# Accountability and Information in Elections 

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#### Abstract

Elections are thought to improve voter welfare through two channels: effective accountability (i.e., providing incentivesforpoliticians to take costly effort) and electoral selection (i.e., retaining politicians with characteristics voters value). We show that there may be a trade-off between these two channels. Higher levels of effective accountability may hinder the voters' ability to learn about the politicians. In turn, this may hinder electoral selection and be detrimental to voter welfare. This is because increasing effective accountability directly impacts how informative governance outcomes are about an incumbent's type. We show that, if politicians' effort and type are local substitutes (resp. complements) in the production of governance outcomes, an increase in effective accountability corresponds to a decrease (resp. increase) in Blackwell (1951) informativeness. We also show that effective accountability can vary even absent institutional variation. In particular, we provide necessary and sufficient conditions for there to be multiple equilibria that differ in terms of both effective accountability and electoral selection. Overall, our findings have implications for voter behavior, the efficacy of institutional reforms, and voter welfare. (JEL D72, D83)


Electoral accountability is an important feature of democratic societies. The hope is that giving citizens formal accountability-i.e., the formal right to retain or replace policymakers-will lead to policy outcomes that are better for citizen welfare.

Formal accountability is thought to be linked to welfare improvements for two reasons (see Fearon 1999). First, when voters have the formal right to retain or replace policymakers, the politicians' actions are shaped by how they anticipate the

[^0]voters will respond. That is, formal accountability creates what we will call effective accountability. Second, voters have the ability to retain politicians whose characteristics are best aligned with their own interests. That is, formal accountability creates electoral selection.

An important precursor for electoral selection is the voters' ability to learn about the politician's characteristics. This paper focuses on how changes in the level of effective accountability impact the voters' ability to do so-i.e., on the informational consequences of changing the level of effective accountability. It shows that such informational effects can have counterintuitive implications for both voter behavior and voter welfare.

To illustrate the informational effect, consider an incumbent politician implementing a new policy. High quality politicians have better ideas and low quality politicians have worse ideas. The ultimate policy outcome depends on both the politician's idea and the effort she puts into implementing the policy. Voters observe neither her quality nor her effort. While the incumbent knows how much effort she puts in, she does not know her own quality-that is, she does not know how good her idea is. Both the incumbent and voter observe the the governance outcome, i.e., the results of the final policy. Thus, the governance outcome serves as a signal of the incumbent's quality. Importantly, this signal depends on the effort she put into implementing the policy.

Notice, in the above, it is governance outcomes and not actions that serve as a signal of the incumbent's characteristics. This is an important point: The trade-off we focus on does not arise because the incumbent has an incentive to signal her type by choosing a costly action. That is, it does not arise because there is correlation between the incumbent's action and her type. In particular, we focus on an informational effect that arises within a signal-jamming model in the spirit of Holmström's (1999) career concerns model. (A long tradition in political economy uses such models to study the agency relationship between a voter and a politician. See, e.g., Lohmann 1998; Persson and Tabellini 2000; Alesina and Tabellini 2007, 2008.) In these models, politicians do not know their types and so all types must choose the same action. Thus, there is no correlation between action and type.

Nonetheless, the voters' ability to learn about the characteristics of politicians will quite generally depend on the level of effective accountability. From the voters' perspective, the governance outcome (which, again, depends on the incumbent's characteristics and actions) is the realization of a statistical experiment that generates information about the incumbent. Different levels of effective accountability generate different statistical experiments. Hence, how informative governance outcomes are about the incumbent depends on the level of effective accountability. In particular, we show that, when the politicians' actions and characteristics are local substitutes (resp. complements) in the production of governance outcomes, an increase in effective accountability corresponds to a decrease (resp. increase) in Blackwell (1951) informativeness. We next preview the implications for voter behavior and voter welfare.

Voter Behavior.-An increase in effective accountability induces an improvement in the distribution of governance outcomes. As a result, when effective
accountability is higher, the voter requires higher levels of public goods to infer neutral news about the incumbent's characteristics. It is then natural to conjecture that, when effective accountability is higher, the voter will hold the incumbent to a more stringent performance standard. But this need not be the case. There is a potentially countervailing effect that arises from the fact that changes in effective accountability alter the informativeness of governance outcomes. Whether this informational effect offsets the direct effect of increasing effective accountability depends on (i) whether actions and characteristics are local complements or substitutes and (ii) the probability that incumbents and challengers are high ability. (We later explain why.) We show that an increase in effective accountability may correspond to a less stringent performance standard.

This fact has important implications for the empirical literature on accountability. In particular, there is an influential body of scholarship that makes inferences about the level of effective accountability by observing facts about voter behavior. (See, e.g., Fiorina 1981; Achen and Bartels 2004, 2006; Healy, Malhotra, and Mo 2010.) The ability to make such inferences rests on the natural conjecture that the voter adopts a performance standard that is monotone in the level of effective accountability (i.e., that the voter holds the incumbent to a more stringent performance standard if there is a higher level of effective accountability). Our results show that the relationship is more subtle; this, in turn, has implications for this empirical literature's attempts to identify the level of effective accountability.

Voter Welfare.-We show that there may be a trade-off between effective accountability and electoral selection. Higher levels of effective accountability may reduce the voters' ability to learn about the characteristics of politicians, thereby hindering electoral selection. When this is the case, there are nontrivial consequences for voter welfare. Specifically, if higher levels of effective accountability correspond to lower levels of informativeness, they can also be associated with lower levels of voter welfare.

Our results show that there is a trade-off between electoral selection and effective accountability if and only if the politician's effort and characteristics are local substitutes (in the production of governance outcomes). So, in the case of complements, higher levels of effective accountability unambiguously improve voter welfare. But, in the case of substitutes, the voter may be worse off with a higher level of effective accountability and a lower level of electoral selection.

This raises the question of how to improve voter welfare. The standard approach is to think of variation in effective accountability as a result of variation in the institutions of formal accountability. That is, varying term length, compensation, campaign finance rules, staff resources, etc., may lead to variation in effective accountability. (See, e.g., Persson and Tabellini 2000; Maskin and Tirole 2004; Besley 2006; Ashworth and Bueno de Mesquita 2006; Gehlbach 2007; Myerson 2006 for theoretical studies and Besley and Case 1995, 2003; Persson and Tabellini 2000; Huber and Gordon 2004; Besley 2004; Alt, Bueno de Mesquita, and Rose 2011; de Janvry, Finan, and Sadoulet 2012; Gagliarducci and Nannicini 2013; Ferraz and Finan 2011 for empirical studies.) Our formal model captures institutional variation only in reduced form-in particular, by varying politicians' benefits of holding office and costs from actions.

But, importantly, we also show that varying institutions is not a prerequisite for varying the level of effective accountability (and electoral selection). Within a fixed set of institutions, there can be variation in the behavior of politicians and voters if there are multiple equilibria that differ in terms of both their level of effective accountability and in electoral selection. We provide necessary and sufficient conditions for this to be the case. We go on to show that any production environment is consistent with such multiplicity: For any production environment, there are preferences of the incumbent that give rise to multiple equilibria that differ in their level of effective accountability.

In our model, equilibrium multiplicity is tied to the voter's desire to select good types. The incumbent's early behavior influences the voter's ability to infer information about the incumbent's characteristics; it is this fact that generates the possibility for multiplicity. (This fact is reminiscent of the multiplicity result in Dewatripont, Jewitt, and Tirole 1999. We discuss the relationship between our result and theirs in Section V.) This raises the possibility that different societies-with identical insti-tutions-might differ in both their level of effective accountability and electoral selection. We discuss these implications further in Section VI.

The paper proceeds as follows. Section I presents a canonical model of electoral accountability. Section II describes the basic properties of equilibrium. Section III characterizes when there is and is not a trade-off between effective accountability and electoral selection and shows that higher levels of effective accountability do not necessarily correspond to higher voter welfare. Section IV describes implications for voter behavior. Section V shows that variation in electoral behavior need not derive from institutional variation by providing necessary and sufficient conditions for the existence of multiple equilibria that differ in their level of effective accountability and electoral selection. Section VI discusses the normative implications of our results.

## I. The Model

There is an Incumbent $(I)$, a Challenger $(C)$, and a Voter $(V)$. We refer to each Politician $(P)$ as "she" and the Voter as "he." In each of two periods, the Voter receives a level of public goods. This level is a function of the effort taken by the Politician in office, the type of the Politician in office, and an idiosyncratic shock.

The Politician in office chooses a level of effort in $A \subseteq \mathbb{R}_{+}$. The set $A$ is closed, with $\underline{a}$ its smallest element. The set of types is $\Theta=\{\underline{\theta}, \bar{\theta}\}$, where $\underline{\theta}$ is the low type and $\bar{\theta}>\underline{\theta}$ is the high type. ${ }^{1}$ Write $\pi_{P} \in(0,1)$ for the probability that Politician $P$ is type $\bar{\theta}$. These probabilities are commonly understood by the players.

The level of public goods produced in a period is a function of a production technology and a random shock. The production function $f: A \times \Theta \rightarrow \mathbb{R}_{+}$is strictly increasing in effort $(a)$ and type $(\theta)$. We will be interested in complementarity and substitutability between effort and type.

[^1]DEFINITION 1: Fix actions $a_{* *}>a_{*}$ :
(i) Effort and type are complements at $a_{*}, a_{* *}$ if $f\left(a_{* *}, \bar{\theta}\right)-f\left(a_{* *}, \underline{\theta}\right)$ $\geq f\left(a_{*}, \bar{\theta}\right)-f\left(a_{*}, \underline{\theta}\right)$.
(ii) Effort and type are substitutes at $a_{*}, a_{* *}$ if $f\left(a_{*}, \bar{\theta}\right)-f\left(a_{*}, \underline{\theta}\right) \geq f\left(a_{* *}, \bar{\theta}\right)-$ $f\left(a_{* *}, \underline{\theta}\right)$.

These definitions of complementarity versus substitutability are local; effort and type may be complements at two levels of effort, but substitutes at other levels of effort. If effort and type are complements (resp. substitutes) at $a_{*}, a_{* *}$ but not substitutes (resp. complements) at $a_{*}, a_{* *}$, we say that effort and type are strict complements (resp. strict substitutes) at $a_{*}, a_{* *}$.

If, in period $t$, the Politician in office chooses effort level $a$, is of type $\theta$, and the random shock is $\epsilon_{t}$, the level of public goods produced in that period is $f(a, \theta)+$ $\epsilon_{t}$. Each $\epsilon_{t}$ is the realization of a random variable. These random variables are independent of each other and are independent of the Politicians' abilities. In particular, each of these random variables is distributed according to an absolutely continuous CDF, $\Phi$, with a continuously differentiable PDF, $\phi$. This distribution satisfies two additional requirements: First, for each $x>x^{\prime} \geq 0$, the associated likelihood ratio defined by

$$
g \mapsto \frac{\phi(g-x)}{\phi\left(g-x^{\prime}\right)}
$$

is onto with nonzero derivatives. Second, the distribution satisfies the (strict) monotone likelihood ratio property (MLRP) relative to all possible realizations of production: If $x>x^{\prime} \geq 0$, then the associated likelihood ratio

$$
\frac{\phi(g-x)}{\phi\left(g-x^{\prime}\right)}
$$

is strictly increasing in $g .2$ At times, we will be interested in specific examples of this framework.

SPECIFIC MODEL ASSUMPTIONS: Call the PDF symmetric if, for each $x \in \mathbb{R}$, $\phi(x)=\phi(-x)$. If $\phi$ is symmetric, the MLRP implies that $\phi$ is single-peaked about zero, i.e., $\phi$ is strictly increasing on $(-\infty, 0)$ and strictly decreasing on $(0, \infty)$. (See Torgersen 1991, theorem 9.4.9). Call the model symmetric if the density is symmetric and $\pi_{I}=\pi_{C}$.

[^2]

Figure 1. Timeline

Prior to the game being played, Nature determines the realizations of each Politician's type and of the random shocks (in all periods). These realizations are not observed by any of the players. Figure 1 depicts the timeline: In the initial governance period, the Incumbent chooses a level of effort $a_{1}$. The choice is not observed by the Voter. Instead, the Voter observes the level of public goods produced, $g_{1}$. This leads to the electoral stage, in which the Voter chooses to reelect the Incumbent or replace her with a Challenger. The winner of the election is the Politician in office in the second governance period. She chooses a level of effort $a_{2}$. Again, the Voter observes the level of public goods produced, $g_{2}$.

The Voter's payoffs are the sum of public goods produced in the two periods. Each Politicians' payoffs depend on both a benefit from holding office and the level of effort chosen while in office. The benefit from holding office is given by $B>0$. The cost of taking effort $a$ is given by a cost function $c(\cdot)$, where $c(\cdot)$ is strictly increasing and $B>c(\underline{a}) \geq 0$. A Politician's payoff in governance period $t$ is 0 if she is not in office and $B-c\left(a_{t}\right)$ if she is in office and chooses effort $a_{t}$. A Politician's payoffs are given by the sum of her payoffs in each governance period.

We will be interested in understanding how changing the Politicians' benefits of re-election and cost function affects the Voter's ability to acquire information and the Voter's welfare. With this in mind, it will be convenient to think of the model as parameterized by $B$ and $c$. Write $\mathcal{M}(B, c)$ for the model in which the benefits of re-election and cost functions take on the values $B$ and $c$.

## A. Comments on the Model

Let us comment on how the features of the model relate to the concepts of interest.
First, in our model, the level of effective accountability corresponds to the level of effort exerted by a politician. Institutions of formal accountability correspond to institutional features that affect the incumbent's electoral incentives to exert effort. In practice, these can correspond to changes in the value of holding office-e.g., compensation, term limits, opportunities for promotion, or (external) opportunities in the lobbying industry (Gagliarducci and Nannicini 2013). They can also correspond to changes in the costliness of effort-e.g., the level of funding for corruption investigations might impact the trade-off between public corruption and public goods provision (Ferraz and Finan 2011). We abstract away from any specific institutional features. Instead, we take a reduced-form approach, modeling changes in the institutions of formal accountability through variation in $B$ and $c$.

An important aspect of the model is the nature of the production technology. In practice, effort and type can be complements or substitutes, depending on the particular environment and the particular level of effort. Take these two examples.

First, suppose the quality of the incumbent reflects her expertise on a particular point of legislation. High quality incumbents have lots of expertise and low quality incumbents have little. Consider a situation where the incumbent is putting in a small amount of effort. A marginal increase in effort might have a larger impact on the legislation for the high quality incumbent since she can use the extra time to fine tune the substance of the legislation, whereas a low quality incumbent would use the extra time to learn basics. Here, effort and type would be strict local complements. On the other extreme, consider an incumbent who is already putting in a large amount of effort. A marginal increase in effort might have a smaller impact on the legislation for the high quality incumbent since she will use the extra time for finishing touches, whereas a low quality incumbent would use the extra time to fine tune the substance of the legislation. Here, effort and type would be strict local substitutes.

Second, suppose that quality reflects a politician's relationship with the bureaucracy. High quality incumbents have a good relationship and low quality incumbents have a poor relationship. Consider a situation where the incumbent is implementing a new ambitious reform. Incumbent effort involves working on designing the reform. High quality incumbents get the bureaucracy to work hard on the reform, while low quality incumbents have a hard time getting the bureaucracy to work on the reform. A marginal increase in effort will then have a larger impact on the reform for the high quality incumbent. Here, effort and type would be strict complements. On the other extreme, consider a situation where the incumbent must monitor the bureaucracy. A marginal increase in monitoring will have a larger impact for the lower quality incumbent, since she can't count on a good relationship to keep the bureaucrats in line. Here, effort and type would be strict substitutes.

## II. Properties of Equilibrium

In this section, we fix a model $\mathcal{M}(B, c)$ and point to basic properties of equilibrium that will be useful in our subsequent analysis. We focus on pure strategy Perfect Bayesian Equilibrium.

The Voter's Re-election Decision.-In the second governance period, there are no electoral benefits from taking costly effort. As such, the Politician in office will choose the lowest possible effort, i.e., $a_{2}=\underline{a}$, independent of the history. So, the Voter's electoral decision depends only on his expectation about the Politicians' types. As a consequence, the Voter adopts a cutoff rule in the space of posterior beliefs. If the Voter expects the Incumbent's first-period choice was $\tilde{a}$ and he observed a level of public goods $g_{1}$, his posterior belief that the Incumbent is of high type is $\operatorname{Pr}_{I}\left(\bar{\theta} \mid g_{1}, \tilde{a}\right)$. The Voter reelects the Incumbent if and only if his posterior,

$$
\operatorname{Pr}_{I}\left(\bar{\theta} \mid g_{1}, \tilde{a}\right)=\frac{\pi_{I} \phi\left(g_{1}-f(\tilde{a}, \bar{\theta})\right)}{\pi_{I} \phi\left(g_{1}-f(\tilde{a}, \bar{\theta})\right)+\left(1-\pi_{I}\right) \phi\left(g_{1}-f(\tilde{a}, \underline{\theta})\right)},
$$

is higher than the threshold $\pi_{C}$.
The fact that the Voter adopts a cutoff rule in the space of posterior probabilities implies that the Voter also adopts a cutoff rule in the space of public goods. To
see this, define the likelihood ratio as a function of public goods and effort, i.e., LR : $\mathbb{R} \times A \rightarrow \mathbb{R}$ with

$$
\operatorname{LR}(g, a)=\frac{\phi(g-f(a, \bar{\theta}))}{\phi(g-f(a, \underline{\theta}))} .
$$

Observe that $\operatorname{Pr}_{I}\left(\bar{\theta} \mid g_{1}, \tilde{a}\right) \geq \pi_{C}$ holds if and only if

$$
\begin{equation*}
\operatorname{LR}\left(g_{1}, \tilde{a}\right)=\frac{\phi\left(g_{1}-f(\tilde{a}, \bar{\theta})\right)}{\phi\left(g_{1}-f(\tilde{a}, \underline{\theta})\right)} \geq \frac{1-\pi_{I}}{\pi_{I}} \frac{\pi_{C}}{1-\pi_{C}} . \tag{1}
\end{equation*}
$$

Since $\operatorname{LR}(\cdot, \tilde{a})$ is onto, there exists some $\hat{g}(\tilde{a})$ satisfying

$$
\begin{equation*}
\operatorname{LR}(\hat{g}(\tilde{a}), \tilde{a})=\frac{\phi(\hat{g}(\tilde{a})-f(\tilde{a}, \bar{\theta}))}{\phi(\hat{g}(\tilde{a})-f(\tilde{a}, \underline{\theta}))}=\frac{1-\pi_{I}}{\pi_{I}} \frac{\pi_{C}}{1-\pi_{C}} . \tag{2}
\end{equation*}
$$

By the MLRP, the left-hand side of equation (1) is strictly increasing in $g_{1}$. So, if the Voter believes the Incumbent's first-period effort is $\tilde{a}$ and the first-period outcome is $g_{1}$, the Voter reelects the Incumbent if $g_{1}>\hat{g}(\tilde{a})$ and replaces her if $g_{1}<\hat{g}(\tilde{a})$.

Example 1: Suppose the model is symmetric. Since $\pi_{I}=\pi_{C}$, equation (2) says that $\hat{g}(a)$ solves $\phi(\hat{g}(a)-f(a, \bar{\theta}))=\phi(\hat{g}(a)-f(a, \underline{\theta}))$. Since, the PDF $\phi$ is symmetric and single-peaked and $\hat{g}(a)-f(a, \underline{\theta})>\hat{g}(a)-f(a, \bar{\theta})$, it follows that $(\hat{g}(a)-f(a, \bar{\theta}))=-(\hat{g}(a)-f(a, \underline{\theta}))$. This implies

$$
\hat{g}(a)=\frac{f(a, \bar{\theta})+f(a, \underline{\theta})}{2} .
$$

Example 2: Let $\phi$ be the PDF of the standard Normal distribution. Then,

$$
\frac{\exp ^{-0.5(\hat{g}(a)-f(a, \bar{\theta}))^{2}}}{\exp ^{-0.5(\hat{g}(a)-f(a, \theta))^{2}}}=\frac{\pi_{C}\left(1-\pi_{I}\right)}{\pi_{I}\left(1-\pi_{C}\right)} .
$$

It follows that

$$
\hat{g}(a)=\frac{\log \left(\frac{\pi_{C}\left(1-\pi_{1}\right)}{\pi_{l}\left(1-\pi_{C}\right)}\right)}{f(a, \bar{\theta})-f(a, \underline{\theta})}+\frac{f(a, \bar{\theta})+f(a, \underline{\theta})}{2} .
$$

Observe that the Voter always uses the same cutpoint in the space of posterior beliefs, namely $\pi_{C}$. However, his cutpoint in the space of public goods depends on his belief about the Incumbent's effort. In Example 1, this cutpoint is increasing in effort. One natural conjecture is that this is always the case: If the Voter expects the Incumbent to choose a higher effort, she holds the Incumbent to a higher benchmark for public good provision, since a larger portion of any success is credited to effort
rather than competence. Section IV will show that this conjecture is incorrect. (That analysis will be based on Example 2.)

The Incumbent's First-Period Choice.-Suppose the Incumbent chooses the action $a$ and the Voter expects the Incumbent to choose action $a_{*}$. The Incumbent is reelected (resp. replaced) if the level of public goods observed, $f\left(a, \theta_{I}\right)+\epsilon_{1}$, exceeds (resp. falls short of) the Voter's threshold $\hat{g}\left(a_{*}\right)$. The probability that the Incumbent is reelected if she chooses $a$ when the Voter expects $a_{*}, \operatorname{Pr}\left(a \mid a_{*}\right)$, is then the probability that $\epsilon_{1} \geq \hat{g}\left(a_{*}\right)-f\left(a, \theta_{I}\right),{ }^{3}$ i.e.,
(3) $\operatorname{Pr}\left(a \mid a_{*}\right)=\pi_{I}\left[1-\Phi\left(\hat{g}\left(a_{*}\right)-f(a, \bar{\theta})\right)\right]+\left(1-\pi_{I}\right)\left[1-\Phi\left(\hat{g}\left(a_{*}\right)-f(a, \underline{\theta})\right)\right]$.

Now consider two levels of effort $a, a^{\prime} \in A$. If the Voter expects the Incumbent to choose effort level $a_{*}$, then the incremental increase in probability of re-election from choosing $a^{\prime}$ instead of $a$ is $\operatorname{IR}\left(a^{\prime}, a \mid a_{*}\right)=\operatorname{Pr}\left(a^{\prime} \mid a_{*}\right)-\operatorname{Pr}\left(a \mid a_{*}\right) \cdot{ }^{4}$ We can construct an equilibrium where the Incumbent chooses the effort $a_{*}$ in the first period if and only if, for each effort $a \in A, \operatorname{IR}\left(a_{*}, a \mid a_{*}\right)(B-c(\underline{a})) \geq c\left(a_{*}\right)-c(a)$, i.e., if and only if, for each $a \in A$, the Incumbent's incremental benefit from choosing $a_{*}$ over $a$ (when the Voter expects $a_{*}$ ) is higher than her incremental cost of choosing $a_{*}$ over $a$.

Set of Equilibria.-Fix a model associated with the benefit of re-election and cost function ( $B, c$ ). In light of the analysis above, we will identify a pure-strategy Perfect Bayesian Equilibrium with the first-period effort level chosen by the Incumbent in that equilibrium. Write $\mathcal{E}(B, c)$ for the set of first-period equilibrium efforts in the game parameterized by $(B, c)$. Say the pair $(B, c)$ justifies $a \in A$ if $a \in \mathcal{E}(B, c)$. Say effort $a$ is justifiable if there exists $(B, c)$ that justifies $a$.

Recall, in any equilibrium, the second-period Politician chooses $\underline{a}$. Thus, the set $\mathcal{E}(B, c)$ represents the set of possible levels of effective accountability in the model $\mathcal{M}(B, c)$. Varying the institutions of formal accountability varies the consequences of holding office or exerting effort and, in so doing, can lead to variation in $(B, c)$. In turn, varying $(B, c)$ can lead to variation in the level of effective accountability, in the sense that it can change the set of first-period equilibrium effort levels. But we will see that, even without varying the institutions of formal accountability (or $(B, c)$ ), we can have variation in the level of effective accountability. That is, for a given model $\mathcal{M}(B, c)$, there may be multiple equilibria that differ in their level of first-period effort.

To understand why this can occur, recall that the Voter reelects the Incumbent if and only if the level of public goods meets some threshold. The threshold depends on the effort the Voter believes the Incumbent chose. This last fact is what generates

[^3]the possibility of multiple equilibria that differ in first-period effort level: Distinct expectations about the Incumbent's first-period behavior are associated with distinct cutoff rules for the Voter. This, in turn, generates distinct first-period incentives for the Incumbent, since the ex ante probability of re-election $\operatorname{Pr}(\cdot \mid \cdot): A \times A \rightarrow[0,1]$ depends both on the Incumbent's effort and the Voter's expectation of the Incumbent's effort. This intuition is familiar from the career concerns literature (Dewatripont, Jewitt, and Tirole 1999). In Section V, we return to the question of when such multiplicity does versus does not obtain, and provide results that go beyond this familiar intuition.

## III. Welfare and Information

One natural hypothesis is that, all else equal, a higher level of effective accountability is necessarily beneficial to the Voter. In this section, we show that this conjecture is incorrect in general, and provide sufficient conditions under which the conjecture is in fact correct.

Consider a model in which there is an equilibrium where the Incumbent plays the first-period effort level $a_{*}$. Expected Voter welfare is the sum of expected first-period Voter welfare and ex ante expected second-period Voter welfare. Expected first-period Voter welfare is

$$
\mathrm{VW}_{1}\left(a_{*}\right)=\operatorname{Pr}(\bar{\theta}) f\left(a_{*}, \bar{\theta}\right)+(1-\operatorname{Pr}(\bar{\theta})) f\left(a_{*}, \underline{\theta}\right)
$$

Ex ante expected second-period Voter welfare is

$$
\mathrm{VW}_{2}\left(a_{*}\right)=\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right) f(\underline{a}, \bar{\theta})+\left(1-\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right)\right) f(\underline{a}, \underline{\theta}),
$$

where $\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right)$ is the equilibrium probability that the Politician in office in the second period is type $\bar{\theta} .{ }^{5}$ Note that $\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right)$ depends on the first-period equilibrium effort and the Voter's equilibrium cutoff rule.

Clearly, higher effective accountability is good for first period welfare. Hence, it can only harm overall welfare if it decreases second period welfare. In turn, higher effective accountability can only decrease second period welfare if it decreases the probability of selecting a high type in the second period, i.e., $\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right)$.

Notice, governance outcomes serve as a signal of the Incumbent's type. The signal depends on the level of effective accountability. Thus, changing the level of effective accountability serves to change the distribution from which the signal is drawn. This, in turn, changes the informativeness of the signal and, so, the Voter's ability to select high types. With this in mind, we begin with a notion of informativeness.

DEFINITION 2 (Lehman 1988): Say $a_{* *}$ is more informationally effective than $a_{*}$ if $\theta>\theta^{\prime}, \Phi\left(g_{* *}-f\left(a_{* *}, \theta\right)\right)=\Phi\left(g_{*}-f\left(a_{*}, \theta\right)\right)$, and $\Phi\left(g_{* *}^{\prime}-f\left(a_{* *}, \theta^{\prime}\right)\right)$ $=\Phi\left(g_{*}-f\left(a_{*}, \theta^{\prime}\right)\right)$ imply $g_{* *} \geq g_{* *}^{\prime}$.

[^4]

Figure 2. Gap for $\theta$ at $g_{*}$

Because $\Theta$ contains two types, $a_{* *}$ is more informative than $a_{*}$ in the sense of Blackwell (1951) if and only if $a_{* *}$ is more informationally effective than $a_{*}$. (See Blackwell and Girshick 1979 or Jewitt 2007.)

Fix a type $\theta$ and observe that each effort level $a$ induces a CDF of governance outcomes, i.e., $\Phi(g-f(a, \theta)) .{ }^{6}$ Figure 2 depicts the CDFs associated with effort levels $a_{* *}>a_{*}$. Notice, $\Phi\left(g-f\left(a_{* *}, \theta\right)\right)<\Phi\left(g-f\left(a_{*}, \theta\right)\right)$. So, for any given $g_{*}$, there exists $g_{* *}>g_{*}$ with $\Phi\left(g_{* *}-f\left(a_{* *}, \theta\right)\right)=\Phi\left(g_{*}-f\left(a_{*}, \theta\right)\right)$. Refer to $g_{* *}-g_{*}$ as the "gap for $\theta$ at $g_{*}$." If $a_{* *}$ is more informationally effective than $a_{*}$, the gap for $\theta$ at $g_{*}$ must be increasing in $\theta$. Intuitively, this can only happen if the difference in productivity between the high and low actions, i.e., $f\left(\theta, a_{* *}\right)-f\left(\theta, a_{*}\right)$, is increasing in $\theta$. This allows us to draw a tight connection between informational effectiveness and local complements/local substitutes. ${ }^{7}$

THEOREM 1: Fix $a_{* *}>a_{*}$ :
(i) If effort and type are complements at $\left(a_{*}, a_{* *}\right)$, then $a_{* *}$ is more informationally effective than $a_{*}$.
(ii) If effort and type are substitutes at $\left(a_{*}, a_{* *}\right)$, then $a_{*}$ is more informationally effective than $a_{* *}$.

[^5]Informational effectiveness has an immediate implication for second-period welfare. (See Proposition 4 in Jewitt 2007.) Thus, Theorem 1 gives the following corollary.

COROLLARY 1: Fix $a_{* *}>a_{*}$, where $a_{*}$ and $a_{* *}$ are each justifiable:
(i) If effort and type are complements at $\left(a_{*}, a_{* *}\right)$, then $\mathrm{VW}_{2}\left(a_{* *}\right) \geq \mathrm{VW}_{2}\left(a_{*}\right)$.
(ii) If effort and type are substitutes at $\left(a_{*}, a_{* *}\right)$, then $\mathrm{VW}_{2}\left(a_{*}\right) \geq \mathrm{VW}_{2}\left(a_{* *}\right)$.

If effort and type are complements at $\left(a_{*}, a_{* *}\right)$, then increasing the level of effective accountability from $a_{*}$ to $a_{* *}$ increases both first- and second-period welfare. If effort and type are substitutes at $\left(a_{*}, a_{* *}\right)$, then increasing the level of effective accountability from $a_{*}$ to $a_{* *}$ increases first-period welfare but decreases second-period welfare.

In light of Corollary 1, we can provide a sufficient condition for total Voter welfare to be increasing in effective accountability.

PROPOSITION 1: Suppose the model is symmetric. If $a_{* *}>a_{*}$ and $a_{*}$ and $a_{* *}$ are each justifiable, then there exists $\underline{\pi}\left[a_{*}, a_{* *}\right], \bar{\pi}\left[a_{*}, a_{* *}\right] \in(0,1)$ so that the following are equivalent:
(i) $\pi_{I} \in\left(0, \underline{\pi}\left[a_{*}, a_{* *}\right]\right) \cup\left(\bar{\pi}\left[a_{*}, a_{* *}\right], 1\right)$;
(ii) $\mathrm{VW}_{1}\left(a_{* *}\right)+\mathrm{VW}_{2}\left(a_{* *}\right) \geq \mathrm{VW}_{1}\left(a_{*}\right)+\mathrm{VW}_{2}\left(a_{*}\right)$.

Proposition 1 focuses on the symmetric model, where the pool of Incumbents and Challengers coincide. If the prior is close to zero or one, the Voter has relatively little to gain from improved selection. In that case, most Politicians are of the same type, so the likelihood that the Politician in office in the second period will be a high type is not very responsive to the first-period outcome. Hence, the first-period welfare effect of high effort dominates any negative second-period welfare effect of high effort.

It is important to note that if effort and type are local substitutes, we may have $\bar{\pi}\left[a_{*}, a_{* *}\right]>\underline{\pi}\left[a_{*}, a_{* *}\right]$. In that case higher levels of effective accountability may decrease overall Voter welfare. The following example shows that this can occur.

Example 3: Let $A=\left\{a, a_{*}, a_{* *}\right\}$, where $a_{* *}>a_{*}>\underline{a}$. The production function $f$ is described by the Table 1. Take $\Phi$ to be the CDF of the standard Normal distribution and let $\pi_{I}=\pi_{C}=0.6$.

Table 1—Production Function

|  | $\underline{\theta}$ | $\bar{\theta}$ |
| :--- | :---: | :---: |
| $a_{* *}$ | 38 | 39 |
| $a_{*}$ | 35 | 37 |
| $\underline{a}$ | 0 | 36 |

Using Example 1, the incremental returns in re-election probabilities satisfy the conditions in Table 2:

Table 2-Incremental Returns in Reelection Probabilities

| $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \in(0.498,0.499)$ | $\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right) \in(0.534,0.535)$ |
| :--- | :--- |
| $\operatorname{IR}\left(a_{*}, a_{* *} \mid a_{*}\right) \in(-0.422,-0.421)$ | $\operatorname{IR}\left(a_{*}, \underline{a} \mid a_{*}\right) \in(0.268,0.269)$ |
| $\operatorname{IR}\left(\underline{a}, a_{*} \mid \underline{a}\right) \in(-0.401,-0.399)$ | $\operatorname{IR}\left(\underline{a}, a_{* *} \mid \underline{a}\right) \in(-0.401,-0.399)$ |

We will consider two models parameterized by $\left(B_{* *}, c_{* *}\right)=\left(10, c_{* *}\right)$ and $\left(B_{*}, c_{*}\right)$ $=\left(5, c_{*}\right)$, where $c_{* *}\left(a_{* *}\right)=c_{*}\left(a_{* *}\right)=5, c_{* *}\left(a_{*}\right)=c_{*}\left(a_{*}\right)=1$, and $c_{* *}(\underline{a})$ $=c_{*}(\underline{a})=0$. It can be verified that $\mathcal{E}\left(10, c_{* *}\right)=\left\{a_{* *}\right\}$ and $\mathcal{E}\left(5, c_{*}\right)=\left\{a_{*}\right\}$. Thus, the model associated with $\left(B_{* *}, c_{* *}\right)$ has a higher level of effective accountability than the model associated with $\left(B_{*}, c_{*}\right)$. But, the Voter's welfare is strictly higher in the model associated with $\left(B_{*}, c_{*}\right)$ than in the model associated with $\left(B_{* *}, c_{* *}\right)$. In particular,

$$
\begin{aligned}
\mathrm{VW}_{1}\left(a_{*}\right)+\mathrm{VW}_{2}\left(a_{*}\right)= & 0.4(35)+0.6(37) \\
& +36[0.6+0.6(0.4)(2 \Phi(1)-1)]>63.69
\end{aligned}
$$

and

$$
\begin{aligned}
\mathrm{VW}_{1}\left(a_{* *}\right)+\mathrm{VW}_{2}\left(a_{* *}\right)= & 0.4(38)+0.6(39) \\
& +36[0.6+0.6(0.4)(2 \Phi(0.5)-1)]<63.51
\end{aligned}
$$

Example 3 illustrates that higher levels of effective accountability may be associated with lower second-period Voter welfare, so much so that it may also be associated with lower levels of total Voter welfare. The key is that the Voter's expected second-period welfare depends on the probability that the Politician in office in the second-period is of high type. This, in turn, depends the Voter's ability to learn about the Incumbent's type, given the (equilibrium) level of first-period effort. Higher levels of effective accountability may hinder the Voter's ability to learn about the Incumbent's type.

## IV. The Re-election Decision and Information

There is a natural conjecture concerning the relationship between the Voter's re-election standard and the level of effective accountability. Intuitively, when there is a higher level of effective accountability, the Voter expects a higher level of effort from the Incumbent and so should use a higher cutoff rule with respect to public goods. Indeed, this was true in the symmetric model of Example 1. But this intuition is incomplete. Changes to effective accountability change the informativeness of governance outcomes. This change in informativeness has an independent effect on the Voter's re-election standard. We will see that the informativeness effect can have counterintuitive implications: higher levels of effective accountability can lead the Voter to use a lower benchmark for re-election.

To understand why this is the case, consider the impact of increasing effort, $a$, on the benchmark, $\hat{g}(a)$. There are two effects. The first is due to an increase in productivity, holding the level of informativeness fixed. Let $\nu(a)$ be the level of public goods that, in equilibrium, provides neutral news about the Incumbent-that is, that leaves the Voter's beliefs about the Incumbent unchanged. Then $\nu(a)$ solves $\operatorname{Pr}_{I}(\bar{\theta} \mid \nu(a), a)=\pi_{I}$. When $\phi$ is symmetric, we can repeat the analysis in Example 1 to get

$$
\nu(a)=\frac{f(a, \bar{\theta})+f(a, \underline{\theta})}{2}
$$

Observe that $\nu(a)$ is increasing in $a$.
The second effect is due to the change in informativeness. Take

$$
\iota(a)=\frac{f(a, \bar{\theta})-f(a, \underline{\theta})}{2}
$$

as a measure of informational effectiveness. By Theorem 1, if $a_{* *}>a_{*}$, then the following equivalence holds: Effort and type are local complements (resp. local substitutes) at $a_{* *}, a_{* *}$ if and only if $\iota\left(a_{* *}\right) \geq \iota\left(a_{*}\right)\left(\right.$ resp. $\left.\iota\left(a_{*}\right) \geq \iota\left(a_{* *}\right)\right)$.

To understand, intuitively, how the informativeness effect works, start by observing that the Voter may receive good news (resp. bad news) about the Incumbent, relative to his prior, and nonetheless replace (resp. retain) her. This can occur if the ex ante probability that the Incumbent is a high type is lower (resp. higher) than the ex ante probability that the Challenger is a high type. With this in mind, observe that: (i) more informative experiments exacerbate good and bad news about the Incumbent, while (ii) less informative experiments temper good and bad news about the Incumbent. If the benchmark associated with a lower level of effort provides good news about the Incumbent (relative to the prior $\pi_{I}$ ), then a more informative experiment will require a lower benchmark for re-election. Likewise, if the benchmark associated with a higher level of effort provides bad news about the Incumbent (relative to the prior $\pi_{I}$ ), then a less informative experiment will require a lower benchmark for re-election.

This argument suggests two instances in which the Voter's benchmark will move in the intuitive direction. The two effects will move in the same direction if either (i) the Voter's benchmark provides bad news about the Incumbent and there are complements or (ii) the Voter's benchmark provides good news about the Incumbent and there are substitutes. The following result bears out this intuition.

THEOREM 2: Suppose $\phi$ is symmetric. Fix $a_{* *}>a_{*}$ :
(i) If $\pi_{I} \geq \pi_{C}$ and effort and type are complements at $a_{* *}, a_{*}$, then $\hat{g}\left(a_{* *}\right)>\hat{g}\left(a_{*}\right)$.
(ii) If $\pi_{C} \geq \pi_{I}$ and effort and type are substitutes at $a_{* *}, a_{*}$, then $\hat{g}\left(a_{* *}\right)>\hat{g}\left(a_{*}\right)$.

Below we present a graphical argument that allows us to separately analyze the two effects discussed above. Doing so has two benefits. First, it makes clear how the proof of Theorem 2 works. Second, it elucidates the two effects, which allow us to
understand the cases not covered by the theorem. These cases allow for the possibility of a counterintuitive change in Voter behavior, which will be demonstrated in Examples 4 and 5 at the end of this section.

For each $a \in A$, we can express the likelihood ratio in terms of the variables $(g, \nu(a), \iota(a))$ :

$$
\operatorname{LR}(g, a)=\frac{\phi(g-\nu(a)-\iota(a))}{\phi(g-\nu(a)+\iota(a))}
$$

Fix $a_{* *}>a_{*}$ where both $a_{*}$ and $a_{* *}$ are justifiable. $\operatorname{Write} \operatorname{LR}_{*}(\cdot)=\operatorname{LR}\left(\cdot, a_{*}\right)$ and $\mathrm{LR}_{* *}(\cdot)=\operatorname{LR}\left(\cdot, a_{* *}\right)$. Figure 3 draws the function $\mathrm{LR}_{*}(\cdot)$. Observe that by the MLRP, it is increasing in $g$. The values $\nu\left(a_{*}\right)$ and $\hat{g}\left(a_{*}\right)$ solve $\operatorname{LR}_{*}\left(\nu\left(a_{*}\right)\right)=1$ and $\operatorname{LR}_{*}\left(\hat{g}\left(a_{*}\right)\right)=\beta$, where we write $\beta$ for the benchmark $\frac{1-\pi_{I}}{\pi_{I}} \frac{\pi_{C}^{*}}{1-\pi_{C}}$. The value $\hat{g}\left(a_{* *}\right)$ also solves $\operatorname{LR}_{* *}\left(\hat{g}\left(a_{* *}\right)\right)=\beta$. Thus, to determine the relationship between $\hat{g}\left(a_{*}\right)$ and $\hat{g}\left(a_{* *}\right)$, we want to understand the relationship between the functions $\mathrm{LR}_{*}(\cdot)$ and $\mathrm{LR}_{* *}(\cdot)$.

To do so, it will be convenient to define an auxiliary function $\tilde{L}: \mathbb{R} \rightarrow \mathbb{R}$ where

$$
\tilde{\mathrm{L}}(g)=\frac{\phi\left(g-\nu\left(a_{* *}\right)-\iota\left(a_{*}\right)\right)}{\phi\left(g-\nu\left(a_{* *}\right)+\iota\left(a_{*}\right)\right)} .
$$

That is, this function is obtained from $\mathrm{LR}_{*}$ by increasing the $\nu\left(a_{*}\right)$ variable to $\nu\left(a_{* *}\right)$, leaving the $\iota\left(a_{*}\right)$ variable fixed. This change can be viewed as a thought experiment where we change the level of productivity expected by the Voter, holding fixed the informativeness of the signal. Figure 3 depicts this function below the function $\mathrm{LR}_{*}$ and intersecting the benchmark $\beta$ at a higher level of public goods $\tilde{g}$. Indeed, this must always be the case.

## LEMMA 1 :

(i) For each $g, \mathrm{LR}_{*}(g)>\tilde{\mathrm{L}}(g)$.
(ii) If $\tilde{\mathrm{L}}(\tilde{g})=\beta$, then $\tilde{g}>\hat{g}\left(a_{*}\right)$.

Now suppose we begin with the function $\tilde{\mathrm{L}}$ and allow the level of informativeness to change, going from $\iota\left(a_{*}\right)$ to $\iota\left(a_{* *}\right)$. Figure 4, panels A and B depict the case of complements, which increases informativeness. In this case, outcomes that were good news about the Incumbent are now better news and outcomes that were bad news about the Incumbent are now worse news. Thus, if $g>\nu\left(a_{* *}\right)$ (resp. $g<\nu\left(a_{* *}\right)$ ) then $\mathrm{LR}_{* *}(g)>\tilde{\mathrm{L}}(g)\left(\right.$ resp. $\left.\mathrm{LR}_{* *}(g)<\tilde{\mathrm{L}}(g)\right)$. Figure 4 panels C and D depict the case of substitutes, which decreases informativeness. This tempers good news about the Incumbent and exacerbates bad news about the Incumbent. Thus, if $g>\nu\left(a_{* *}\right)$ (resp. $g<\nu\left(a_{* *}\right)$ ), then $\mathrm{LR}_{* *}(g)<\tilde{\mathrm{L}}(g)\left(\right.$ resp. $\left.\mathrm{LR}_{* *}(g)>\tilde{\mathrm{L}}(g)\right)$.

## Panel A. $\beta<1$



Panel B. $\beta>1$


Figure 3. Relationship between $\hat{g}\left(a_{*}\right), \nu\left(a_{*}\right)$, and $\tilde{g}$

## Panel A. Complements and $\beta<1$



Panel C. Substitutes and $\beta<1$


Panel B. Complements and $\beta>1$


Panel D. Substitutes and $\beta>1$


Figure 4. Cutoff Rule: Complements and Substitutes

LEMMA 2: Suppose $\phi$ is symmetric:
(i) Suppose effort and type are complements at $a_{*}, a_{* *}$. Then $g>\nu\left(a_{* *}\right)$ implies $\mathrm{LR}_{\tilde{*}}(g)>\tilde{\mathrm{L}}(g), g<\nu\left(a_{* *}\right)$ implies $\mathrm{LR}_{* *}(g)<\tilde{\mathrm{L}}(g)$, and $\mathrm{LR}_{* *}\left(\nu\left(a_{* *}\right)\right)$ $=\tilde{\mathrm{L}}\left(\nu\left(a_{* *}\right)\right)$.
(ii) Suppose effort and type are substitutes at $a_{*}, a_{* *}$. Then $g>\nu\left(a_{* *}\right)$ implies $\mathrm{LR}_{* *}(g)<\tilde{\mathrm{L}}(g), g<\nu\left(a_{* *}\right)$ implies $\mathrm{LR}_{* *}(g)>\tilde{\mathrm{L}}(g)$, and $\mathrm{LR}_{* *}\left(\nu\left(a_{* *}\right)\right)$ $=\tilde{\mathrm{L}}\left(\nu\left(a_{* *}\right)\right)$.

Recall, the auxiliary function $\tilde{\mathrm{L}}$ meets the benchmark $\beta$ at some level of public goods $\tilde{g}>\hat{g}\left(a_{*}\right)$. If, at the benchmark $\beta, \mathrm{LR}_{* *}$ is to the right of $\tilde{\mathrm{L}}$, then we also have that $\hat{g}\left(a_{* *}\right) \geq \tilde{g}>\hat{g}\left(a_{*}\right)$. Refer to Figure 4. When effort and type are complements at $a_{*}, a_{* *}$, then $\mathrm{LR}_{* *}$ is to the right of $\tilde{\mathrm{L}}$ at the benchmark $\beta$ provided the benchmark is less than 1 . When effort and type are substitutes at $a_{*}, a_{* *}$, then $\mathrm{LR}_{* *}$ is to the right of $\tilde{\mathrm{L}}$ at the benchmark $\beta$ provided the benchmark is above 1 . Using the fact that $1 \geq \beta$ if and only if $\pi_{I} \geq \pi_{C}$, this gives Theorem 2 .

We have seen that, if the Voter's benchmark provides good news about the Incumbent and there are strict complements or the Voter's benchmark provides bad news about the Incumbent and there are strict substitutes, then the neutral news and informativeness effects pull the Voter's benchmark in opposite directions. It remains the be shown that the informativeness effect can be sufficiently strong to lead to the counterintuitive result: an increase in effective accountability can lead to a less stringent re-election rule. Example 4 shows this is possible in the case of complements and Example 5 shows it in the case of substitutes.

Example 4: Let $A=\{1,2\}, \underline{\theta}=1$ and $\bar{\theta}=2$. Let the production function be $f(a, \theta)=a \cdot \theta$, so that effort and type are complements. Take $\Phi$ to be the CDF of the standard Normal distribution and let $\pi_{I}=0.1$ and $\pi_{C}=0.8$.

We will show that there are models $\mathcal{M}\left(B_{* *}, c_{* *}\right)$ and $\mathcal{M}\left(B_{*}, c_{*}\right)$ with $2 \in$ $\mathcal{E}\left(B_{* *}, c_{* *}\right), 1 \in \mathcal{E}\left(B_{*}, c_{*}\right)$, and $\hat{g}(1)>\hat{g}(2)$. In light of the analysis of Example 2, take

$$
\hat{g}(a)=\frac{\log \left(\frac{0.8(0.9)}{0.1(0.2)}\right)}{a}+\frac{3 a}{2}
$$

for each $a \in A$. This implies that

$$
\hat{g}(1) \in(5.08,5.09) \quad \text { and } \quad \hat{g}(2) \in(4.79,4.80)
$$

With this, observe that the incremental returns in re-election probabilities satisfy

$$
\operatorname{IR}(2,1 \mid 2) \in(0.023,0.024) \quad \text { and } \quad \operatorname{IR}(2,1 \mid 1) \in(0.014,0.015)
$$

Take $\left(B_{* *}, c_{* *}\right)=\left(10, c_{* *}\right) \quad$ and $\quad\left(B_{*}, c_{*}\right)=\left(3, c_{*}\right)$, where $c_{* *}(2)=c_{*}(2)$ $=0.1, c_{* *}(1)=c_{*}(1)=0$. It can then be verified that $\mathcal{E}\left(10, c_{* *}\right)=\{2\}$ and $\mathcal{E}\left(3, c_{*}\right)=\{1\}$.

Example 5: The environment is as in Example 4, with two exceptions. The production function is now $f(a, \theta)=\sqrt{a+\theta}$, so that effort and type are substitutes.

The ex ante probabilities that the Incumbent and Challenger are the high types are $\pi_{I}=0.6$ and $\pi_{C}=0.4$.

We will show that there are models $\mathcal{M}\left(B_{* *}, c_{* *}\right)$ and $\mathcal{M}\left(B_{*}, c_{*}\right)$ with $2 \in$ $\mathcal{E}\left(B_{* *}, c_{* *}\right), 1 \in \mathcal{E}\left(B_{*}, c_{*}\right)$, and $\hat{g}(1)>\hat{g}(2)$. In light of the analysis of Example 2, take

$$
\hat{g}(a)=\frac{\log \left(\frac{0.4^{2}}{0.6^{2}}\right)}{\sqrt{a+2}-\sqrt{a+1}}+\frac{\sqrt{a+2}+\sqrt{a+1}}{2}
$$

for each $a \in A$. Now,

$$
\hat{g}(1) \in(-0.98,-0.97) \quad \text { and } \quad \hat{g}(2) \in(-1.17,-1.16)
$$

With this, observe that the incremental returns in re-election probabilities satisfy

$$
\operatorname{IR}(2,1 \mid 2) \in(0.001,0.002) \quad \text { and } \quad \operatorname{IR}(2,1 \mid 1) \in(0.003,0.004)
$$

Take $\left(B_{* *}, c_{* *}\right)=\left(100, c_{* *}\right)$ and $\left(B_{*}, c_{*}\right)=\left(1, c_{*}\right)$, where $c_{* *}(2)=c_{*}(2)=0.1$, $c_{* *}(1)=c_{*}(1)=0$. It can be verified that $\mathcal{E}\left(100, c_{* *}\right)=\{2\}$ and $\mathcal{E}\left(1, c_{*}\right)=\{1\}$.

## V. Variation in Effective Accountability

Up until now, we have attempted to understand the effect of higher levels of effective accountability on the Voter's welfare and the Voter's re-election strategy. In the examples we have seen, variation in effective accountability resulted from variation in the institutions of formal accountability (reflected as variations in $(B, c)) \cdot{ }^{8}$ In this section, we will see that variation in effective accountability can arise even without variation in the institutions of formal accountability. Such variation can arise if, within a given model $\mathcal{M}(B, c)$, there are multiple equilibria that differ in their level of effective accountability-i.e., if, for a given model $\mathcal{M}(B, c)$, the set $\mathcal{E}(B, c)$ has at least two levels of effort.

Section II pointed to why such multiplicity might arise: Distinct expectations about the Incumbent's first-period behavior are associated with distinct cutoff rules for the Voter. This generates distinct first-period incentives for the Incumbent. But, while suggestive that this may lead to a situation of multiplicity, the multiplicity is not inevitable, as Example 10 in Appendix D illustrates.

In what follows, we fix a production function and beliefs (about type and noise). We ask: Does there exist a benefit of re-election and a cost function so that there are multiple equilibria that differ in their first-period effort levels? Theorem 3 provides a condition on the production technology and beliefs that is necessary and sufficient for an affirmative answer to this question. Proposition 2 points out that any production function is consistent with multiple equilibria that differ in their first-period effort levels.

[^6]To understand what this approach buys, note that, for a given production function and beliefs, all equilibria may involve the same first-period effort level. (Again, refer to Example 11.) However, for the very same production function and beliefs but a slightly perturbed cost function, there may be multiple equilibria that differ in their first-period effort levels. See Example 10 in Appendix D.

Why is the approach we take of interest? We view both the Incumbent's benefit from re-election and the cost of effort as fundamentally subjective. The set of institutions will influence material benefits of re-election (e.g., salary, prestige, and so on) and material costs of higher effort (e.g., foregone rents, time not devoted to other policy areas, and so on). But the institutions cannot pin down the Politician's utility from these material outcomes. On the other hand, for a given set of institutions, the analyst may have intuitions or empirical knowledge about the nature of the production function or the beliefs. For instance, the analyst may think that it is quite likely that there are many high types in the pool of potential Politicians and so it is quite likely that any given Politician is a high type. Or, in a particular application, the analyst may think effort and type are complements/substitutes. Thus, it is of interest to understand conditions on production and beliefs that imply that there is a benefit of re-election and cost function so that, in the associated game, there are multiple equilibria that differ in terms of first-period effort.

## A. Necessary and Sufficient Conditions for Multiplicity

Say the pair $\left(a_{*}, a_{* *}\right)$ is justifiable if there exists some pair $(B, c)$ so that $\left\{a_{*}, a_{* *}\right\}$ $\subseteq \mathcal{E}(B, c)$. (Observe that $a_{*}$ and $a_{* *}$ can each be justifiable even if the pair $\left(a_{*}, a_{* *}\right)$ is not justifiable, i.e., we may have $a_{*} \in \mathcal{E}\left(B_{*}, c_{*}\right)$, $a_{* *} \in \mathcal{E}\left(B_{* *}, c_{* *}\right)$, but no pair $(B, c)$ with $\left\{a_{*}, a_{* *}\right\} \subseteq \mathcal{E}(B, c)$.)

Begin by fixing a benefit of re-election and cost function, $(B, c)$. Observe that $(B, c)$ justifies $a_{*}$ if and only if $\operatorname{IR}\left(a_{*}, a \mid a_{*}\right)(B-c(\underline{a})) \geq c\left(a_{*}\right)-c(a)$, for all $a \in A$. So, if $\left\{a_{*}, a_{* *}\right\} \subseteq \mathcal{E}(B, c)$, then

$$
\begin{equation*}
\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \frac{c\left(a_{* *}\right)-c\left(a_{*}\right)}{B-c(\underline{a})} \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right) . \tag{4}
\end{equation*}
$$

(Here we use the fact that $-\operatorname{IR}\left(a_{*}, a_{* *} \mid a_{*}\right)=\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$.) This trivial argument gives a necessary condition for the pair $\left(a_{*}, a_{* *}\right)$ to be justifiable, namely

$$
\begin{equation*}
\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right) . \tag{5}
\end{equation*}
$$

To interpret this necessary condition, take $a_{* *}>a_{*}$. If the Voter uses the cutoff rule $\hat{g}\left(a_{* *}\right)\left(\operatorname{resp} . \hat{g}\left(a_{*}\right)\right)$, then $\operatorname{IR}\left(a_{* *}, a_{* *} \mid a_{* *}\right)\left(\operatorname{resp} . \operatorname{IR}\left(a_{* *}, a_{* *} \mid a_{*}\right)\right)$ is the incremental increase in the probability of re-election in moving from the lower to the higher effort. Equation (5) says that the incremental increase in the probability of re-election, in moving from the lower effort $a_{*}$ to the higher effort $a_{* *}$ is higher when the Voter uses the cutoff rule $\hat{g}\left(a_{* *}\right)$ versus when the Voter uses the cutoff rule $\hat{g}\left(a_{*}\right)$.

We are interested in providing a sufficient condition for a pair of distinct effort levels $\left(a_{*}, a_{* *}\right)$ to be justifiable. Observe that if equation (5) is satisfied, then we can find a benefit of re-election and a cost function $(B, c)$ satisfying equation (4). If
$a_{*}$ and $a_{* *}$ are the only possible effort levels, equation (4) suffices to conclude that $(B, c)$ justifies both $a_{*}$ and $a_{* *}$. But, when there are three or more levels of effort, equation (4) is no longer sufficient: For a given $(B, c)$, equation (4) may be satisfied even if $(B, c)$ does not justify either of $a_{*}, a_{* *}$, or both. Specifically, equation (4) says that if there is an equilibrium where $a_{*}\left(\right.$ resp. $\left.a_{* *}\right)$ is played, then the Incumbent does not have an incentive to deviate to $a_{* *}\left(\right.$ resp. $\left.a_{*}\right)$. It is silent about deviating to some effort level $a \neq a_{*}, a_{* *}$. (Example 12 in Appendix D illustrates that there may very well be an incentive to deviate to some alternate effort level.)

In sum, if the local condition $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$ is satisfied, it is trivial to construct a pair $(B, c)$ so that equation (4) is satisfied. However, equation (4) is not sufficient for $(B, c)$ to justify either of $a_{*}$ or $a_{* *}$. Nonetheless, a surprising fact is true: Whenever the local condition $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$ is satisfied, we can construct some pair $(B, c)$ that justifies both $a_{*}$ and $a_{* *}$. That is, the local condition is sufficient for multiplicity even though equilibrium depends on global incentive compatibility.

THEOREM 3: Fix $a_{* *}, a_{*} \in A$, so that $a_{* *}>a_{*}$. The following are equivalent:
(i) $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$.
(ii) There exists $a(B, c)$ that justifies both $a_{*}$ and $a_{* *}$.
(iii) There is an uncountable set of pairs $(B, c)$, viz. $C$, so that $(i)$ if $(B, c) \in C$, then $\left\{a_{*}, a_{* *}\right\} \subseteq \mathcal{E}(B, c)$, and (ii) distinct elements of the set $C$ represent different preferences.

Observe that the requirement of $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$ is identified from the technological environment-i.e., the production technology and distributions of types and noise-alone. Thus, parts (i)-(ii) of Theorem 3 provide a necessary and sufficient condition for the technological environment to be consistent with multiple equilibria that differ in their level of effective accountability. There are preferences of politicians that justify (at least) two distinct effort levels if and only if there are effort levels $a_{* *}>a_{*}$, so that $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$. Part (iii) says that such preferences are non-degenerate. If there is some preference profile $(B, c)$ that justifies both $a_{*}$ and $a_{* *}$, then there are an uncountable set of distinct preferences that each justify both $a_{*}$ and $a_{* *}$.

Focus on the equivalence between parts (i) and (ii). One direction is trivial: It is immediate that part (ii) implies part (i). The converse is not obvious. When part (i) holds, it is trivial to construct a pair $(B, c)$ so that equation (4) holds. However, as we have seen, that is not sufficient for an equilibrium. In particular, if there is an equilibrium of $(B, c)$ with a first-period effort level $a_{*}$, then $\operatorname{IR}\left(a_{*}, a \mid a_{*}\right) \geq \frac{c\left(a_{*}\right)-c(a)}{B}$ for all $a \in A$. Thus, there are $|A|-1$ inequalities associated with any equilibrium level of effort-for multiplicity there are at least $2(|A|-1)$. This can result in uncountably many inequalities. Part (i), instead, verifies a single inequality.

## B. Understanding Multiplicity

Theorem 3 provides a necessary and sufficient condition for there to exist some pair $(B, c)$ that justifies two levels of effort. But it is silent on the extent to which those conditions put restrictions on cost functions or production functions that are consistent with multiplicity. In this section, we address some natural questions that arise about such restrictions. We then use that discussion to relate Theorem 3 to the multiplicity result in Dewatripont, Jewitt, and Tirole (1999).

Convex Costs.-If $\left(a_{*}, a_{* *}\right)$ is justifiable, then there exists some pair $(B, c)$ so that $\left\{a_{*}, a_{* *}\right\} \in \mathcal{E}(B, c)$. Notice, in the statement, there is no restriction on how we choose the cost function $c$. In particular, we do not require that $c$ be convex. At a conceptual level, we see no reason to restrict attention to convex cost functions (in this application). Nonetheless, it is useful to note that the result does not require that we choose a non-convex cost function. Example 11 in Appendix D illustrates this point: It gives an example of an environment where there are multiple equilibria that differ in their level of effective accountability. In that example, the production function is additive and the cost function is convex.

Production Technologies.-Theorem 3 provides a necessary and sufficient condition for the environment to be consistent with multiple equilibria that differ in their level of effective accountability. The condition is a joint requirement on production and beliefs. Importantly, any production function can satisfy the condition (for some set of beliefs). In particular:

PROPOSITION 2: Suppose $\phi$ is symmetric. Fix effort levels $a_{* *} \neq a_{*}$. There exists a nonempty open set $\mathbb{P}\left[a_{*}, a_{* *}\right] \subseteq[0,1] \times[0,1]$ so that the pair $\left(a_{*}, a_{* *}\right)$ is justifiable if $\left(\pi_{I}, \pi_{C}\right) \in \mathbb{P}\left[a_{*}, a_{* *}\right]$.

Proposition 2 says that for any production technology, any two effort levels $a_{*}, a_{* *}$, and any (symmetric) distribution of noise, we can find some distribution of types so that both $a_{*}$ and $a_{* *}$ are justifiable. This distribution of types will typically depend on the productivity of the particular effort levels relative to the productivity of type. To clarify this point, focus on the case where the Incumbent and Challenger are drawn from the same pool of candidates, so that $\pi_{I}=\pi_{C}$.

PROPOSITION 3: Suppose the model is symmetric, with $\pi_{I}=\pi_{C}=\pi$ and fix $a_{* *} \neq a_{*}$. There exists $a \hat{\pi}\left[a_{*}, a_{* *}\right] \in(0,1)$ so that the pair $\left(a_{*}, a_{* *}\right)$ is justifiable if and only if $\pi \geq \hat{\pi}\left[a_{*}, a_{* *}\right]$. Moreover:
(i) If effort and type are strict complements at $a_{*}, a_{* *}$, then $\hat{\pi}\left[a_{*}, a_{* *}\right]<\frac{1}{2}$.
(ii) If effort and type are both complements and substitutes at $a_{*}, a_{* *}$, then $\hat{\pi}\left[a_{*}, a_{* *}\right]=\frac{1}{2}$.
(iii) If effort and type are strict substitutes at $a_{*}, a_{* *}$, then $\hat{\pi}\left[a_{*}, a_{* *}\right]>\frac{1}{2}$.


Figure 5. Pairs $\left(\pi_{I}, \pi_{C}\right)$ so that ( $a_{*}, a_{* *}$ ) Is Justifiable

Proposition 3 focuses on the case when the Incumbent and Challenger are drawn from the same pool of candidates. It says that the pair $\left(a_{*}, a_{* *}\right)$ is justifiable if the pool of candidates is sufficiently weighted toward high quality types. The phrase "sufficiently weighted" is determined by $\hat{\pi}\left[a_{*}, a_{* *}\right]$, which in turn depends on whether effort and type are local substitutes or local complements.

The conclusions of Proposition 3 do not hinge on the Politicians being drawn from the same distribution. Proposition 4 in Appendix D shows that if they are drawn from "similar" distributions, the conclusions obtain.

Figure 5 panel B illustrates Propositions $2-3$ in the context of an example. Specifically, $\bar{\theta}=2, \underline{\theta}=1$, and the production function is parameterized by $f(a, \theta)$ $=(0.25 a+4 \theta)^{r}$. (The PDF $\phi$ is the standard Normal distribution.) The figures illustrate the set of $\left(\pi_{I}, \pi_{C}\right)$ so that the pair $\left(a_{*}, a_{* *}\right)=(1,2)$ is justifiable. In the context of this example, the requirement that $\pi_{C}$ is close to $\pi_{I}$ does not appear particularly demanding.

Connection to Dewatripont, Jewitt, and Tirole (1999).-In the context of wagebased incentives, Dewatripont, Jewitt, and Tirole (1999, henceforth, DJT) also show a multiplicity result. That result imposes assumptions on the distribution of types and the production technology. In particular, they consider a parameterized production technology that exhibits strict complementarities between the agent's effort and ability. Theorem 3 says that, in our context, multiplicity (associated with different levels of effective accountability) requires a particular form of complementaritythat the incremental return to high effort must be larger when the Voter expects high effort. This type of complementarity is generated endogenously through Voter learning. In particular, Proposition 2 says that strict complementarities between the Incumbent's effort and ability are neither necessary nor sufficient for this type of complementarity: Effort and ability may be strict substitutes and still two effort levels can be justifiable.

At a conceptual level, there is an important difference between the nature of our multiplicity results versus that in DJT. DJT fix a convex cost function as part of the description of the model. Conditional upon observing an outcome, the analysis endogenously chooses a benefit; the benefit is constrained to equal the conditional
expectation of the agent's type. Here, we also choose a benefit, but the nature of the choice is quite different: We choose a benefit as part of the description of the model and we do not constrain the benefit to equal the conditional expectation of the agent's type. Each of these choices is fit for the question at hand: In DJT, the benefit is a wage and the choice of wage is a strategic variable. Thus, it makes sense that the wage is chosen after observing variables and is equal to the conditional expectation of the agent's type. Here, however, the benefit reflects a subjective utility from holding office. Thus, the benefit is prior to the model and there is no reason the benefit ought to equal the conditional expectation of the Politician's type. Likewise, we do not impose a restriction that the chosen cost function must be convex; we allow for a more general class of preferences. (That said, we have seen that multiplicity can be consistent with convex cost functions.)

## C. Welfare and Behavior Results

Above we showed that variation in effective accountability due to variation in institutions can have counterintuitive effects on both Voter welfare and Voter behavior. Here we show that these results extend to the case where the variation in effective accountability derives from equilibrium multiplicity.

Welfare and Multiplicity.-Example 3 illustrated that increasing effective accountability, by changing institutions of formal accountability, may decrease Voter welfare. We observe that the same effect can happen in a setting where the variation in effective accountability comes from equilibrium multiplicity.

Example 6 (Example 3, Revisited): Consider the model in Example 3, now parameterized by $(B, c)$, where $B=10, c\left(a_{* *}\right)=5, c\left(a_{*}\right)=0.5$, and $c(\underline{a})=0$. It can be verified that $\left\{a_{*}, a_{* *}\right\} \subseteq \mathcal{E}(B, c)$. And, as shown in Example 3, $V W_{1}\left(a_{*}\right)+V W_{2}\left(a_{*}\right)>V W_{1}\left(a_{* *}\right)+V W_{2}\left(a_{* *}\right)$.

When is it the case that, within a given model $\mathcal{M}(B, c)$, higher levels of effective accountability are associated with higher levels of Voter welfare? Corollary 1 says that, when effort and type are (local) complements, higher levels of effective accountability are associated with higher levels of Voter welfare. Propositions 1 and 3 speak to the case when effort and type are (local) substitutes-at least when $\pi_{I}=\pi_{C}=\pi$. Proposition 3 says that, in this case, the pair $\left(a_{*}, a_{* *}\right)$ is justifiable if and only if $\pi$ meets some threshold $\hat{\pi}\left[a_{*}, a_{* *}\right]$, where $\hat{\pi}\left[a_{*}, a_{* *}\right]>\frac{1}{2}$. Proposition 1 adds that, in this case, $a_{* *}$ leads to higher Voter welfare if either $\pi$ is sufficiently large, i.e., greater than some $\bar{\pi}\left[a_{*}, a_{* *}\right]$, or sufficiently small, i.e., less than some $\underline{\pi}\left[a_{*}, a_{* *}\right]$. Certainly then, if $\pi$ is greater than $\max \left\{\hat{\pi}\left[a_{*}, a_{* *}\right], \bar{\pi}\left[a_{*}, a_{* *}\right]\right\}$, we have that, within a given model, the equilibrium with a higher level of effective accountability will be the equilibrium with a higher level of Voter welfare. However, we do not know if $\underline{\pi}\left[a_{*}, a_{* *}\right]>\hat{\pi}\left[a_{*}, a_{* *}\right]$ and so do not know if the conclusion only holds when $\pi_{I}$ $=\pi_{C}$ is sufficiently high. ${ }^{9}$

[^7]Re-election Rules and Multiplicity.—Examples 4-5 illustrated that increasing the level of effective accountability, by changing the institutions of formal accountability, may result in the Voter using a less stringent voting rule.

In the case of complements, the same effect can arise in a setting where the variation in effective accountability comes from equilibrium multiplicity.

Example 7 (Example 4, Revisited): Consider the model in Example 4, now parameterized by $(B, c)$, where $B=10, c(2)=2, c(1)=0$. It can be verified that $\{1,2\} \subseteq \mathcal{E}(B, c)$. And, as shown in Example 4, $\hat{g}(1)>\hat{g}(2)$.

Recall Example 5 showed that, in the case of substitutes, variation in effective accountability can lead the Voter to adopt a less stringent voting rule. In the case of substitutes, we do not know if the same conclusion holds when the variation in effective accountability is due to equilibrium multiplicity. (We have neither been able to construct an analogous example nor prove that such an example does not exist.) It, thus, remains an open question.

## VI. Normative Implications

The results in Section V highlight that there can be variation in effective accountability, even if there is no variation in institutions. Variation in players' expectations alone are sufficient to yield different levels of effective accountability. This fact has important normative implications, both for evaluating the performance of societies and institutions and for evaluating voter behavior.

## A. Non-Monotone Welfare Effects of Institutional Reform

It is important to recall that all of our welfare results depend on whether effort and type are local complements or substitutes. As such, the implications of variation in effective accountability for voter welfare may differ depending on the magnitude of the initial level of effort and the change in the incumbent's effort. For instance, there may be settings where effort and type are local complements at $\left(a_{*}, a_{* *}\right)$ and local substitutes at $\left(a_{* *}, a_{* * *}\right)$. (Indeed, we saw such an example in Section I.) Likewise, there may be settings in which effort and type are local complements at ( $a_{*}, a_{* *}$ ) and local substitutes at ( $\left.a_{*}, a_{* * *}\right)$.

This has important implications for thinking about the welfare impact of changes in institutions of formal accountability. Such changes may, in principle, have different effects depending on the extent to which they change $B$ or $c$. For example, suppose a small increase in an incumbent's salary leads to small increase in effort, while a large increase leads to a large increase in effort. If effort and type are local complements in the former case and local substitutes in the latter case, voter welfare would be higher for the small change but might be lower with the large change. This suggests the possibility that variation in an institutional parameter (e.g., B) could have non-monotone effects on voter welfare.

## B. Evaluating Society and Institutions: Accountability Traps

Our multiplicity and welfare results give rise to a concept that we will call an accountability trap. Fix a $\mathcal{M}(B, C)$. We say that the model is consistent with an accountability trap if there are (at least) two equilibria of the model which differ in terms of both their level of effective accountability and their level of Voter welfare. A society is caught in an accountability trap if it is playing an equilibrium with a lower level of Voter welfare than some other equilibrium. ${ }^{10}$ Taken together, Corollary 1 and Proposition 3 show the possibility of an accountability trap.

The idea of accountability traps helps make sense of a number of empirical observations: Countries with similar democratic institutions display considerable variation in the quality of governance outcomes. Further, in many societies with relatively bad governance, voters do not harshly sanction poorly performing politicians. (See, e.g., Bardhan 1997; Chang, Golden, and Hill 2010; Golden 2006.) These observations are consistent with the idea that different societies, while sharing the same set of institutions, can differ in their expectations about governance. Cynical voters can have low expectations for government performance and, consequently, politicians are cynical and do not bother to work hard on the voters' behalf. The result is poor governance outcomes that are not harshly sanctioned. Indeed, such outcomes are often associated with such cynicism. (See, e.g., the discussion in Bardhan 1997.)

The possibility of accountability traps has important implications for improving effective accountability and voter welfare. The literature has presumed that improving voter welfare requires improving the level of effective accountability and, in turn, that increasing the level of effective accountability requires changing institutions. There are two difficulties with this statement. First, a society may be in an accountability trap that involves a high level of effective accountability relative to the level that would maximize voter welfare. In that case, improving voter welfare may require lowering the level of effective accountability. Second, in the presence of an accountability trap, the first-order difficulty faced by the society may not be the current set of institutions. It may instead be societal expectations that leave it trapped in a situation with low voter welfare. Hence, institutional reform may not be effective-even a reform that moves to an institution that, on average, has been associated with a higher level of voter welfare in structurally similar societies.

How then can voter welfare be improved? The answer depends on the underlying source of the accountability trap. In this paper, accountability traps arise because the politician's early behavior influences the voter's ability to infer information about the politician's characteristics. To escape such an accountability trap, a society must shift expectations. And, indeed, improvements in effective accountability do appear consistent with such shifts. As Golden (2006) notes, improvements in accountability often happen suddenly, "as part of a wave of public revulsion."

[^8]
## C. Evaluating Voter Behavior

The primary goal of electoral accountability is to ensure that politicians have incentives to exert effort that benefit voters (Key 1966; Pitkin 1967; Fiorina 1981; Achen and Bartels 2004, 2006; Healy, Malhotra, and Mo 2010). Voters are meant to achieve this goal within a given institutional setting. This raises the question: How well are voters doing with respect to the goal of creating incentives for politicians to take costly effort on their behalf? The question is of interest in political philosophy (Pitkin 1967; Manin, Przeworski, and Stokes 1999; Mansbridge 2003, 2009) and empirical political economy (Fiorina 1981; Achen and Bartels 2004, 2006; Healy, Malhotra, and Mo 2010; Manacorda, Miguel, and Vigorito 2011; Bagues and Esteve-Volart 2016). A formal literature has developed to address this question.

The early formal literature addressed this by focusing on "pure moral hazard" models (Barro 1973; Ferejohn 1986; Austen-Smith and Banks 1989). In those models, there is no candidate heterogeneity, so voters are indifferent between incumbents and challengers. As such, any re-election rule is a best response for the voter. Given this, evaluating voter behavior involves asking how close voters come to using the rule that provides the politician with the most powerful incentives.

A critique of this literature, due to Fearon (1999), argues that this is the wrong benchmark. Choosing the re-election rule that provides the most powerful incentives is generally not credible: If the voter believes there is any difference in an incumbent's and challenger's expected future performance (i.e., if there is any payoff-relevant candidate heterogeneity), then sequential rationality requires him to elect the politician who is expected to perform better. Put differently, Fearon points out that the benchmark by which the voter is evaluated must incorporate the voter's desire to select good types. ${ }^{11}$ A subsequent literature has taken Fearon's argument to suggest that, when politicians are heterogenous, the desire to select good types uniquely pins down voter behavior. The conclusion is that the (historically important) question of evaluating voter behavior is trivial.

Our multiplicity results show that this conclusion is unwarranted. When candidates are heterogeneous, there is a more subtle way to evaluate voter performance: Are voters using the re-election rule that creates the strongest possible incentives relative to the set of all re-election rules consistent with an equilibrium? Theorem 3 and Propositions 2 and 3 say that this set may be nontrivial and so there remains scope for evaluating voter performance.

That said, our results are suggestive that the standard evaluative procedure may be problematic. The literature presumes that increasing effective accountability is necessarily desirable from the perspective of voter welfare. But, as we showed in Examples 3 and 6, this need not be the case: When effort and type are strict substitutes, a higher level of effective accountability reduces the quality of electoral selection. This creates the possibility that the overall quality of governance (as measured in voter welfare) is higher with a lower level of effective accountability.

[^9]If so, increasing effective accountability may not be a desirable goal. As such, our analysis suggests a rethinking of the evaluative criteria that underlies the existing literature-away from the level of effective accountability and towards a more comprehensive measure of voter welfare.

Theorems 1 and 2 also point to subtleties in how observed voter behavior may be mapped into evaluative measures. For instance, suppose we observe a voter adopting a more lenient benchmark by which to hold politicians accountable. The natural conclusion is that this reflects a decline in voter performance. However, our results show that this need not be the case. When $\pi_{C}>\pi_{I}$, it may well be welfare improving for the voter to adopt a more lenient benchmark. In the case of substitutes, this is because the more lenient benchmark corresponds to a lower level of effective accountability which, in turn, can improve voter welfare. In the case of complements, this is because the more lenient benchmark can correspond to a higher level of effective accountability which, in turn, improves voter welfare.

## Appendix A. Proofs for Section III

## PROOF OF THEOREM 1:

For each $\theta \in \Theta$, write $\Phi_{* *, \theta}$ (resp. $\Phi_{*, \theta}$ ) for the CDF defined by $\Phi_{* *, \theta}(g)=\Phi\left(g-f\left(a_{* *}, \theta\right)\right)$ (resp. $\Phi_{*, \theta}(g)=\Phi\left(g-f\left(a_{*}, \theta\right)\right)$ ). Since $\Phi_{* *, \theta}$ is strictly increasing on $\mathbb{R}$, for each $x \in(0,1), \Phi_{* *, \theta}^{-1}(x)$ is well-defined. Observe that $a_{* *}$ is more effective than $a_{*}$ if and only if, for each $g \in \mathbb{R},\left(\Phi_{* *, \theta}\right)^{-1}\left(\Phi_{*, \theta}(g)\right)$ is nondecreasing in $\theta$. With this, it suffices to show the following: For each $g$, $\left(\Phi_{* *, \theta}\right)^{-1}\left(\Phi_{*, \theta}(g)\right)=f\left(a_{* *}, \theta\right)-f\left(a_{*}, \theta\right)+g$.

Start by noting that, for each $x \in(0,1)$,

$$
\begin{equation*}
\Phi_{* *, \theta}^{-1}(x)=f\left(a_{* *}, \theta\right)+\Phi^{-1}(x) \tag{A1}
\end{equation*}
$$

To see this, fix $y=\Phi_{* *, \theta}^{-1}(x)$ and observe that $\Phi_{* *, \theta}(y)=\Phi_{* *, \theta}\left(\Phi_{* *, \theta}^{-1}(x)\right)=x$. So, $x=\Phi\left(y-f\left(a_{* *}, \theta\right)\right)$. Since $\Phi$ is injective, $\Phi^{-1}(x)=y-f\left(a_{* *}, \theta\right)$. Alternatively, $\Phi_{* *, \theta}^{-1}(x)=y=\Phi^{-1}(x)+f\left(a_{* *}, \theta\right)$. This establishes equation (A1).

Next observe that

$$
\begin{aligned}
\left(\Phi_{* *, \theta}\right)^{-1}\left(\Phi_{*, \theta}(g)\right) & =f\left(a_{* *}, \theta\right)+\Phi^{-1}\left(\Phi_{*, \theta}(g)\right) \\
& =f\left(a_{* *}, \theta\right)+\Phi^{-1}\left(\Phi\left(g-f\left(a_{*}, \theta\right)\right)\right) \\
& =f\left(a_{* *}, \theta\right)-f\left(a_{*}, \theta\right)+g
\end{aligned}
$$

where the first equality is by equation (A1), the second is by definition of $\Phi_{*, \theta}$, and the last equality uses the fact that $\Phi$ is injective. This establishes the desired result.

## A. Results for Symmetric Model

Fix some symmetric model $\mathcal{M}(B, c)$ where $\pi_{I}=\pi_{C}=\pi$. Consider an equilibrium associated with the first-period effort level $a_{*}$. Write $\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right)$ for the probability that the Politician in office in the second period is type $\bar{\theta}$. Note, the event "the Politician in office in the second period is type $\bar{\theta}$ " can be realized in one of three ways: (i) the first-period Incumbent is type $\bar{\theta}$ and is reelected, (ii) the first-period Incumbent and the Challenger are both type $\bar{\theta}$ and the first-period Incumbent is not reelected, or (iii) the first-period Incumbent is type $\underline{\theta}$, she is not reelected, and the Challenger is type $\bar{\theta}$. Thus,

$$
\begin{aligned}
\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right)= & \pi \operatorname{Pr}\left(f\left(a_{*}, \bar{\theta}\right)+\epsilon_{1} \geq \hat{g}\left(a_{*}\right)\right) \\
& +\pi^{2} \operatorname{Pr}\left(f\left(a_{*}, \bar{\theta}\right)+\epsilon_{1} \leq \hat{\mathrm{g}}\left(a_{*}\right)\right) \\
& +(1-\pi) \pi \operatorname{Pr}\left(f\left(a_{*}, \underline{\theta}\right)+\epsilon_{1} \leq \hat{g}\left(a_{*}\right)\right) .
\end{aligned}
$$

(This implicitly uses the fact that the random variable generating $\epsilon_{1}$ is atomless, i.e., so that $\operatorname{Pr}\left(f\left(a_{*}, \theta\right)+\epsilon_{1}=\hat{g}\left(a_{*}\right)\right)=0$.) Using the fact that $\operatorname{Pr}\left(f\left(a_{*}, \theta\right)+\epsilon_{1}\right.$ $\left.\geq \hat{g}\left(a_{*}\right)\right)=1-\Phi\left(\hat{g}\left(a_{*}\right)-f\left(a_{*}, \theta\right)\right)$,

$$
\operatorname{Pr}\left(\theta_{P 2}=\bar{\theta} \mid a_{*}\right)=\pi+(1-\pi) \pi\left[2 \Phi\left(\frac{f\left(a_{*}, \bar{\theta}\right)-f\left(a_{*}, \underline{\theta}\right)}{2}\right)-1\right] .
$$

Fix some $a \in A$. It will be convenient to introduce a function $\mathrm{VW}_{1}[a]:[0,1] \rightarrow \mathbb{R}$ and $\mathrm{VW}_{2}[a]:[0,1] \rightarrow \mathbb{R}$ so that

$$
\mathrm{VW}_{1}[a](p)=[f(a, \bar{\theta})-f(a, \underline{\theta})] p+f(a, \underline{\theta})
$$

and

$$
\begin{aligned}
\mathrm{VW}_{2}[a](p)= & (f(\underline{a}, \bar{\theta})-f(\underline{a}, \underline{\theta}))\left[p+(1-p) p\left(2 \Phi\left(\frac{f(a, \bar{\theta})-f(a, \underline{\theta})}{2}\right)-1\right)\right] \\
& +f(\underline{a}, \underline{\theta}) .
\end{aligned}
$$

Then, define $\mathrm{VW}[a]=\mathrm{VW}_{1}[a]+\mathrm{VW}_{2}[a]$. Note, if there is an equilibrium where $a$ is the first-period effort level and $p \in(0,1)$ is $p=\pi$, then $\mathrm{VW}_{1}[a](p)$ is first-period expected Voter welfare, $\mathrm{VW}_{2}[a](p)$ is second-period ex ante expected Voter welfare and VW $[a](p)$ is expected Voter welfare.

In what follows, we will fix a production function $f$ and effort levels $a_{* *}>a_{*}$. It will be convenient to adopt the notation (for the output of production) described in Figure A1. So, $\rho_{*}=f\left(a_{*}, \bar{\theta}\right)-f\left(a_{*}, \underline{\theta}\right)>0, \rho_{* *}=f\left(a_{* *}, \bar{\theta}\right)-f\left(a_{* *}, \underline{\theta}\right)>0$, and $\psi_{* *}=f\left(a_{* *}, \underline{\theta}\right)-f\left(a_{*}, \underline{\theta}\right)>0$.

LEMMA 3: $\psi_{* *}+\rho_{* *}-\rho_{*}>0$.

|  | $\underline{\theta}$ | $\bar{\theta}$ |
| :--- | :---: | :---: |
| $a_{* *}$ | $\psi_{*}+\psi_{* *}$ | $\psi_{*}+\psi_{* *}+\rho_{* *}$ |
| $a_{*}$ | $\psi_{*}$ | $\psi_{*}+\rho_{*}$ |

Figure A1. The Function $f$

## PROOF:

Note that $f\left(a_{* *}, \bar{\theta}\right)-f\left(a_{*}, \bar{\theta}\right)=\psi_{* *}+\rho_{* *}-\rho_{*}$, from which the claim follows $■$.

## PROOF OF PROPOSITION 1:

It will be convenient to adopt the notation for the production function described in Figure A1. Suppose $(B, c)$ justifies $a_{* *}$ and $a_{*}$, where $a_{* *}>a_{*}$. Consider the difference (in welfare) function $\Delta\left[a_{* *}, a_{*}\right]=\operatorname{VW}\left[a_{* *}\right]-\mathrm{VW}\left[a_{*}\right]$. Note, this is a continuous function with $\Delta\left[a_{* *}, a_{*}\right](0)=\psi_{* *}>0$ and (by Lemma 3) $\Delta\left[a_{* *}, a_{*}\right](1)=\psi_{* *}+\rho_{* *}-\rho_{*}>0$. Moreover, the derivative of the difference function with respect to $p$ is

$$
\left(\rho_{* *}-\rho_{*}\right)+2[f(\underline{a}, \bar{\theta})-f(\underline{a}, \underline{\theta})]\left[\Phi\left(\frac{\rho_{* *}}{2}\right)-\Phi\left(\frac{\rho_{*}}{2}\right)\right](1-2 p) .
$$

Thus, if effort and type are strict substitutes (respectively, strict complements) at $a_{*}, a_{* *}$, the difference function is strictly decreasing (respectively, strictly increasing) on $\left(0, p_{\min }\right) \neq \emptyset\left(\right.$ respectively, $\left.\left(0, p_{\max }\right) \neq \emptyset\right)$ and strictly increasing (respectively, strictly decreasing $)$ on $\left(p_{\min }, 1\right) \neq \emptyset$ (respectively, $\left.\left(p_{\max }, 1\right) \neq \emptyset\right)$. And, if effort and type are neither strict substitutes nor strict complements at $a_{*}, a_{* *}$, this is a constant function. It follows that there exists $\underline{\pi}\left[a_{*}, a_{* *}\right], \bar{\pi}\left[a_{*}, a_{* *}\right] \in(0,1)$ so that $p \in\left(0, \underline{\pi}\left[a_{*}, a_{* *}\right]\right) \cup\left(\bar{\pi}\left[a_{*}, a_{* *}\right], 1\right)$ if and only if $\Delta\left[a_{* *}, a_{*}\right](p) \geq 0$.

## Appendix B. Proofs for Section IV

Prior to presenting the results in this Appendix, we will establish a useful equivalence: Fix a real-valued random variable that is distributed according to some distribution full support differentiable PDF $\psi$. Define $L: \mathbb{R}^{3} \rightarrow \mathbb{R}$ so that

$$
L\left(y, x, x^{\prime}\right):=\frac{\psi(y-x)}{\psi\left(y-x^{\prime}\right)} .
$$

Say $\psi$ satisfies the strict MLRP if, for each $x>x^{\prime}, L\left(\cdot, x, x^{\prime}\right)$ is strictly increasing. Observe that, if $\psi$ satisfies the strict MLRP and each $L\left(\cdot, x, x^{\prime}\right)$ has nonzero derivatives, then the derivative of each $L\left(\cdot, x, x^{\prime}\right)$ with respect to $y$ is strictly positive (at each point).

## LEMMA 4:

(i) If $\psi$ satisfies the strict MLRP and each likelihood ratio $L\left(\cdot, x, x^{\prime}\right)$ has nonzero derivatives, then $\log \circ \psi$ is strictly concave.
(ii) If $\log \circ \psi$ is strictly concave, then $\psi$ satisfies the strict MLRP and each likelihood ratio $L\left(\cdot, x, x^{\prime}\right)$ has nonzero derivatives.

REMARK 1: For each $\tilde{y}$ and $x>x^{\prime}$,

$$
\left.\frac{\partial L\left(\cdot, x, x^{\prime}\right)}{\partial y}\right|_{\tilde{y}}=\frac{\psi^{\prime}(\tilde{y}-x) \psi\left(\tilde{y}-x^{\prime}\right)-\psi(\tilde{y}-x) \psi^{\prime}\left(\tilde{y}-x^{\prime}\right)}{\psi\left(\tilde{y}-x^{\prime}\right)^{2}}>0
$$

if and only if

$$
(\log \circ \psi)^{\prime}(\tilde{y}-x)=\frac{\psi^{\prime}(\tilde{y}-x)}{\psi(\tilde{y}-x)}>\frac{\psi^{\prime}\left(\tilde{y}-x^{\prime}\right)}{\psi\left(\tilde{y}-x^{\prime}\right)}=(\log \circ \psi)^{\prime}\left(\tilde{y}-x^{\prime}\right) .
$$

## PROOF OF LEMMA 4:

First suppose that $\psi$ satisfies the strict MLRP and each likelihood ratio $L\left(\cdot, x, x^{\prime}\right)$ has nonzero derivatives. Then, for each $\tilde{y},\left.\frac{\partial L\left(\cdot, x, x^{\prime}\right)}{\partial y}\right|_{\tilde{y}}>0$. For each $z^{\prime}>z$, choose $x>x^{\prime}$ and $y^{\prime}$ so that $z^{\prime}=y^{\prime}-x^{\prime}$ and $z=y^{\prime}-x$. It follows from Remark 1 that $(\log \circ \psi)^{\prime}\left(z^{\prime}\right)<(\log \circ \psi)^{\prime}(z)$ and so $\log \circ \psi$ is concave.

Next suppose $\log \circ \psi$ is strictly concave. The fact that $\psi$ satisfies the strict MLRP is standard. (E.g., in Lehmann and Romano 2005, Example 8.2.1 replace weak inequality with strict inequality.) Remark 1 completes the proof.

## PROOF OF LEMMA 1:

Observe that, for each $g$, there exists some $g>g^{\prime}$ so that $g=g^{\prime}+$ $\left(\nu\left(a_{* *}\right)-\nu\left(a_{*}\right)\right)$. Since

$$
\tilde{\mathrm{L}}(g)=\frac{\phi\left(g^{\prime}-\nu\left(a_{*}\right)-\iota\left(a_{*}\right)\right)}{\phi\left(g^{\prime}-\nu\left(a_{*}\right)+\iota\left(a_{*}\right)\right)}=\mathrm{LR}_{*}\left(g^{\prime}\right)
$$

and $\mathrm{LR}_{*}(g)>\mathrm{LR}_{*}\left(g^{\prime}\right)$, it follows that $\mathrm{LR}_{*}(g)>\tilde{\mathrm{L}}(g)$. Since $\tilde{\mathrm{L}}$ is strictly increasing, $\tilde{\mathrm{L}}(\tilde{g})=\beta$ implies that $\tilde{g}>\hat{g}\left(a_{*}\right)$.

LEMMA 5: Suppose $\phi$ is symmetric. Consider a function : $\mathbb{R} \times \mathbb{R}_{+} \rightarrow \mathbb{R}_{*}$ given by

$$
\mathcal{L}(x, \delta)=\frac{\phi(x-\delta)}{\phi(x+\delta)}
$$

(i) $\mathcal{L}$ is increasing in $\delta$ if $x>0$.
(ii) $\mathcal{L}$ is decreasing in $\delta$ if $x<0$.

## PROOF OF LEMMA 5:

First, define $\underline{Q}(x, \delta)=x-\delta$ and $\bar{Q}(x+\delta)$. Note that

$$
\log (x, \delta)=\log \phi(\underline{Q}(x, \delta))-\log \phi(\bar{Q}(x, \delta)) .
$$

Now, differentiate with respect to $\delta$ to get

$$
\begin{aligned}
\frac{\partial}{\partial \delta} \log \mathcal{L}(x, \delta) & =\frac{\phi^{\prime}(\underline{Q}(x, \delta))}{\phi(\underline{Q}(x, \delta))} \frac{\partial \underline{Q}(x, \delta)}{\partial \delta}-\frac{\phi^{\prime}(\bar{Q}(x, \delta))}{\phi(\bar{Q}(x, \delta))} \frac{\partial \bar{Q}(x, \delta)}{\partial \delta} \\
& =\frac{\phi^{\prime}(x-\delta)}{\phi(x-\delta)}-\frac{\phi^{\prime}(x+\delta)}{\phi(x+\delta)}
\end{aligned}
$$

Since $\phi$ is symmetric, $\phi^{\prime}(y)=-\phi^{\prime}(-y)$. So, for all $\delta, \frac{\partial}{\partial \delta} \log \mathcal{L}(0, \delta)=0$. With this, the claim will follow if $\frac{\partial}{\partial \delta} \log \mathcal{L}(x, \delta)$ is increasing in $x$.

Differentiating $\log (x, \delta)$,

$$
\begin{aligned}
\frac{\partial^{2}}{\partial \delta \partial x} \log \mathcal{L}(x, \delta) & =-\frac{d}{d \underline{Q}}\left(\frac{\phi^{\prime}(\underline{Q}(x, \delta))}{\phi(\underline{Q}(x, \delta))}\right) \frac{\partial \underline{Q}(x, \delta)}{\partial x}-\frac{d}{d \bar{Q}}\left(\frac{\phi^{\prime}(\bar{Q}(x, \delta))}{\phi(\bar{Q}(x, \delta))}\right) \frac{\partial \bar{Q}(x, \delta)}{\partial x} \\
& =-\frac{d}{d \underline{Q}}\left(\frac{\phi^{\prime}(\underline{Q}(x, \delta))}{\phi(\underline{Q}(x, \delta))}\right)-\frac{d}{d \bar{Q}}\left(\frac{\phi^{\prime}(\bar{Q}(x, \delta))}{\phi(\bar{Q}(x, \delta))}\right)
\end{aligned}
$$

By Lemma 4, $\phi$ is strictly $\log$ concave. From this, the last equivalence is strictly positive.

## PROOF OF LEMMA 2:

Immediate from Lemma 5.

## PROOF OF THEOREM 2:

By Lemma 1, it suffices to show that $\hat{g}\left(a_{* *}\right) \geq \tilde{g}$. To show this, it suffices to show that $\mathrm{LR}_{* *}(\tilde{g}) \leq \tilde{\mathrm{L}}(\tilde{g})=\beta$. If so, then using the MLRP and the fact that $\mathrm{LR}_{* *}\left(\hat{g}\left(a_{* *}\right)\right)$ $=\beta$, it follows that $\hat{g}\left(a_{* *}\right) \geq \tilde{g}$.

Observe that, by symmetry of the PDF,

$$
\tilde{\mathrm{L}}\left(\nu\left(a_{* *}\right)\right)=\frac{\phi\left(-\iota\left(a_{*}\right)\right)}{\phi\left(\iota\left(a_{*}\right)\right)}=1=\frac{\phi\left(-\iota\left(a_{* *}\right)\right)}{\phi\left(\iota\left(a_{* *}\right)\right)}=\operatorname{LR}_{* *}\left(\nu\left(a_{* *}\right)\right) .
$$

So, if $\pi_{I} \geq \pi_{C}$ (resp. $\pi_{C} \geq \pi_{I}$ ) and effort and type are complements (resp. substitutes) at $a_{* *}, a_{*}$, Lemma 2 gives that $\mathrm{LR}_{* *}(\tilde{g}) \leq \tilde{\mathrm{L}}(\tilde{g})=\beta$, as desired.

## Appendix C. Formal Accountability

Are there systematic changes in the institutions of formal accountability that serve to increase effective accountability? The main text provides examples (Examples 3-4) where higher levels of $B$ correspond to higher levels of effective accountability. But, this monotonicity need not hold more generally, as the next two examples show. The first example has global complements; the second has global substitutes.

Example 8: Let $A=\{0,1,2\}$. The production function $f$ is described by Table C1. Take $\Phi$ to be the CDF of the standard Normal distribution and let $\pi_{I}=\pi_{C}=0.5$.

Using Example 1, the incremental returns satisfy the bounds in Table C2.
We will consider two models parameterized by $\left(B_{* *}, c_{* *}\right)=\left(1, c_{* *}\right)$ and $\left(B_{*}, c_{*}\right)=\left(1.26, c_{*}\right)$, where $c_{* *}(2)=c_{*}(2)=0.65, c_{* *}(1)=c_{*}(1)=0.041$, and $c_{* *}(0)=c_{*}(0)=0$. It can be verified that $\mathcal{E}\left(1, c_{* *}\right)=\{0,1\}$ and $\mathcal{E}\left(1.26, c_{*}\right)$ $=\{0\}$. Thus, the model associated with $\left(B_{* *}, c_{* *}\right)=\left(1, c_{* *}\right)$ has an equilibrium with a higher level of effective accountability than any equilibrium of the model associated with $\left(B_{*}, c_{*}\right)=\left(1.26, c_{* *}\right)$, even though $B_{* *}<B_{*}$.

Table C1—Production Function

|  | $\underline{\theta}$ | $\bar{\theta}$ |
| :---: | :--- | :---: |
| 2 | 6.5 | 11 |
| 1 | 0.5 | 4.9 |
| 0 | 0 | 4 |

Table C2—Incremental Returns in Reelection Probabilities

| $\operatorname{IR}(1,0 \mid 0) \in(0.0324,0.0325)$ | $\operatorname{IR}(1,0 \mid 1) \in(0.0466,0.0467)$ | $\operatorname{IR}(1,0 \mid 2) \in(0.000029,0.00003)$ |
| :--- | :--- | :--- |
| $\operatorname{IR}(2,0 \mid 0) \in(0.4999,0.5001)$ | $\operatorname{IR}(2,0 \mid 1) \in(0.5466,0.5467)$ | $\operatorname{IR}(2,0 \mid 2) \in(0.4999,0.5001)$ |
| $\operatorname{IR}(2,1 \mid 0) \in(0.4675,0.4676)$ | $\operatorname{IR}(2,1 \mid 1) \in(0.49996,0.49997)$ | $\operatorname{IR}(2,1 \mid 2) \in(0.49996,0.49998)$ |

Notice, since this example has strict substitutes, it is unambiguous that Voter welfare is lower in an equilibrium with lower effective accountability. Thus, in this example, increasing the rewards to office can lead to reduction in Voter welfare even though the Voter does not have to pay for that increased reward.

Example 9: Let $A=\{0,1,2\}$. The production function $f$ is described by Table C3. Take $\Phi$ to be the CDF of the standard Normal distribution and let $\pi_{I}=0.6$ and $\pi_{C}=0.5$.

Table C3-Production Function

|  | $\underline{\theta}$ | $\bar{\theta}$ |
| :---: | :---: | :---: |
| 2 | 8 | 9 |
| 1 | 5 | 7 |
| 0 | 0 | 4 |

Using Example 1, the incremental returns satisfy the bounds of Table C4.
We will consider two models parameterized by $\left(B_{* *}, c_{* *}\right)=\left(1, c_{* *}\right)$ and $\left(B_{*}, c_{*}\right)=\left(1.3, c_{*}\right)$, where $c_{* *}(2)=c_{*}(2)=1.04, \quad c_{* *}(1)=c_{*}(1)=0.57$, and $c_{* *}(0)=c_{*}(0)=0$. It can be verified that $\mathcal{E}\left(1, c_{* *}\right)=\{0,1\}$ and $\mathcal{E}\left(5, c_{*}\right)$ $=\{0\}$. Thus, the model associated with $\left(B_{* *}, c_{* *}\right)=\left(1, c_{* *}\right)$ has an equilibrium
with a higher level of effective accountability than any equilibrium of the model associated with $\left(B_{*}, c_{*}\right)=\left(1.3, c_{* *}\right)$, even though $B_{* *}<B_{*}$.

We can reparameterize Examples 3, 4, 8, and 9 to show that, holding $B$ fixed and decreasing $c$ also does not lead to systematic changes in effective accountability.

Table C4-Incremental Returns in Reelection Probabilities

| $\operatorname{IR}(1,0 \mid 0) \in(0.3987,0.3988)$ | $\operatorname{IR}(1,0 \mid 1) \in(0.5946,0.5947)$ | $\operatorname{IR}(1,0 \mid 2) \in(0.0824,0.0825)$ |
| :--- | :--- | :--- |
| $\operatorname{IR}(2,0 \mid 0) \in(0.3991,0.3992)$ | $\operatorname{IR}(2,0 \mid 1) \in(0.9723,0.9724)$ | $\operatorname{IR}(2,0 \mid 2) \in(0.6753,0.6754)$ |
| $\operatorname{IR}(2,1 \mid 0) \in(0.0003,0.0004)$ | $\operatorname{IR}(2,1 \mid 1) \in(0.3777,0.3778)$ | $\operatorname{IR}(2,1 \mid 2) \in(0.5928,0.5929)$ |

## Appendix D. Examples and Proofs for Section V

We begin with the examples mentioned in the main text.
Example 10: Take $A=\{0,1,2\}, \bar{\theta}=2, \underline{\theta}=1$, and $f(a, \theta)=a+\theta$. Let $\pi_{I}=\pi_{C}=\frac{3}{4}$ and let $\Phi$ be the standard Normal distribution. Then $\hat{g}(a)=a+1.5$ (see Example 1) and $\operatorname{IR}\left(a, a^{\prime} \mid a\right)$ satisfies:

| $\operatorname{IR}(0,1 \mid 0) \in(-0.2771,-0.2770)$ | $\operatorname{IR}(0,2 \mid 0) \in(-0.3830,-0.3829)$ |
| :--- | :--- |
| $\operatorname{IR}(1,0 \mid 1) \in(0.3476,0.3477)$ | $\operatorname{IR}(1,2 \mid 1) \in(-0.2771,-0.2770)$ |
| $\operatorname{IR}(2,0 \mid 2) \in(0.5440,0.5441)$ | $\operatorname{IR}(2,1 \mid 2) \in(0.3476,0.3477)$ |

Consider the model $\mathcal{M}(B, c)$, where $B=1$ and $c(a)=0.2 a^{2}$. There is no equilibrium where the Incumbent chooses a first-period level of effort $a \in\{0,2\}$. (Observe that $c(0)-c(1)=-0.2>\operatorname{IR}(0,1 \mid 0)=\operatorname{IR}(0,1 \mid 0)(B-c(0))$ and $c(2)-c(0)=0.8>\operatorname{IR}(2,0 \mid 2)=\operatorname{IR}(2,0 \mid 2)(B-c(0))$.

However, we can construct an equilibrium where the Incumbent chooses a first-period level of effort $a_{*}=1$. (Observe that $\operatorname{IR}(1,0 \mid 1)(B-c(0))$ $=\operatorname{IR}(1,0 \mid 1)>0.2=c(1)-c(0) \quad$ and $\quad \operatorname{IR}(1,2 \mid 1)(B-c(0))=\operatorname{IR}(1,2 \mid 1)$ $>-0.6=c(1)-c(2)$.

We next point out that, by perturbing the cost of effort, we have multiple equilibria that differ in their levels of effective accountability.

Example 11: Again, consider the environment in Example 10. Now change only the cost function so that $c(a)=0.3 a^{2}$. We can still construct an equilibrium where the Incumbent chooses the first-period level of effort $a_{*}=1$, since $\operatorname{IR}(1,0 \mid 1)$ $\times(B-c(0))=\operatorname{IR}(1,0 \mid 1)>0.3=c(1)-c(0) \quad$ and $\quad \operatorname{IR}(1,2 \mid 1)(B-c(0))$ $=\operatorname{IR}(1,2 \mid 1)>-0.9=c(1)-c(2)$. But now we can also construct an equilibrium where the Incumbent chooses the first-period level of effort $a_{* *}=0$, since $\operatorname{IR}(0,1 \mid 0)(B-c(0))=\operatorname{IR}(0,1 \mid 0)>-0.3=c(0)-c(1)$ and $\operatorname{IR}(0,2 \mid 0)(B-c(0))=\operatorname{IR}(0,2 \mid 0)>-1.2=c(0)-c(2)$.

Example 12: Again consider the environment in Example 10, but let $\pi_{I}=\pi_{C}=0.85$. Then $\operatorname{IR}\left(a, a^{\prime} \mid a\right)$ satisfies the following bounds:

| $\operatorname{IR}(0,1 \mid 0) \in(-0.2630,-0.2629)$ | $\operatorname{IR}(0,2 \mid 0) \in(-0.3507,-0.3506)$ |
| :--- | :--- |
| $\operatorname{IR}(1,0 \mid 1) \in(0.3617,0.3618)$ | $\operatorname{IR}(1,2 \mid 1) \in(-0.2630,-0.2629)$ |
| $\operatorname{IR}(2,0 \mid 2) \in(0.5763,0.5764)$ | $\operatorname{IR}(2,1 \mid 2) \in(0.3617,0.3618)$ |

Consider the model $\mathcal{M}(B, c)$, where $B=1$ and $c(a)=0.1 a^{2}$. Observe that

$$
\operatorname{IR}(2,0 \mid 2) \geq \frac{c(2)-c(0)}{B-c(0)}=0.4 \geq-\operatorname{IR}(0,2 \mid 0)=\operatorname{IR}(2,0 \mid 0)
$$

So equation (4) is satisfied for effort levels $a_{*}=0$ and $a_{* *}=2$. We might then conclude that $\{0,2\} \subseteq \mathcal{E}(B, c)$. But, in fact, $0 \notin \mathcal{E}(B, c)$.

A strategy of the Incumbent can be written as $s_{I}=\left(a, s_{I, 2}\right)$, where $a$ is a first-period level of effort and $s_{I, 2}$, a second-period plan, maps each first-period level of public goods plus a decision to reelect the Incumbent to a second-period level of effort. Since the Incumbent's second-period information set can only be reached by a decision to reelect, we suppress reference to the decision and simply write $s_{I, 2}(g)$. A strategy of the Voter $s_{V}$ maps each level of public goods provided (in the first period) to a re-election decision. Thus, $s_{V}(g)=1\left(s_{V}(g)=0\right)$ represents the fact that the Voter reelects (replaces) the Incumbent if $g$ is the level of first-period public goods. A strategy for the Challenger $s_{C}$ maps each level of first-period public goods provided and a decision to replace the Incumbent to a second period level of effort. Since the Challenger's second-period information set can only be reached by a decision to replace, we suppress reference to the decision and simply write $s_{C}(g)$.

LEMMA 6: The pair $(B, c)$ justifies $a_{*}$ if and only if $c(a) \geq \operatorname{IR}\left(a, a_{*} \mid a_{*}\right)(B-c(\underline{a}))+$ $c\left(a_{*}\right)$ for each $a \in A$.

## PROOF:

If $(B, c)$ justifies $a_{*}$, then

$$
\operatorname{Pr}\left(a_{*} \mid a_{*}\right)(B-c(\underline{a}))+\left(B-c\left(a_{*}\right)\right) \geq \operatorname{Pr}\left(a \mid a_{*}\right)(B-c(\underline{a}))+(B-c(a))
$$

for each level of effort $a \in A$. From this the claim follows. Conversely, suppose that $c(a) \geq \operatorname{IR}\left(a, a_{*} \mid a_{*}\right)(B-c(\underline{a}))+c\left(a_{*}\right)$ for each $a \in A$. Construct $\left(s_{I}, s_{V}, s_{C}\right)$ so that (i) $s_{I}=\left(a_{*}, s_{I, 2}\right)$, (ii) for each realization of public goods $g, s_{I, 2}(g)$ $=s_{C}(g)=\underline{a}$, and (iii) $s_{V}(g)=1$ if and only if $g \geq \hat{g}\left(a_{*}\right)$. It is readily verified that $\left(s_{I}, s_{V}, s_{C}\right)$ is a perfect Bayesian equilibrium.

LEMMA 7: Fix some $a_{*}$ and some $a>a^{\prime}$.
(i) $\operatorname{Pr}\left(a \mid a_{*}\right)>\operatorname{Pr}\left(a^{\prime} \mid a_{*}\right)$.
(ii) $\operatorname{IR}\left(a, a^{\prime} \mid a_{*}\right)>0$.
(iii) $\operatorname{IR}\left(a_{*}, a^{\prime} \mid a_{*}\right)>\operatorname{IR}\left(a_{*}, a \mid a_{*}\right)$.

## PROOF:

Part (i) follows from the fact that $f$ is increasing in $a$. Part (ii) follows from part (i). Part (iii) follows from part (i), since $\operatorname{IR}\left(a_{*}, a^{\prime} \mid a_{*}\right)=\operatorname{Pr}\left(a_{*} \mid a_{*}\right)-\operatorname{Pr}\left(a^{\prime} \mid a_{*}\right)>$ $\operatorname{Pr}\left(a_{*} \mid a_{*}\right)-\operatorname{Pr}\left(a \mid a_{*}\right)=\operatorname{IR}\left(a_{*}, a \mid a_{*}\right)$.

Before coming to the proof of Theorem 3 we provide a sketch of the argument. Consider the case where $a_{*}=\underline{a}$. The idea will be to fix a benefit of re-election, viz. $B$, and constants $n_{*}$ and $n_{* *}$. The constants will turn out to be the costs associated with the high and the lowest effort levels-i.e., when we later choose a cost function $c$ that justifies both $a_{*}$ and $a_{* *}$, it will satisfy $c\left(a_{*}\right)=n_{*}$ and $c\left(a_{* *}\right)=n_{* *}$. As such, we fix $B>n_{*}>0$ and (in light of the necessity condition given by equation (4)), we fix $n_{* *}>n_{*}$ so that

$$
\begin{equation*}
\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \frac{n_{* *}-n_{*}}{B-n_{*}} \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right) \tag{D1}
\end{equation*}
$$

Suppose, in fact, that $c$ is a cost function with $c\left(a_{*}\right)=n_{*}$ and $c\left(a_{* *}\right)=n_{* * *}$. Notice that, if $(B, c)$ justifies $a_{*}$, it must be that $c(\cdot)$ lies above the function $N\left(\cdot, a_{*}\right)$ : $A \rightarrow \mathbb{R}$, where

$$
N\left(a, a_{*}\right)=\operatorname{IR}\left(a, a_{*} \mid a_{*}\right)\left(B-n_{*}\right)+n_{*} .
$$

If this condition were not satisfied, the Incumbent would have an incentive to deviate from $a_{*}$ to an alternate effort level. Analogously, if $(B, c)$ justifies $a_{* *}$, it must be that $c(\cdot)$ lies above the function $N\left(\cdot, a_{* *}\right): A \rightarrow \mathbb{R}$ with

$$
N\left(a, a_{* *}\right)=\operatorname{IR}\left(a, a_{* *} \mid a_{* *}\right)\left(B-n_{*}\right)+n_{* *} .
$$

Figure D1 illustrates the functions $N\left(\cdot, a_{*}\right)$ and $N\left(\cdot, a_{* *}\right)$. They are each strictly increasing. Moreover, by equation (D1), $N\left(a_{*}, a_{*}\right)=n_{*} \geq N\left(a_{*}, a_{* *}\right)$ and $N\left(a_{* *}, a_{* *}\right)=n_{* *} \geq N\left(a_{* *}, a_{*}\right)$. Thus, taking $c$ to be the upper envelope of the functions, $(B, c)$ justifies both $a_{*}$ and $a_{* *}$.

## PROOF OF THEOREM 3: PART (i) IF AND ONLY IF PART (ii):

The fact that part (ii) implies part (i) follows immediately from Lemma 6. We show that part (i) implies part (ii). Suppose that $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$ and we will show that we can construct $(B, c)$ that justifies both $a_{* *}$ and $a_{*}$.

To do so, it will be useful to fix certain constants: First choose $B$ and $\underline{n}$ so that $B>\underline{n}>0$. If $a_{*}=\underline{a}$, fix $n_{*}=\underline{n}$. If $a_{*} \neq \underline{a}$, fix $n_{*}$ so that:
$\mathbf{i}_{*} \quad n_{*}>\underline{n} ;$
$\mathbf{i i}_{*} \quad \underline{n}+\operatorname{IR}\left(a_{*}, \underline{a} \mid a_{*}\right)(B-\underline{n})>n_{*} ;$ and


Figure D1
$\mathbf{i i i}_{*} \quad \underline{n}+\left[\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)-\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)\right](B-\underline{n})>n_{*}$.
The fact that requirements $\mathrm{i}_{*}-\mathrm{i} \mathrm{i}_{*}$ can be satisfied simultaneously follows from Lemma 7(ii). The fact that requirements $\mathrm{i}_{*}-\mathrm{iii}_{*}$ can be satisfied simultaneously follows from $\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)>\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right)(\operatorname{Lemma} 7(i i i))$ and $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq$ $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$ (by assumption), so that $\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)>\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$.

Now fix $n_{* *}$ such that
$\mathbf{i}_{* *} \quad n_{* *} \geq n_{*}+\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)(B-\underline{n}) ;$
$\mathbf{i i}_{* *} \quad n_{*}+\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right)(B-\underline{n}) \geq n_{* *} ;$ and
$\mathbf{i i i}_{* *} \quad \underline{n}+\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)(B-\underline{n}) \geq n_{* *}$ with strict inequality if $a_{*} \neq \underline{a}$.
The fact that requirements $\mathrm{i}_{* *}-\mathrm{i} \mathrm{i}_{* *}$ can be satisfied simultaneously follows from the assumption that $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$. Condition iii ${ }_{* *}$ follows from condition $\mathrm{ii}_{* *}$, if $a_{*}=\underline{a}$. The fact that requirements $\mathrm{i}_{* *}-\mathrm{iii}_{* *}$ can be satisfied simultaneously when $a_{*}>\underline{a}$ follows from condition $\mathrm{iii}_{*}$ above. Note, it follows from Lemma 7 and $\mathrm{i}_{* *}$ that $n_{* *}>n_{*}$.

Construct a function $N: A \times\left\{a_{*}, a_{* *}\right\} \rightarrow \mathbb{R}$ so that $N\left(a, a_{*}\right)=\operatorname{IR}\left(a, a_{*} \mid a_{*}\right)$ $\times(B-\underline{n})+n_{*}$ and $N\left(a, a_{* *}\right)=\operatorname{IR}\left(a, a_{* *} \mid a_{* *}\right)(B-\underline{n})+n_{* *}$. It follows from Lemma 7 that $N\left(\cdot, a_{*}\right)$ and $N\left(\cdot, a_{* *}\right)$ are strictly increasing in $a$. Moreover,

- $\underline{n} \geq \max \left\{N\left(\underline{a}, a_{*}\right), N\left(\underline{a}, a_{* *}\right)\right\} ;$
- $n_{*}=N\left(a_{*}, a_{*}\right) \geq N\left(a_{*}, a_{* *}\right)$;
- $n_{* *}=N\left(a_{* *}, a_{* *}\right) \geq N\left(a_{* *}, a_{*}\right)$.

The first of these follows from requirement $\mathrm{ii}_{*}$ on $n_{*}$ and requirement $\mathrm{iii}_{* *}$ on $n_{* *} \cdot 12$ The second of these follows from requirement $\mathrm{ii}_{* *}$ on $n_{* *}$. The third of these follows from requirement $i_{* *}$ on $n_{* *}$.

Now let $\hat{N}: A \rightarrow \mathbb{R}$ be the upper envelope of $N\left(\cdot, a_{*}\right)$ and $N\left(\cdot, a_{* *}\right)$, i.e., $\hat{N}(a)=\max \left\{N\left(a, a_{*}\right), N\left(a, a_{* *}\right)\right\}$ for each $a \in A$. It is strictly increasing. Moreover, it satisfies

- $\hat{N}(\underline{a}) \leq \underline{n}$,
- $\hat{N}\left(a_{*}\right)=n_{*}$, and
- $\hat{N}\left(a_{* *}\right)=n_{* *}$.

It follows that we can construct a strictly increasing function $c: A \rightarrow \mathbb{R}$ that lies everywhere above $\hat{N}$, i.e., for each $a \in A, c(a) \geq \hat{N}(a)$, with

- $c(\underline{a})=\underline{n}$,
- $c\left(a_{*}\right)=n_{*}$, and
- $c\left(a_{* *}\right)=n_{* *}$.

Applying Lemma 6 we get that the $(B, c)$ constructed justifies both $a_{* *}$ and $a_{*}$.
Preferences $(B, c)$ and $\left(B^{\prime}, c^{\prime}\right)$ represent different preferences over risk if there is no $(x, y) \in \mathbb{R} \times \mathbb{R}_{+}$and so that $c^{\prime}(a)=x+y c(a)$ for all $a \in A$.

PROOF OF THEOREM 3: PART (iii) IF AND ONLY IF PART (i):
It is immediate that part (iii) implies part (ii). Fix $a_{* *}>a_{*}$, so that the pair $\left(a_{*}, a_{* *}\right)$ is justifiable and we will show that part (iii) is satisfied.

By the proof that part (i) implies part (ii), we can find a part $(B, c)$ that justifies both $a_{*}$ and $a_{* *}$ and satisfies the following requirements:
$\mathbf{i}_{*} \quad$ if $a_{*} \neq \underline{a}, c\left(a_{*}\right)>c(\underline{a}) ;$
$\mathbf{i i}_{*} \quad$ if $a_{*} \neq \underline{a}, c(\underline{a})+\operatorname{IR}\left(a_{*}, \underline{a} \mid a_{*}\right)(B-c(\underline{a}))>c\left(a_{*}\right) ;$
iii $_{*} \quad$ if $a_{*} \neq \underline{a}, c(\underline{a})+\left[\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)-\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)\right](B-c(\underline{a}))>c\left(a_{*}\right) ;$
$\mathbf{i}_{* *} \quad c\left(a_{* *}\right) \geq c\left(a_{*}\right)+\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)(B-c(\underline{a})) ;$
$\mathbf{i i}_{* *} \quad c\left(a_{*}\right)+\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right)(B-c(\underline{a})) \geq c\left(a_{* *}\right) ;$ and
$\mathbf{i i i}_{* *} \quad c(\underline{a})+\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)(B-c(\underline{a})) \geq c\left(a_{* *}\right)$ with strict inequality if $a_{*} \neq \underline{a}$.
Let $\alpha=\frac{c\left(a_{* *}\right)-c\left(a_{*}\right)}{B-c(\underline{a})}$ and observe that

$$
\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \alpha \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)>0
$$

${ }^{12}$ Of course, $\underline{n}=N\left(\underline{a}, a_{*}\right)$ if $a_{*}=\underline{a}$.

For a given $\varepsilon>0$, define

- $B^{\prime}=B+2 \varepsilon$,
- $c^{\prime}\left(a_{* *}\right)=c\left(a_{* *}\right)+\varepsilon(\alpha+1)$,
- $c^{\prime}\left(a_{*}\right)=c\left(a_{*}\right)+\varepsilon$, and
- $c^{\prime}(\underline{a})=c(\underline{a})+\varepsilon$.
(Observe that $c^{\prime}\left(a_{* *}\right)>c^{\prime}\left(a_{*}\right)$.) We will show that we can construct some pair $(\hat{B}, \hat{c})$ so that $\hat{B}=B^{\prime}, \hat{c}\left(a_{* *}\right)=c^{\prime}\left(a_{* *}\right), \hat{c}\left(a_{*}\right)=c^{\prime}\left(a_{*}\right), \hat{c}(\underline{a})=c^{\prime}(\underline{a})$, and $(\hat{B}, \hat{c})$ justifies $\left(a_{*}, a_{* *}\right)$. Since $(\hat{B}, \hat{c})$ represents different preferences than $(B, c)$ and $\varepsilon>0$ is chosen arbitrarily, this establishes the desired result.

First, observe we have the following properties:
$\mathbf{i}_{*}^{\prime} \quad$ if $a_{*} \neq \underline{a}, c^{\prime}\left(a_{*}\right)>c^{\prime}(\underline{a}) ;$
$\mathbf{i i}_{*}^{\prime} \quad$ if $a_{*} \neq \underline{a}, c^{\prime}(\underline{a})+\operatorname{IR}\left(a_{*}, \underline{a} \mid a_{*}\right)\left(B^{\prime}-c^{\prime}(\underline{a})\right)>c^{\prime}\left(a_{*}\right) ;$
$\mathbf{i i i}_{*}^{\prime} \quad$ if $a_{*} \neq \underline{a}, c^{\prime}(\underline{a})+\left[\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)-\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)\right]\left(B^{\prime}-c^{\prime}(\underline{a})\right)>c^{\prime}\left(a_{*}\right)$.
Property $i_{*}^{\prime}$ is immediate. Property $i i_{*}^{\prime}$ follows from $\left(i i_{*}\right)$. Property $i i i_{*}^{\prime}$ follows from $\left(i i i_{*}\right)$. Next observe that
$\mathbf{i}_{* *}^{\prime} \quad c^{\prime}\left(a_{* *}\right) \geq c^{\prime}\left(a_{*}\right)+\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)\left(B^{\prime}-c^{\prime}(\underline{a})\right) ;$
$\mathbf{i i}_{* *}^{\prime} \quad c^{\prime}\left(a_{*}\right)+\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right)\left(B^{\prime}-c^{\prime}(\underline{a})\right) \geq c^{\prime}\left(a_{* *}\right) ;$ and
$\mathbf{i i i}_{* *}^{\prime} \quad c^{\prime}(\underline{a})+\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right)\left(B^{\prime}-c^{\prime}(\underline{a})\right) \geq c^{\prime}\left(a_{* *}\right) \quad$ with $\quad$ strict inequality if $a_{*} \neq \underline{a}$.

Property $i_{* *}^{\prime}$ follows from $\left(i_{* *}\right)$ and the fact that $\alpha \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$. Property $i i_{* *}^{\prime}$ follows from $\left(i i_{* *}\right)$ and the fact that $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \alpha$. Part $i i i_{* *}^{\prime}$ follows from $\left(i i i_{* *}\right)$ and the fact that $\operatorname{IR}\left(a_{* *}, \underline{a} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \alpha$.

We next turn to Propositions 2 and 3. Define numbers

$$
\begin{aligned}
X \equiv & {\left[\Phi\left(\hat{g}\left(a_{* *}\right)-f\left(a_{*}, \bar{\theta}\right)\right)-\Phi\left(\hat{g}\left(a_{*}\right)-f\left(a_{*}, \bar{\theta}\right)\right)\right] } \\
& -\left[\Phi\left(\hat{g}\left(a_{* *}\right)-f\left(a_{* *}, \bar{\theta}\right)\right)-\Phi\left(\hat{g}\left(a_{*}\right)-f\left(a_{* *}, \bar{\theta}\right)\right)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
Y \equiv & {\left[\Phi\left(\hat{g}\left(a_{* *}\right)-f\left(a_{* *}, \underline{\theta}\right)\right)-\Phi\left(\hat{g}\left(a_{*}\right)-f\left(a_{* *}, \underline{\theta}\right)\right)\right] } \\
& -\left[\Phi\left(\hat{g}\left(a_{* *}\right)-f\left(a_{*}, \underline{\theta}\right)\right)-\Phi\left(\hat{g}\left(a_{*}\right)-f\left(a_{*}, \underline{\theta}\right)\right)\right] .
\end{aligned}
$$

LEMMA 8: If $\pi_{I}=\pi_{C}=\pi$, then the following are equivalent:
(i) $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right) \geq \operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right) ;$
(ii) $\pi X \geq(1-\pi) Y$.

In what follows, we provide conditions on the production technology that imply $\pi X \geq(1-\pi) Y$ when $\pi=\pi_{I}=\pi_{C}$ is sufficiently high. To do so, it will be useful to have two properties of $\Phi$.

LEMMA 9: Let $\phi$ be symmetric:
(i) $\Phi(x)=1-\Phi(-x)$.
(ii) If $y, z>0$ and $x \in(-(y+z), y)$, then $\Phi(x+z)-\Phi(x)>\Phi(y+z)-\Phi(y)$.

## PROOF:

Part (i) follows from symmetry, since

$$
1-\Phi(-x)=1-\int_{-\infty}^{-x} \phi(q) d q=1-\int_{x}^{\infty} \phi(q) d q=\int_{-\infty}^{x} \phi(q) d q=\Phi(x)
$$

For part (ii), fix $z>0$ and note

$$
\Phi(x+z)-\Phi(x)=\int_{x}^{x+z} \phi(q) d q
$$

By single-peakedness, $\phi$ is strictly increasing on $(-\infty, 0)$ and strictly decreasing on $(0, \infty)$. Thus, if $x \in[0, y)$, it is immediate that $\Phi(x+z)-\Phi(x)>\Phi(y+z)-$ $\Phi(y)$. If $x \in(-(y+z), 0)$, then

$$
\Phi(x+z)-\Phi(x)>\Phi(-y-z+z)-\Phi(-y-z)=\Phi(y+z)-\Phi(y)
$$

where the equality follows from the Part (i) of this Lemma.

In what follows, we will fix a production function $f$ and effort levels $a_{* *}>a_{*}$. It will be convenient to adopt the notation (for the output of production) described in Figure A1.

LEMMA 10: If $\phi$ is symmetric, then $X>0$.

## PROOF:

Applying Lemma 9(i), $X$ can be written as

$$
X=\left[\Phi\left(\frac{1}{2} \rho_{* *}\right)-\Phi\left(\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}\right)\right]-\left[\Phi\left(\rho_{* *}+\psi_{* *}-\frac{1}{2} \rho_{*}\right)-\Phi\left(\frac{1}{2} \rho_{*}\right)\right] .
$$

Write $d=\rho_{* *}+\psi_{* *}-\rho_{*}$ and note that, by Lemma 3, $d>0$. We can then rewrite $X$ as

$$
\begin{aligned}
X= & {\left[\Phi\left(\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}+d\right)-\Phi\left(\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}\right)\right] } \\
& -\left[\Phi\left(\frac{1}{2} \rho_{*}+d\right)-\Phi\left(\frac{1}{2} \rho_{*}\right)\right] .
\end{aligned}
$$

Thus, by Lemma 9(ii), $X>0$ provided (i) $\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}>-\frac{1}{2} \rho_{*}-d$ and (ii) $\frac{1}{2} \rho_{*}>\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}$. Condition (i) is immediate and Condition (ii) is by Lemma 3.

LEMMA 11: If $\phi$ is symmetric, then $Y>0$.

## PROOF:

Note, $Y$ can be written as

$$
Y=\left[\Phi\left(\frac{1}{2} \rho_{*}\right)-\Phi\left(\frac{1}{2} \rho_{*}-\psi_{* *}\right)\right]-\left[\Phi\left(\frac{1}{2} \rho_{* *}+\psi_{* *}\right)-\Phi\left(\frac{1}{2} \rho_{* *}\right)\right] .
$$

By Lemma 3, $\frac{1}{2} \rho_{* *}>\frac{1}{2} \rho_{*}-\psi_{* *}$. Since $\psi_{* *}>0$ and $\frac{1}{2} \rho_{*}-\psi_{* *}>-\frac{1}{2} \rho_{* *}-\psi_{* *}$, we can apply Lemma 9 (ii) to conclude that $Y>0$.

LEMMA 12: Suppose the model is symmetric with $\pi_{I}=\pi_{C}=\pi$. If $\pi^{\prime}>\pi$ and $\pi X \geq(1-\pi) Y$, then $\pi^{\prime} X \geq\left(1-\pi^{\prime}\right) Y$.

## PROOF:

Immediate from Lemmata 10-11.

The following is immediate from Lemmata 8 and 12.
REMARK 2: Suppose the model is symmetric with $\pi_{I}=\pi_{C}=\pi$. Then $\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}\right)-\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}\right)$ is strictly increasing in $\pi$.

## PROOF OF PROPOSITION 3:

Take $\hat{\pi}\left[a_{*}, a_{* *}\right]=\frac{Y}{Y+X}$. By Lemmata $10-11, \hat{\pi}\left[a_{*}, a_{* *}\right] \in(0,1)$. Now, the pair $\left(a_{*}, a_{* *}\right)$ is justifiable if and only if $\pi \geq \hat{\pi}\left[a_{*}, a_{* *}\right]$.

To conclude the proof, we show: (i) if effort and type are strict complements at $a_{*}, a_{* *}$, then $Y-X<0$; (ii) if effort and type are neither strict complements nor strict substitutes at $a_{*}, a_{* *}$, then $X=Y$; and (iii) if effort and type are strict substitutes at $a_{*}, a_{* *}$, then $X-Y<0$. In what follows we set $d=\frac{1}{2}\left(\rho_{* *}-\rho_{*}\right)$. Then $d>0$ (resp. $-d>0)$ if effort and type are strict complements (resp. substitutes) at $a_{*}, a_{* *}$.

First, suppose that effort and type are strict complements at $a_{*}, a_{* *}$. Note,

$$
\begin{aligned}
Y-X= & {\left[\Phi\left(\frac{1}{2} \rho_{* *}+\psi_{* *}+d\right)-\Phi\left(\frac{1}{2} \rho_{* *}+\psi_{* *}\right)\right] } \\
& -\left[\Phi\left(\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}+d\right)-\Phi\left(\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}\right)\right]
\end{aligned}
$$

Since effort and type are strict complements at $a_{*}, a_{* *}, d>0$. Observe that $\frac{1}{2} \rho_{* *}+$ $\psi_{* *}>0$ and $\rho_{*}-\psi_{* *}-\frac{1}{2} \rho_{* *}>-\left(\frac{1}{2} \rho_{* *}+\psi_{* *}+d\right)=-\rho_{* *}-\psi_{* *}+\frac{1}{2} \rho_{*}$. So, by Lemma 9(ii), $Y-X<0$.

Next, suppose that effort and type are neither strict substitutes nor strict complements at $a_{*}, a_{* *}$. Then, using the fact that $\rho_{*}=\rho_{* *}$, it is immediate that $X=Y$.

Finally, suppose that effort and type are strict substitutes at $a_{*}, a_{* *}$. Note

$$
\begin{aligned}
X-Y= & {\left[\Phi\left(\rho_{* *}+\psi_{* *}-\frac{1}{2} \rho_{*}-d\right)-\Phi\left(\rho_{* *}+\psi_{* *}-\frac{1}{2} \rho_{*}\right)\right] } \\
& -\left[\Phi\left(\frac{1}{2} \rho_{*}-\psi_{* *}-d\right)-\Phi\left(\frac{1}{2} \rho_{*}-\psi_{* *}\right)\right]
\end{aligned}
$$

Since effort and type are strict substitutes at $a_{*}, a_{* *},-d>0$. By Lemma 3, $\rho_{* *}+$ $\psi_{* *}-\frac{1}{2} \rho_{*}>0$. Moreover, $\frac{1}{2} \rho_{*}-\psi_{* *}>-\left(\rho_{* *}+\psi_{* *}-\frac{1}{2} \rho_{*}-d\right)=-\frac{1}{2} \rho_{* *}-$ $\psi_{* *}$. By Lemma 9(ii), $X-Y<0$.

PROPOSITION 4: Suppose $\phi$ is symmetric. Fix efforts $a_{* *} \neq a_{*}$ and let $\pi \in$ $\left(\hat{\pi}\left[a_{*}, a_{* *}\right], 1\right)$. There exists some open neighborhood of $(\pi, \pi) \in[0,1]^{2}$, viz. $N(\pi, \pi)$, so that, if $\left(\pi_{I}, \pi_{C}\right) \in N(\pi, \pi)$, then $\left(a_{*}, a_{* *}\right)$ is justifiable.

## PROOF:

Observe that $\hat{g}$ is a function, not only of $a$, but of $\pi_{I}$ and $\pi_{C}$. It will be convenient to write $\hat{g}: A \times[0,1] \times[0,1] \rightarrow \mathbb{R}$, so that, for each $\left(a, \pi_{I}, \pi_{C}\right), \hat{g}\left(a, \pi_{I}, \pi_{C}\right)$ :

$$
\operatorname{LR}\left(\hat{g}\left(a, \pi_{I}, \pi_{C}\right), a\right)=\frac{\pi_{C}}{1-\pi_{C}} \frac{1-\pi_{I}}{\pi_{I}}
$$

Thus, for each $a, a^{\prime} \in A, \operatorname{IR}\left(a, a^{\prime} \mid \cdot\right): A \times[0,1] \times[0,1] \rightarrow \mathbb{R}$ is also a function of $\left(a, \pi_{I}, \pi_{C}\right)$.

It suffices to show that, for each $\left(a, a^{\prime}\right)$ and each $\left(a_{*}, \cdot, \cdot\right), \operatorname{IR}\left(a, a^{\prime} \mid a_{*}, \cdot, \cdot\right)$ : $[0,1] \times[0,1] \rightarrow \mathbb{R}$ is continuous in $\left(\pi_{I}, \pi_{C}\right)$. If so, by Proposition 3, for each $(\tilde{\pi}, \tilde{\pi}) \in\left(\hat{p}\left[a_{*}, a_{* *}\right], 1\right)^{2}$,

$$
\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}, \tilde{\pi}, \tilde{\pi}\right)-\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}, \tilde{\pi}, \tilde{\pi}\right)>0
$$

By continuity, for each $(\tilde{\pi}, \tilde{\pi}) \in\left(\hat{p}\left[a_{*}, a_{* *}\right], 1\right)^{2}$, there is some open neighborhood $N$ of $(\tilde{\pi}, \tilde{\pi})$ so that, for all $\left(\pi_{I}, \pi_{C}\right) \in N$,

$$
\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{* *}, \pi_{I}, \pi_{C}\right)-\operatorname{IR}\left(a_{* *}, a_{*} \mid a_{*}, \pi_{I}, \pi_{C}\right)>0
$$

By Theorem 3, the claim follows.
To show that $\operatorname{IR}\left(a, a^{\prime} \mid a_{*}, \cdot, \cdot\right)$ is continuous in $\left(\pi_{I}, \pi_{C}\right)$, it suffices to show that, for each $a, \hat{g}(a, \cdot \cdot \cdot)$ is continuous in $\left(\pi_{I}, \pi_{C}\right)$. If so, then by continuity of $\Phi$, $\operatorname{IR}\left(a, a^{\prime} \mid a_{*}, \cdot, \cdot\right)$ it is continuous in $\left(\pi_{I}, \pi_{C}\right)$.

Write $\widehat{\mathrm{LR}}: \mathbb{R} \times A \times[0,1] \times[0,1] \rightarrow \mathbb{R}$ so that

$$
\widehat{\mathrm{LR}}\left(g, a, \pi_{I}, \pi_{C}\right)=\frac{\phi(g-f(a, \bar{\theta}))}{\phi(g-f(a, \underline{\theta}))}-\frac{1-\pi_{I}}{\pi_{I}} \frac{\pi_{C}}{1-\pi_{C}}
$$

Observe that, for each $a, \widehat{\operatorname{LR}}(\cdot, a, \cdot, \cdot): \mathbb{R} \times[0,1] \times[0,1] \rightarrow \mathbb{R}$ is differentiable in $g$ and continuous in $\left(g, \pi_{I}, \pi_{C}\right)$. (This follows from the fact that $\phi$ is continuously differentiable.) Moreover, $\widehat{\mathrm{LR}}(\cdot, a)$ is differentiable with a nonzero derivatives (since LR is). Thus, it follows from the Implicit Function Theorem that each $\hat{g}(a, \cdot, \cdot)$ is continuous in $\left(\pi_{I}, \pi_{C}\right)$.

## PROOF OF PROPOSITION 2:

Immediate from Proposition 4.

## REFERENCES

Achen, Christopher H., and Larry M. Bartels. 2004. "Blind Retrospection: Electoral Responses to Drought, Flu, and Shark Attacks." https://www.ethz.ch/content/dam/ethz/special-interest/gess/cis/ international-relations-dam/Teaching/pwgrundlagenopenaccess/Weitere/AchenBartels.pdf.
Achen, Christopher H., and Larry M. Bartels. 2006. "It Feels Like We're Thinking: The Rationalizing Voter and Electoral Democracy." Paper presented at the Annual Meeting of the American Political Science Association, Philadelphia, PA, August 30-September 3.

- Alesina, Alberto, and Guido Tabellini. 2007. "Bureaucrats or Politicians? Part I: A Single Policy Task." American Economic Review 97 (1): 169-79.
- Alesina, Alberto, and Guido Tabellini. 2008. "Bureaucrats or politicians? Part II: Multiple policy tasks." Journal of Public Economics 92 (3-4): 426-47.
- Alt, James, Ethan Bueno de Mesquita, and Shanna Rose. 2011. "Disentangling Accountability and Competence in Elections: Evidence from U.S. Term Limits." Journal of Politics 73 (1): 171-86.
- Ashworth, Scott, and Ethan Bueno de Mesquita. 2006. "Delivering the Goods: Legislative Particularism in Different Electoral and Institutional Settings." Journal of Politics 68 (1): 168-79.
Austen-Smith, David, and Jeffrey Banks. 1989. "Electoral Accountability and Incumbency." In Models of Strategic Choice in Politics, edited by Peter C. Ordeshook, 121-50. Ann Arbor: University of Michigan Press.
- Bagues, Manuel, and Berta Esteve-Volart. 2016. "Politicians' Luck of the Draw: Evidence from the Spanish Christmas Lottery." Journal of Political Economy 124 (5): 1269-94.
Bardhan, Pranab. 1997. "Corruption and Development: A Review of Issues." Journal of Economic Literature 35 (3): 1320-46.
- Barro, Robert J. 1973. "The control of politicians: An economic model." Public Choice 14 (1): 19-42.
- Besley, Timothy. 2004. "Paying Politicians: Theory and Evidence." Journal of the European Economic Association 2 (2-3): 193-215.
Besley, Timothy. 2006. Principled Agents? The Political Economy of Good Government. The Lindahl Lectures. Oxford: Oxford University Press.
- Besley,Timothy, and Anne Case. 1995. "Does Electoral Accountability Affect Economic Policy Choices? Evidence from Gubernatorial Term Limits." Quarterly Journal of Economics 110 (3): 769-98.

Besley, Timothy, and Anne Case. 2003. "Political Institutions and Policy Choices: Evidence from the United States." Journal of Economic Literature 41 (1): 7-73.
Blackwell, David. 1951. "Comparison of Experiments." In Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, edited by Jerzy Neyman, 93-102. Berkeley: University of California Press.
Blackwell, David, and M. A. Girshick. 1979. Theory of Games and Statistical Decisions. Mineola, NY: Dover Publications, (Orig. pub. 1954).
-Chang, Eric C. C., Miriam A. Golden, and Seth J. Hill. 2010. "Legislative Malfeasance and Political Accountability." World Politics 62 (2): 177-220.
de Janvry, Alain, Frederico Finan, and Elisabeth Sadoulet. 2012. "Local Electoral Incentives and Decentralized Program Performance." Review of Economics and Statistics 94 (3): 672-85.
DDewatripont, Mathias, Ian Jewitt, and Jean Tirole. 1999. "The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies." Review of Economic Studies 66 (1): 199-217.
Fearon, James D. 1999. "Electoral Accountability and the Control of Politicians: Selecting Good Types versus Sanctioning Poor Performance." In Democracy, Accountability, and Representation, edited by Adam Przeworski, Susan C. Stokes, and Bernard Manin, 55-97. Cambridge: Cambridge University Press.

- Ferejohn, John. 1986. "Incumbent performance and electoral control." Public Choice 50 (1-3): 5-25.

Ferraz, Claudio, and Frederico Finan. 2011. "Electoral Accountability and Corruption: Evidence from the Audits of Local Governments." American Economic Review 101 (4): 1274-1311.
Fiorina, Morris P. 1981. Retrospective Voting in American National Elections. New Haven, CT: Yale University Press.
-Gagliarducci, Stefano, and Tommaso Nannicini. 2013. "Do Better Paid Politicians Perform Better? Disentangling Incentives from Selection." Journal of the European Economic Association 11 (2): 369-98.
-Gehlbach, Scott. 2007. "Electoral Institutions and the National Provision of Local Public Goods." Quarterly Journal of Political Science 2 (1): 5-25.
Golden, Miriam. 2006. "Some Puzzles of Political Corruption in Modern Advanced Democracies." https://www.researchgate.net/publication/228652944_Some_Puzzles_of_Political_Corruption_ in_Modern_Advanced_Democracies.
-Healy, Andrew J., Neil Malhotra, and Cecilia Hyunjung Mo. 2010. "Irrelevant events affect Voters’ evaluations of government performance." Proceedings of the National Academy of Sciences of the United States of America 107 (29): 12804-09.
-Holmström, Bengt. 1999. "Managerial Incentive Problems: A Dynamic Perspective." Review of Economic Studies 66 (1): 169-82.
-Huber, Gregory A., and Sanford C. Gordon. 2004. "Accountability and Coercion: Is Justice Blind when It Runs for Office?" American Journal of Political Science 48 (2): 247-63.
Jewitt, Ian. 2007. "Information Order in Decision and Agency Problems." Unpublished.
Key, V. O., Jr. 1966. The Responsible Electorate: Rationality in Presidential Voting, 1936-1960. Cambridge, MA: Belknap Press.
Landa, Dimitri. 2010. "Selection Incentives and Accountability Traps: A Laboratory Experiment." https://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID1646550_code464312. pdf? abstractid $=1277281 \&$ mirid $=1$.
-Lehmann, E. L. 1988. "Comparing Location Experiments." Annals of Statistics 16 (2): 521-33.
Lehmann, Erich L., and Joseph P. Romano. 2005. Testing Statistical Hypotheses. New York: Springer-Verlag.
Lohmann, Susanne. 1998. "An Informational Rationale for the Power of Special Interests." American Political Science Review 92 (4): 809-27.
-Manacorda, Marco, Edward Miguel, and Andrea Vigorito. 2011. "Government Transfers and Political Support." American Economic Journal: Applied Economics 3 (3): 1-28.
Manin, Bernard, Adam Przeworski, and Susan C. Stokes. 1999. "Elections and Representation." In Democracy, Accountability, and Representation, edited by Adam Przeworski, Susan C. Stokes, and Bernard Manin, 29-54. Cambridge: Cambridge University Press.
-Mansbridge, Jane. 2003. "Rethinking Representation." American Political Science Review 97 (4): 515-28.
-Mansbridge, Jane. 2009. "A 'Selection Model' of Political Representation." Journal of Political Philosophy 17 (4): 369-98.
-Maskin, Eric, and Jean Tirole. 2004. "The Politician and the Judge: Accountability in Government." American Economic Review 94 (4): 1034-54.
-Meirowitz, Adam. 2007. "Probabilistic Voting and Accountability in Elections with Uncertain Policy Constraints." Journal of Public Economic Theory 9 (1): 41-68.

- Myerson, Roger B. 2006. "Federalism and Incentives for Success of Democracy." Quarterly Journal of Political Science 1 (1): 3-23.
Persson, Torsten, and Guido Tabellini. 2000. Political Economics: Explaining Economic Policy. Cambridge: MIT Press.
Pitkin, Hanna Fenichel. 1967. The Concept of Representation. Berkeley, CA: University of California Press.
Schwabe, Rainer. 2009. "Reputation and Accountability in Repeated Elections." Paper presented at the Annual Meeting of the American Political Science Association, Toronto, ON, September 3-6.
-Snyder, James M., Jr., and Michael M. Ting. 2008. "Interest groups and the electoral control of politicians." Journal of Public Economics 92 (3-4): 482-500.
Torgersen, Erik. 1991. Comparison of Statistical Experiments. Encyclopedia of Mathematics and Its Applications. Cambridge: Cambridge University Press.


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8. Elliott Ash, W. Bentley MacLeod. 2021. Reducing partisanship in judicial elections can improve judge quality: Evidence from U.S. state supreme courts. Journal of Public Economics 201, 104478. [Crossref]
9. Jan Klingelhöfer. 2021. Competitive elections, incumbency advantage, and accountability. Economic Theory 71:4, 1397-1428. [Crossref]
10. Brian Blankenship, Ryan Kennedy, Johannes Urpelainen, Joonseok Yang. 2021. Barking Up the Wrong Tree: How Political Alignment Shapes Electoral Backlash from Natural Disasters. Comparative Political Studies 54:7, 1163-1196. [Crossref]
11. Hao Hong, Tsz-Ning Wong. 2020. Authoritarian election as an incentive scheme. Journal of Theoretical Politics 32:3, 460-493. [Crossref]
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15. Joshua A Strayhorn. 2019. Plausible deniability. Journal of Theoretical Politics 31:4, 600-625. [Crossref]
16. Haritz Garro. 2019. Terrorism prevention with reelection concerns and valence competition. Journal of Theoretical Politics 31:3, 330-369. [Crossref]
17. Catherine E. Vries, Hector Solaz. Corruption and Electoral Accountability: Avenues for Future Research 1-13. [Crossref]
18. Federica Izzo, Torun Dewan, Stephane Wolton. 2018. Cumulative Knowledge in the Social Sciences: The Case of Improving Voters' Information. SSRN Electronic Journal. [Crossref]
19. Liqun Liu. 2018. Reasonable Doubt and Beyond. SSRN Electronic Journal . [Crossref]
20. Scott Ashworth, Ethan Bueno de Mesquita. 2017. Unified versus Divided Political Authority. The Journal of Politics 79:4, 1372-1385. [Crossref]
21. Andreas Grunewald, Emanuel Hansen, Gert PPnitzsch. 2017. Political Selection and the Optimal Concentration of Political Power. SSRN Electronic Journal 125. . [Crossref]

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[^1]:    ${ }^{1}$ Most of the results in the paper extend to an arbitrary type set $\Theta$. The notable exception is Theorem 2.

[^2]:    ${ }^{2}$ Observe that this is a requirement that is independent of the particular production function-put differently, it is a requirement that can be satisfied by any production function. Lemma 4 states that the distribution satisfies strict MLRP and the likelihood ratios have nonzero derivatives if and only if $\log \circ \phi$ is strictly concave. (Weak versions of this equivalence are standard. See, e.g., Torgersen 1991, Theorem 9.4.9.)

[^3]:    ${ }^{3}$ Here we use the fact that the random variable generating $\epsilon_{1}$ is atomless. This follows from the fact that $\Phi$ is continuous.
    ${ }^{4}$ Observe that, as a matter of computation, we can and do define the function $\hat{g}(a)$ from equation (2), irrespective of whether $a$ is a first-period equilibrium effort. With this, we can and do define the function $\operatorname{IR}\left(a^{\prime}, a \mid a_{*}\right)$ (by way of equation (3)) for each triple of efforts $\left(a^{\prime}, a, a_{*}\right)$, i.e., irrespective of whether $a_{*}$ is a first-period equilibrium effort.

[^4]:    ${ }^{5}$ The term $\operatorname{Pr}\left(\theta_{P 2}=\theta \mid a_{*}\right)$ can be expressed in terms of primitives of the model. See the Appendix.

[^5]:    ${ }^{6}$ Note, the probability that the realization of $g$ is less than some $\tilde{g}$ is $\Phi(\tilde{g}-f(a, \theta))$.
    ${ }^{7}$ Theorem 1 applies verbatim, when $\Theta$ has more than two types. However, in that case, informational effictiveness is no longer equivalent to Blackwell informativeness: If $a_{* *}$ is more Blackwell informative than $a_{*}$, then it is also more informationally effective than $a_{*}$. But, the converse need not hold.

[^6]:    ${ }^{8}$ Appendix C shows that variations in the institutions of formal accountability can lead to changes in effective accountability despite not leading to systematic changes in effective accountability.

[^7]:    ${ }^{9}$ The bound $\underline{\pi}\left[a_{*}, a_{* *}\right]$ is not determined constructively.

[^8]:    ${ }^{10}$ Our use of the phrase "accountability traps" is similar to, but distinct from, the use of the phrase in Landa (2010). Landa uses the term to mean that the society is caught in an equilibrium that has a lower level of voter welfare than some other pair of re-election rule and politician behavior, independent of whether the other re-election rule is optimal for the Voter given the politician's behavior. By contrast, our concept only applies when that other pair forms an equilibrium.

[^9]:    ${ }^{11}$ Notable exceptions to this view are Meirowitz (2007), Snyder and Ting (2008), and Schwabe (2009). In their models, equilibria exist in which the voter need not select good types. The key is that, in their models, the voter is indifferent between different types of politicians.

