer of a division $(\alpha, 1-\alpha)$

Player 2 says accepts or rejects

If Player 2 accepts, each makes a payoff equal to her share of the dollar

If Player 2 rejects, each makes 0

The Ultimatum Game in a Tree



Subgame Perfect Nash Equilibrium

Subgame Perfect Nash Equilibrium is a refinement of Nash Equilibrium

It rules out equilibria that rely on incredible threats in a dynamic environment

All SPNE are identified by backward induction