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Public debt in economies with heterogeneous agents☆

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A B S T R A C T

We study public debt in competitive equilibria in which a government chooses transfers and taxes optimally and in addition decides how thoroughly to enforce debt contracts. If the government enforces perfectly, asset inequality is determined in an optimum competitive equilibrium but the level of government debt is not. Welfare increases if private debt contracts are not enforced. Borrowing frictions let the government gather monopoly rents that come from issuing public debt without facing competing private borrowers. Regardless of whether the government chooses to enforce private debt contracts, the level of initial government debt does not affect an optimal allocation.

1. Introduction

> *If, indeed, the debt were distributed in exact proportion to the taxes to be paid so that every one should pay out in taxes as much as he received in interest, it would cease to be a burden. . . . if it were possible, there would be [no] need of incurring the debt. For if a man has money to loan the Government, he certainly has money to pay the Government what he owes it.* Simon Newcomb (1865, p.85)

Understanding whether a government's debt is too high or too low requires knowing who owes what, when, to whom. That impels studying balance sheets of both creditors and debtors as well as the budget sets that appear in a coherent economic model and leads to distinguishing superficial from substantive features by tracking and properly consolidating assets and liabilities. We seek features of government debt that affect continuation allocations and prices. For that purpose, this paper studies an economy with people who differ in their productivities and a government that administers a non-linear tax on labor earnings. Agents and the government trade one-period bonds. There is no capital. The economy starts with an exogenously given distribution of debt across agents and the government. Taxes are restricted by agents' abilities to pay. Public policies are chosen at time 0 i.e., the government *commits*.

The structure of budget constraints implies that the cross-section distribution of initial net assets, not gross assets, affects the set of feasible allocations that can be implemented in competitive equilibria. An increase in initial government debt that

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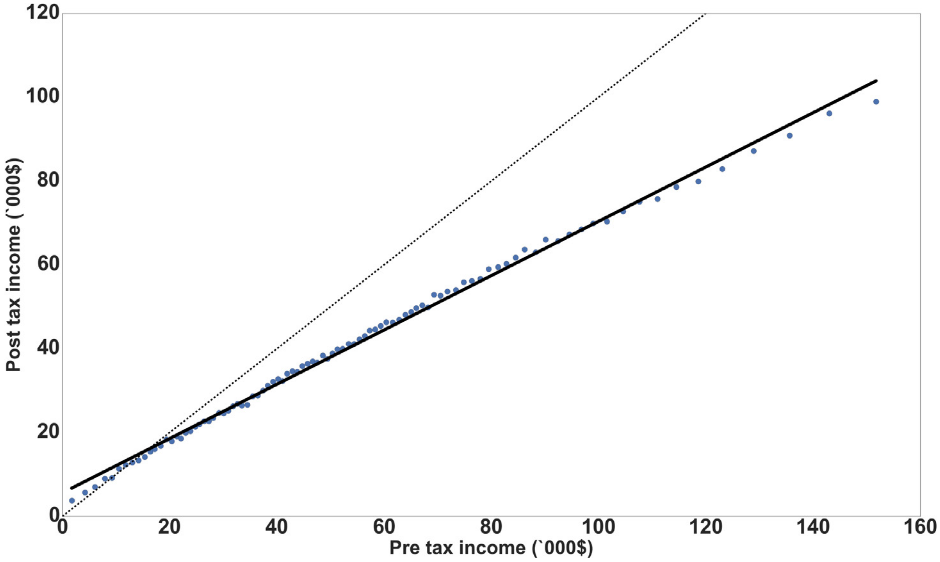

Fig. 1. Representation of U.S. tax and transfer system from the PSID 2000-06 and TAXSIM. Source: Heathcote et al. (2017). The dotted line is the 45° line and the solid line is a linear fit.

is shared equally among all agents leaves the distribution of net assets unchanged and therefore also leaves an equilibrium allocation unaltered. This outcome embodies ideas proclaimed by Newcomb (1865). The same logic applies to models with and without physical capital, with complete or incomplete asset markets, and with more general tax structures. On the other hand, if the government cares about redistribution, an increase in initial government debt that is held mostly by agents with high labor earnings decreases welfare. In our setting, the correlation of initial debt holdings and labor earnings affects the welfare cost of public debt.

The role of government debt depends on how well private debt contracts are enforced. If both tax and debt obligations are enforced perfectly, then agents' abilities to borrow are restricted only by their abilities to repay their debts and an optimal level of government debt is indeterminate. In this case, any sequence of government debts is optimal and a version of Ricardian equivalence holds even though taxes distort private agents' decisions. Nevertheless, it turns out that in this case the dynamics of asset inequality share qualitative features with the dynamics of public debt in representative agent models that exogenously rule out transfers.

We also show that welfare under an optimal policy increases if the government commits not to enforce private debt contracts in ways that produce the outcome that agents can borrow only up to an exogenous ad hoc debt limit. In this case, government debt provides an additional instrument to affect equilibrium allocations. Welfare gains under an optimal policy come from monopoly power on the asset market that the government acquires by restricting the ability of private agents to provide liquidity. What matters for this result is not the size of the debt limit per se, but the agents' inabilities to use anticipated transfers to relax current borrowing constraints. An optimal government debt is determined by a trade-off between gains from exploiting monopoly rents and costs from distorting agents' intertemporal marginal rates of substitution.

A sizable literature about government debt and Ricardian equivalence goes back at least to Barro (1974). It is well understood that in representative agent economies the role of government debt hinges on whether lump sum taxes are allowed. But there is no inherent economic reason to rule out lump sum taxes in those models. Furthermore, the proportional labor taxes often assumed in representative agent models do not approximate data well because transfers are such a large part of modern tax systems (see, e.g., Fig. 1). In our model, agents are heterogeneous, taxes are restricted by agents' resources, and the government chooses taxes to maximize a weighted average of agents' lifetime utilities.

Werning (2007) obtained counterparts to our results about net versus gross asset positions in a complete markets economy with heterogeneous agents, an affine tax structure, and transfers that are unrestricted in sign. Because he allowed unrestricted taxation of initial assets, the initial distribution of assets played no role in the model. Our Lemma 2 and its corollaries extend Werning's results by showing that all distributions of gross assets among private agents and the government that imply the same net asset positions lead to the same equilibrium allocation, a conclusion that holds beyond complete markets. While Werning (2007) characterized optimal allocations and distortions in complete market economies, Werning (2012) investigated how precautionary savings motives that incomplete markets impart both to private agents and to a benevolent government affect optimal allocations.[1]

[1] Other recent pertinent papers include Azzimonti et al. (2008a, 2008b) and Correia (2010). These papers study optimal policies in economies with agents heterogeneous in skills and initial assets.

Our results on desirability of weak enforcement of private debt contracts build on insights of Yared (2012, 2013), who showed that it may be optimal not to undo agents' borrowing frictions even when a government can undo them. Bassetto (2014) studied the roles of taxation and debt limits in heterogeneous agent economies in which transfers are ruled out. Broner et al. (2010) and Broner and Ventura (2011, 2016) explored incentives of the government to enforce private debt contracts in the context of international finance.

This paper is organized as follows. In Section 2, we describe a baseline environment in which taxes are restricted to be affine functions of labor earnings and agents are heterogeneous in labor earnings but face no idiosyncratic uncertainty. In Section 3, we study an economy in which agents' abilities to borrow are restricted only by their abilities to pay. In Section 4, we study an economy in which agents face more stringent borrowing constraints. We show that our results extend to richer tax systems constrained only by informational frictions in Section 5.

2. Environment

Time is discrete and infinite. There are I types of agents each of mass n_i for $i \in \{1, 2, \ldots, I\}$ with $\sum_{i=1}^{I} n_i = 1$. Preferences of an agent of type i over stochastic processes for consumption $\{c_{i,t}\}_t$ and labor supply $\{l_{i,t}\}_t$ are ordered by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i(c_{i,t}, l_{i,t}), \tag{1}$$

where $\mathbb{E}_t$ is a mathematical expectations operator conditioned on time t information and β a discount factor. We assume that U^i is increasing and concave in $(c, -l)$. The labor supply of agent i lies in a set $\left[0, \bar{L}_i\right]$. We allow $\bar{L}_i$ to be infinite.

Uncertainty is summarized by a shock s_t governed by an irreducible Markov process that takes values in a finite set S. We let $s^t = (s_0, \ldots, s_t)$ denote a history of shocks having joint probability density $\pi_t(s^t)$. A boldface letter $\boldsymbol{x}$ denotes a sequence $\left\{x_t\left(s^t\right)\right\}_{t\geq 0, s^t}$. We write $s^{t'} \in s^{t''}$ for $t'' > t'$ if the first t' elements of $s^{t''}$ constitute $s^{t'}$. When it does not cause confusion, we use x_t to denote a random variable that depends on s^t. Finally, we define a set of infinite histories $\mathbb{S}^\infty$ such that $s^\infty \in \mathbb{S}^\infty$ satisfies $\pi_t(s^t) > 0$ for all $s^t \in s^\infty$.

Shock s_t affects government expenditures $g_t(s_t)$ and individuals' productivities $\{\theta_{i,t}(s_t)\}_i$. An agent of type i who supplies l_i units of labor produces $y_i \equiv \theta_i(s_t) l_i$ units of output. Feasible allocations satisfy

$$\sum_{i=1}^{I} n_i c_{i,t} + g_t = \sum_{i=1}^{I} n_i \theta_{i,t} l_{i,t}. \tag{2}$$

Agents trade riskless one-period zero coupon bonds with each other and the government. At date t, history s^t the price is denoted by $q_t(s^t)$. Let the cumulation of past prices at t, s^t be $Q_t(s^t) \equiv \prod_{k \leq t, s^k \in s^t} q_k(s^k)$. We denote asset holdings of agents and the government in period t by $\{b_{i,t}\}_i$ and B_t, respectively. We use a convention that negative values denote net indebtedness of the agent or of the government. Agents and the government begin with assets $\{b_{i,-1}\}_{i=1}^{I}$ and B_{-1}, respectively. Asset holdings satisfy market clearing conditions

$$\sum_{i=1}^{I} n_i b_{i,t} + B_t = 0 \text{ for all } t \geq -1. \tag{3}$$

In each period, the government collects $\mathcal{T}_t\left(y_{i,t}\right)$ from agent i, where $y_{t,i} = \theta_{i,t} l_{i,t}$. To be comparable to the literature, we assume throughout most of this section that $\mathcal{T}_t$ is an affine function

$$\mathcal{T}_t(y_t) = -T_t + \tau_t y_t. \tag{4}$$

Affine tax functions approximate actual tax and transfer programs pretty well; see Fig. 1, adapted from Heathcote et al. (2017). As will be indicated in our proofs, our results extend to more general non-linear income tax schedules $\mathcal{T}_t(y_t)$ and to even richer tax systems. We discuss these later.

With affine taxes, the government budget constraint is

$$g_t + q_t B_t = \tau_t \sum_{i=1}^{I} n_i \theta_{i,t} l_{i,t} + B_{t-1} - T_t. \tag{5}$$

A government's preferences over stochastic process for consumption and work are ordered by

$$\mathbb{E}_0 \sum_{i=1}^{I} n_i \omega_i \sum_{t=0}^{\infty} \beta^t U_t^i(c_{i,t}, l_{i,t}), \tag{6}$$

where $\omega_i \geq 0$, $\sum_{i=1}^{I} \omega_i = 1$ is a set of Pareto weights.

A type i agent's budget constraint at $t \geq 0$ is

$$c_{i,t} + q_t b_{i,t} = (1 - \tau_t)\theta_{i,t} l_{i,t} + b_{i,t-1} + T_t. \tag{7}$$

In competitive equilibrium, agent i maximizes utility (1) by choosing sequences $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ that satisfy budget constraints (7). Without restrictions on debt holdings, this problem is ill-posed because it allows agents to achieve infinite utility by running Ponzi schemes. To rule out explosive debt paths, we restrict sequences $\boldsymbol{b}_i$ to be bounded from below. Later we consider more stringent constraints on private debt.

In the spirit of Lucas and Stokey (1983), we study government policies $(\boldsymbol{\tau}, \boldsymbol{T}, \boldsymbol{B})$ that maximize welfare criterion (6) in a competitive equilibrium, given an initial distribution of assets $\left(\left\{b_{i,-1}\right\}_i, B_{-1}\right)$.

We want to answer two questions: (a) how does the level of the initial government debt B_{-1} affect welfare in an optimal equilibrium and (b) what determines properties of an optimal path of government debt $\boldsymbol{B}$. The first question is about legacy costs of past debt. The second question is about whether an optimal level of government debt exists and, if it does, how quickly the government should converge to it.

That agents are heterogeneous affects our answers. In a representative agent economy, the answers to these questions depend on whether lump-sum taxes are available (see Barro, 1974). If agents really are identical, there is little reason to rule out lump sum taxes. Authors of representative agent models typically justify ruling out lump-sum taxes by explicitly or implicitly alluding to unmodeled heterogeneity in the form of the presence of a subset of poor agents who cannot afford to pay lump-sum taxes. We model the presence of such poor agents explicitly and are able to study an optimal tax policy while allowing lump-sum taxes and transfers. Because our transfers $\boldsymbol{T}$ are anonymous, the budget constraints of the poorest agents restrict the sign and magnitude of lump-sum taxes or transfers.

Answers to our two questions depend partly on borrowing constraints. We interpret such constraints as arising from the inability or disinclination of a government to punish agents who default on their obligations. As a benchmark, we start with the loosest borrowing limits: the so-called "natural borrowing limits" that allow agents to borrow any amounts that are feasible for them to repay in all future states. We interpret these limits as indicating the presence of a government that is willing and able to impose the harshest punishments on agents who default. We then discuss implications of stricter limits on private borrowing.

3. Optimal debt under natural debt limits

We begin with definitions.

Definition 1. An *allocation* is a sequence $\{\boldsymbol{c}_i, \boldsymbol{l}_i\}_i$. An *asset profile* is a sequence $(\{\boldsymbol{b}_i\}_i, \boldsymbol{B})$. A *price process* is a sequence $\boldsymbol{q}$. A *tax policy* is a sequence $(\boldsymbol{\tau}, \boldsymbol{T})$.

Definition 2. A *competitive equilibrium with natural debt limits* given initial assets $\left(\left\{b_{i,-1}\right\}_i, B_{-1}\right)$ is a $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ such that (i) $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ maximize (1) subject to (7) and $\boldsymbol{b}_i$ is bounded below for all i; (ii) constraints (2), (3), and (5) are satisfied.

Definition 3. An *optimal* competitive equilibrium with natural debt limits given initial asset $\left(\left\{b_{i,-1}\right\}_i, B_{-1}\right)$ is a competitive equilibrium with natural debt limits that maximizes (6).

A discussion of our terminology is useful. Aiyagari (1994) popularized the natural debt limit terminology. He considered an economy with finite after tax endowments and utility functions defined over non-negative consumption. When the equilibrium interest rate $\frac{1}{q_t}$ is strictly greater than one[2] Aiyagari required that an agent's debt not exceed the present value of his maximum after-tax income at a worst shock sequence.[3] Formally, the maximum income of agent i in state s^t is $Y_{i,t}(s^t) \equiv \max\left\{\left(1-\tau_t\left(s^t\right)\right)\theta_{i,t}\left(s^t\right)\bar{L}_i, 0\right\} + T_t\left(s^t\right)$ and the present value of his maximum income at a worst shock sequence is

$$D_t\left(\mathbf{Y}_i; s^t\right) \equiv \inf_{s^\infty \in \mathbb{S}^\infty : s^t \in s^\infty} \sum_{k>t, s^k \in s^\infty} \frac{Q_{k-1}\left(s^{k-1}\right)}{Q_t\left(s^t\right)} Y_{i,k}\left(s^k\right). \tag{8}$$

The natural debt limit requires that if agents i's consumption is bounded below and $\boldsymbol{Q}$ is a summable sequence, then agent i's assets are constrained by

$$b_{i,t}\left(s^t\right) \geq -D_t\left(\mathbf{Y}_i; s^t\right) \text{ for all } t, s^t. \tag{9}$$

The following lemma indicates that our definition of competitive equilibrium extends Aiyagari's notion of borrowing constraints to situations in which his definition of a natural debt limit is ill-posed.

Lemma 1. *Suppose that U^i is defined only for $c \geq 0$, $\mathbf{Y}_i$ is bounded above and bounded below away from zero, and $\boldsymbol{Q}$ is a strictly positive summable sequence. Then $\boldsymbol{b}_i$ satisfies the natural debt limit if and only if $\boldsymbol{b}_i$ is bounded below.*

Proof. See online appendix 1 □

Our definition of an optimal competitive equilibrium allows the government to optimize over taxes and transfers $(\boldsymbol{\tau}, \boldsymbol{T})$. Since competitive equilibria are well defined only over $(\boldsymbol{\tau}, \boldsymbol{T})$ systems under which all consumers can afford to pay (i.e.,

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[2] This condition can be relaxed to require that $\boldsymbol{Q}$ is a strictly positive and summable sequence, meaning that $\sum_{t,s^t} Q_t(s^t)$ exists.

[3] When the gross interest rate is less than one so that the present value of income is infinite he imposed an explicit lower bound on debt.

for which each consumer's budget set is nonempty), this definition endogenously imposes restrictions on admissible tax policies.

We start with an important result.

Lemma 2. *Given* $(\{b_{i,-1}\}_i, B_{-1})$, *let* $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ *be a competitive equilibrium with natural debt limits. For any bounded sequences* $\{\hat{\boldsymbol{b}}_i\}_i$ *and* $\{\hat{b}_{i,-1}\}_i$ *that satisfy*

$$\hat{b}_{i,t} - \hat{b}_{I,t} = b_{i,t} - b_{I,t} \text{ for all } t \geq -1, i \in [1, 2, \ldots, I-1] \tag{10}$$

there exist sequences $(\hat{\boldsymbol{T}}, \hat{\boldsymbol{B}})$ *such that* $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\}_i, \hat{\boldsymbol{B}}, \boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}})$ *is a competitive equilibrium with natural debt limits given* $(\{\hat{b}_{i,-1}\}_i, \hat{B}_{-1})$.

Proof. For any bounded $\{\hat{\boldsymbol{b}}_i\}_i$ let $\Delta_t \equiv \hat{b}_{I,t} - b_{I,t}$ for all $t \geq -1$. Define, for all $t \geq -1$,

$$\hat{T}_t = T_t + q_t \Delta_t - \Delta_{t-1}, \quad \hat{B}_t = B_t + \Delta_t. \tag{11}$$

The sequence $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\}_i, \hat{\boldsymbol{B}}, \boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}})$ satisfies (2), (3), and (5), so it remains only to show that $(\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i)$ is the optimal choice given $(\boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}})$. Observe that $(\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i)$ satisfies budget constraint

$$\begin{aligned} c_{i,t} &= (1-\tau_t)\theta_{i,t} l_{i,t} + b_{i,t-1} - q_t b_{i,t} + T_t \\ &= (1-\tau_t)\theta_{i,t} l_{i,t} + (b_{i,t-1} - b_{I,t-1}) - q_t(b_{i,t} - b_{I,t}) + T_t + b_{I,t-1} - q_t b_{I,t} \\ &= (1-\tau_t)\theta_{i,t} l_{i,t} + \left(\hat{b}_{i,t-1} - \hat{b}_{I,t-1}\right) - q_t\left(\hat{b}_{i,t} - \hat{b}_{I,t}\right) + T_t + b_{I,t-1} - q_t b_{I,t} \\ &= (1-\tau_t)\theta_{i,t} l_{i,t} + \hat{b}_{i,t-1} - q_t \hat{b}_{i,t} + \hat{T}_t. \end{aligned}$$

Suppose that $(\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i)$ is not an optimal choice for consumer i, in the sense that there exists some other sequence $\left(\boldsymbol{c}'_i, \boldsymbol{l}'_i, \boldsymbol{b}'_i\right)$ that provides consumer i higher utility given $\left(\boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}}\right)$. The sequence $\left(\boldsymbol{c}'_i, \boldsymbol{l}'_i, \boldsymbol{b}'_i - \boldsymbol{\Delta}\right)$ satisfies (7) and (9) given $(\boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ and provides strictly higher utility than $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$. Therefore, $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ cannot be a part of a competitive equilibrium $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$, a contradiction. □

We answer the two questions posed in Section 2 with two propositions that follow from Lemma 2.

Proposition 1. *For any pair* B'_{-1}, B''_{-1}, *there are asset profiles* $\{b'_{i,-1}\}_i$ *and* $\{b''_{i,-1}\}_i$ *such that optimum equilibrium allocations under natural debt limits starting from* $(\{b'_{i,-1}\}_i, B'_{-1})$ *are the same as those starting from* $(\{b''_{i,-1}\}_i, B''_{-1})$. *These asset profiles satisfy*

$$b'_{i,-1} - b'_{I,-1} = b''_{i,-1} - b''_{I,-1} \text{ for all } i. \tag{12}$$

Proposition 1 asserts that it is not total government debt but how its ownership is distributed that affects equilibrium allocations. To understand why, suppose that we increase an initial level of government debt from 0 to some arbitrary level B'_{-1}. If transfers $\boldsymbol{T}$ were held fixed, the government would want to increase tax rates $\boldsymbol{\tau}$ to collect a present value of revenues sufficient to repay B'_{-1}. Since dead-weight losses are convex in the tax rate, higher levels of debt would then impose disproportionately larger distortions, which makes higher levels of debt particularly bad. But this conclusion changes if we allow the government to adjust transfers. To find optimal transfers, we need to know how holdings of government debt B'_{-1} are distributed. Suppose that agents hold equal amounts of the additional debt B'_{-1}. In this case, each unit of debt repayment achieves the same redistribution as one unit of transfers. Since the original level of transfers at zero government debt is optimal, the best policy for the government with debt B'_{-1} is to reduce transfers by exactly the amount of the increase in per capita debt. As a result, both the distorting taxes $\boldsymbol{\tau}$ and allocations remain unchanged. This example illustrates ideas expressed by Newcomb (1865, p.85) in the quotation with which we began this paper.

This logic is sensitive to the assumption that holdings of additional government debt are equal across agents. Suppose instead that the government debt is owned disproportionately by high-earnings agents so that inequality is higher in economies with higher government debt; the optimal fiscal response would typically call for an increase in both tax rates $\boldsymbol{\tau}$ and transfers $\boldsymbol{T}$. The conclusion would be the opposite if government debt were to be disproportionately owned by low-earnings agents.[4]

Proposition 1 cautions against comparing debt burdens across countries based purely on aggregate quantities like debt to GDP ratios. If governments want to redistribute from high-earning to low-earning agents, public debt that is held widely by private agents or government agencies typically will be less distorting than public debt held by agents in the right tail of the earning distribution or by foreign investors. Similarly, our result warns against lumping together explicit debt and implicit debt (such as Social Security obligations) into one aggregate number without adjusting for heterogeneity across holdings of the various types of debts.

Another implication of Lemma 2 is that a path of government debt in the optimal competitive equilibrium with natural debt limits is indeterminate.

[4] It is straightforward to extend our analysis to an open economy with foreign holdings of domestic debt. The more government debt is owned by the foreigners, the higher are the distortions the government will need to impose.

Proposition 2 (Ricardian equivalence). *Suppose that an optimal equilibrium with a natural debt limit given $\left(\left\{b_{i,-1}\right\}_i, B_{-1}\right)$ exists. Then any bounded **B** is part of an optimal competitive equilibrium.*

Lemma 2 and its implications in the form of Propositions 1 and 2 are true in more general environments too. For example, we can allow agents to trade all conceivable Arrow securities and still show that equilibrium allocations depend only on agents' net asset positions. Our results also hold in economies with capital and with arbitrary non-linear income tax schedules $\mathcal{T}_t(y_t)$.

Results in this section suggest that the presence or absence of distorting taxes or incomplete markets is by itself insufficient to imply anything about the level public debt or its welfare costs. In contrast to representative agent models such as Barro (1974, 1979), in our heterogeneous agent setting, both the slope τ_t and the intercept T_t are distorting, but the path of debt in the optimal allocation is indeterminate. The discussion also sheds light on the role of debt in Woodford (1990) and Aiyagari et al. (2002) who allow for lump-sum taxes but feature incomplete markets. In those models, additional restrictions in the form of borrowing limits and ad hoc costs of lump-sum taxes generate motives that ultimately pin down a path of debt. We investigate the role of such assumptions in Section 4.

3.1. Numerical example

To illustrate that it is not total government debt but the distribution of debt across people that matters, we study how the correlation between debt holdings and labor earnings affects an optimal tax rate and output.

Labor earnings and holdings of government debt are highly correlated in the U.S. We use data from the 2013 wave of the Survey of Consumer Finance (SCF) with the sample restricted to married households. The SCF provides information on households' total labor earnings, as well as hours worked by primary and secondary earners. From these data, we construct average household wages. To measure households' holdings of government debt, we sum direct holdings plus indirect holdings through government bond mutual funds (taxable and nontaxable), saving bonds, money market accounts, and components of retirement accounts that are invested in government bonds. We use these variables to compute a least squares regression

$$debt_i = \hat{\delta}_0 + \hat{\delta}_1 wage_i + \epsilon_i \tag{13}$$

and obtain $\hat{\delta}_0 = 14.63$ with s.e. of 2.15, $\hat{\delta}_1 = 0.93$ with s.e. of 0.024, and an R^2 of 7.8%.[5]

To study how the correlation between debt and wages affects an optimal tax rate and output, we assume isoelastic preferences

$$U(c_i, l_i) = \frac{c^{1-\sigma}}{1-\sigma} - \psi\left(\frac{l^{1+\gamma}}{1+\gamma}\right) \tag{14}$$

and no uncertainty. We set set utility function parameters σ, γ, β equal to 1,2,0.96. We choose $I = 20$ with equal measures of agents in each group and fix a time-invariant distribution $\{\theta_i\}_i$ that replicates average wages per quintile for each of 20 wage quintiles in the 2013 SCF. We summarize the distribution of debt holdings $\{b_i\}_i$ with an affine function $b_{i,-1} = \delta_0 + \delta_1\theta_i$.

As a baseline, we set (δ_0, δ_1) equal to our estimated regression coefficients, $(\hat{\delta}_0, \hat{\delta}_1)$. We evaluate welfare using Pareto weights $\omega_i \propto \theta_i^{-\alpha}$. We assume that government expenditures are time-invariant and pick (g, α, ψ) jointly to match a federal government expenditures to output ratio of 12%, the average federal labor tax rate of 24% estimated by Barro and Redlick (2011), and a total debt to total labor earnings ratio of 0.92 that we infer from the 2013 SCF.

Given our assumptions, for any (δ_0, δ_1) the optimal tax rate and output are constant for all $t \geq 1$. It follows from Proposition 1 that neither the optimal tax rate nor optimal output depends on the value of δ_0 because any change in debt holdings due to changes in δ_0 leaves net asset inequality unchanged. But net asset inequality is affected by changes in δ_1. Fig. 2 shows comparative statics of optimal tax rates and outputs as functions of δ_1. Higher values of δ_1 correspond to distributions of bond holdings that are more concentrated with productive agents. We see that an optimal plan responds to a more top heavy distribution of bond holdings by raising the tax rate in order to redistribute towards poor agents. A higher tax distortion makes output decline.

We can use these graphs to interpret effects from changes in government debt that are distributed proportionally to initial benchmark debt holdings. Suppose that, relative to the benchmark economy, government debt is increased by $\triangle$ percent and that this additional debt is distributed to households proportionally to their benchmark debt holdings. The new distribution of debt is described by parameters $(\delta_0, \delta_1) = (\hat{\delta}_0(1+\triangle), \hat{\delta}_1(1+\triangle))$. Since δ_0 does not affect the tax rate or output, the ratio $\delta_1/\hat{\delta}_1$ captures the effect of a $\delta_1/\hat{\delta}_1$-fold increase in government debt distributed proportionally. In this experiment, the level of debt seems to matter, but that is because we are simultaneously changing both the total amount of debt and the net distribution of debt holdings. If government debt doubles and debt is distributed proportionally to the holdings of debt in the data, Fig. 3 indicates that output drops by 2 percentage points and that the tax rate increases by 4 percentage points as we vary $\Delta \in [-0.5, 0.5]$.

[5] We measure wages in units of thousand dollars per annual hours and average hours per year supplied by the household are measured using the 2013 SCF. Bond holdings are measured in thousand dollars.

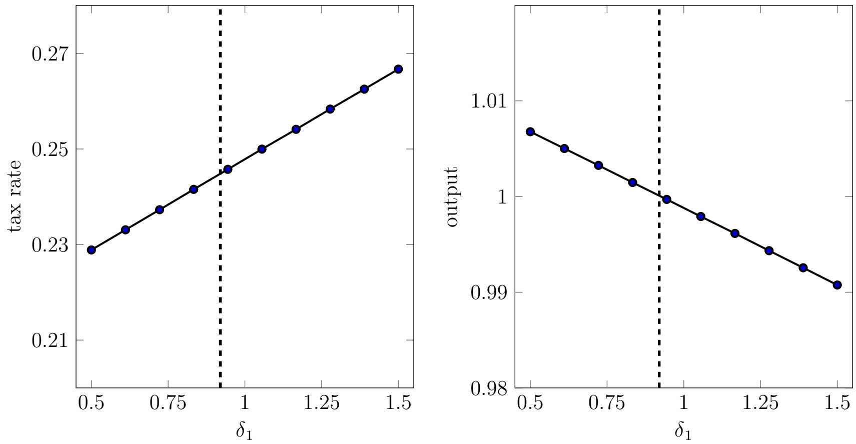

Fig. 2. Comparative statics for tax rate, τ_1 and output y_1 normalized by its value in the baseline across values for δ_1, where δ_1 changes the initial distribution of debt following $b_{i,-1} = \delta_0 + \delta_1\theta_i$. The dashed line is the baseline calibration with $\delta_1 = 0.93$.

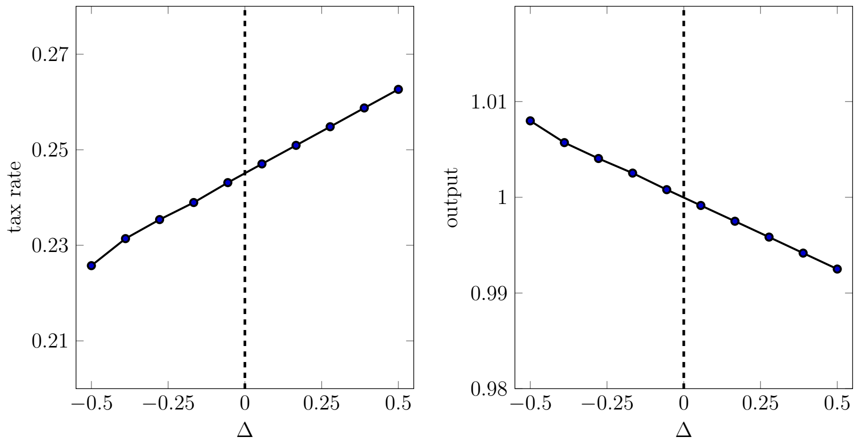

Fig. 3. Comparative statics for tax rate, τ_1 and output y_1 normalized by its value in the baseline across values for Δ, where Δ changes the initial distribution of debt following $b_{i,-1} = \hat{\delta}_0(1+\Delta) + \hat{\delta}_1(1+\Delta)\theta_i$ and $\hat{\delta}_0 = 14.63$ and $\hat{\delta}_1 = 0.93$. The dashed line recovers the baseline calibration for $\Delta = 0$.

4. Imperfect debt enforcement and ad hoc borrowing constraints

The analysis of the previous section closely follows the Ramsey tradition of answering normative questions. At the outset we specify sequences of instruments available to the government ($\boldsymbol{\tau}$, $\boldsymbol{T}$, and $\boldsymbol{B}$ in our case) and assume that the government commits to those sequences in period -1. Optimizing over a set of competitive equilibria associated with those sequences implicitly assumes that the government has the ability to pick the equilibrium with the highest welfare from that set. That is, the government has a *technology* that allows it perfectly to implement an equilibrium allocation associated with its policies.

To elaborate the implementation issue, consider a situation in which agents make choices that render some budget constraints violated, for example, by some agents not working enough to be able to meet their tax liabilities. An implicit enforcement technology assumption would require the government to impose punishments sufficiently harsh to prevent agents from making such choices “off-equilibrium”. If consumption is bounded by 0 and $\lim_{c\to 0} U^i(c,l) = -\infty$ for all i, l, it is sufficient to specify that the government commits to seizing all of an agent’s labor and asset income in a period in which he cannot pay its prescribed taxes. But if the utility function is bounded from below, additional non-pecuniary punishments may be needed to implement an allocation.

The same assumption of perfect enforcement would extend to repayment of private debts – agents never fail to repay their debts in equilibrium presumably because the punishments for not doing so are sufficiently severe. Thus, the equilibrium definition in Section 3 indirectly requires not only that the government has the ability to enforce payments, but also that it uses its ability to enforce both tax and debt payments.

In this section, we stay within the boundaries of a conventional Ramsey analysis but focus on whether it is desirable for the government to enforce both tax and debt obligations and whether it can improve welfare by committing to enforce some type of payments and not others. We represent the government's enforcement choice in a simple form by assuming that agents can borrow up to an ad hoc debt limit

$$b_{i,t} \geq -\underline{b} \tag{15}$$

for some exogenously given $\underline{b} \geq 0$. We interpret these constraints as arising from imperfect government debt enforcement: the government imposes an arbitrary high punishment on agents if they default on any debt less than $\underline{b}$ and no punishment for any default on debt over $\underline{b}$; the case $\underline{b} = 0$ is interpreted as the government's refusing to enforce any private debt contracts. The natural debt limit considered in the previous section is a limit that arises when agents are punished for any debt default.[6] Note that we maintain the assumption that the government enforces tax liabilities perfectly: thus, we study whether it is optimal to enforce taxes and debt contracts differentially.[7]

Definition 4. A *competitive equilibrium with an ad hoc debt limit* given initial assets $\left(\left\{b_{i,-1}\right\}_i, B_{-1}\right)$ is a $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ such that (i) $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ maximize (1) subject to (7) and (15) for all i; and (ii) constraints (2), (3), and (5) are satisfied.

To understand what determines the path of debt, we first show that, in general, it is optimal for the government *not* to enforce private contracts. Restricting private borrowing allows the government more flexibility in managing its debt service costs. Indeed, an optimal path of debt is pinned down by these considerations. This outcome contrasts with to alternative accounts that emphasize that a government should issue debt to *increase* liquidity because there is a lack of other means of savings. We begin with our main proposition for this section.[8]

Proposition 3. *If there are tax policies that support an allocation* $(\boldsymbol{c}, \boldsymbol{l})$ *as a competitive equilibrium allocation with a natural debt limit, then there are tax policies that support* $(\boldsymbol{c}, \boldsymbol{l})$ *as a competitive equilibrium allocation with an ad hoc debt limit for any* $\underline{b}$*. If* $(\boldsymbol{c}, \boldsymbol{l})$ *can be supported as a competitive equilibrium allocation with an ad hoc debt limit* $\underline{b}'$*, it can also be supported as a competitive equilibrium allocation with ad hoc debt limit* $\underline{b}''$ *for any* $\underline{b}''$*.*

Proof. Let $\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i$ be a competitive equilibrium allocation and debt with a natural debt limit. Let $\Delta_t \equiv \max_i \left\{\underline{b} - b_{i,t}\right\}$. Define $\hat{b}_{i,t} \equiv b_{i,t} + \Delta_t$ for all t. By Lemma 2, $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ is also a competitive equilibrium allocation with natural debt limits. Moreover, by construction $\hat{b}_{i,t} - \underline{b} = b_{i,t} + \Delta_t - \underline{b} \geq 0$. Therefore, $\hat{\boldsymbol{b}}_i$ satisfies (15). Since agents' budget sets are smaller in the economy with ad hoc debt limits and since $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ lies in this smaller budget set, then $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ is also an optimal choice for agents in the economy with exogenous borrowing constraints $\underline{b}$. Since all market clearing conditions are satisfied, $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ is a competitive equilibrium allocation and asset profile.

To prove the second assertion, let $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ be a competitive equilibrium with debt limit $\underline{b}'$. Define $\Delta_t \equiv \underline{b}' - \underline{b}''$ and construct $\left(\hat{T}, \hat{B}\right)$ as in (11), $\hat{b}_{i,t} = b_{i,t} + \Delta_t$ for all i, t. Then by using arguments from Lemma 2 we can show that $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\}_i, \hat{\boldsymbol{B}}, \boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}})$ is a competitive equilibrium with debt limit $\underline{b}''$. □

A remarkable implication of Proposition 3 is not only that the government finds it optimal to treat transfers and debt differently, but that the weakest possible enforcement of private debt contracts is optimal. Without loss of generality, we can assume that agents cannot borrow.

Corollary 1. *Welfare in an optimum equilibrium with ad hoc debt limits is higher than welfare in the optimum equilibrium with natural debt limits. This is true for any debt limit* $\underline{b}$*.*

A crucial difference between outcomes with the ad hoc debt limits studied in this section and natural debt limits in the previous section is how they depend on the tax policy. While the lower bound on debt is endogenous and depends on the government tax-transfer policy Section 3 discussion of natural debt limits, it is exogenous with the ad hoc debt limits of this section. The presence of a policy invariant debt limit here implies that changing the timing of transfers can change the set of agents who are up against their borrowing limits. This power lets the government increase welfare.

The critical feature being exploited here is an asymmetric enforcement of taxes and private debt. If the government allowed agents to use prospective transfers as collateral for private borrowing, then by postponing transfers the government would relax agents' borrowing constraints and undo its ability to increase welfare by pushing some people against their borrowing constraints. Asymmetry between enforcement of debt obligations and tax obligations seems to be common in practice. For example, in the U.S. it is illegal to use future social security payments as collateral and it is typically easier to discharge unsecured debt than tax liabilities through bankruptcy.

[6] We believe that another fruitful way to study the role of debt is to drop the full commitment assumption and explicitly specify strategies for all histories for agents and the government as was done by Bassetto (2002) in a closely related context of monetary economics and the fiscal theory of price level. See also Bassetto (2005).

[7] Bryant and Wallace (1984) describe how a government can use borrowing constraints as part of a welfare-improving policy to finance exogenous government expenditures. Sargent and Smith (1987) describe Modigliani–Miller theorems for government finance in a collection of economies in which borrowing constraints on classes of agents produce the rate of return discrepancies that Bryant and Wallace manipulate.

[8] Our proposition builds on Yared (2012); 2013), who showed that a planner may find it optimal not to undo agents' borrowing constraints even when doing so is feasible.

Others have also studied Ramsey policies in economies with ad hoc constraints (15) and pointed out that Ricardian equivalence fails and consequently that the optimal debt is determined.[9] Thus, in the context of the results of Section 3, our Proposition 2 would generally not hold when agents are subject to the ad hoc constraint (15). In the following example we investigate the sources of welfare gains that come from limiting agents' opportunities to borrow.

Example 1. Suppose that there are two types of agents with equal mass. Agent 1 cannot work and orders preferences by $c_{1,\,t}$. Agent 2 orders preferences by $u(c_{2,t} - \frac{1}{1+\gamma} l_{2,t}^{1+\gamma})$ with $\gamma > 0$. Agent2′s productivity satisfies $\theta_{2,t} = 1$ if t is even and $\theta_{2,t} = 0$ if t is odd. There are no government expenditures. The government puts Pareto weight 1 on agent 1's utility. All agents start with no initial assets.

Consider first the optimum equilibrium when debt enforcement is perfect and agents face a natural debt limit. In this case, agent1′s preferences imply that the equilibrium sets $q_t = \beta$ for all t. The government's objective function makes it want to maximize the present value of tax revenues, evaluated at the price system implied by $q_t = \beta$ for all t. Given agent 2's preferences, the optimal tax rate is $\tau_t = \bar{\tau}$ for all t, where $\bar{\tau}$ is the top of the Laffer curve tax rate, namely, $\bar{\tau} = \frac{\gamma}{1+\gamma}$ which implies a labor supply $\bar{l} = (\frac{1}{1+\gamma})^{1/\gamma}$. These findings imply that welfare with natural debt limits is $\sum_t \beta^t T_t = \frac{1}{2}\frac{\bar{Z}}{1-\beta^2}$ where $\bar{Z} = \gamma(\frac{1}{1+\gamma})^{1+1/\gamma}$. By Lemma 2, the timing of transfers is indeterminate. For example, the welfare optimum can be attained by setting $b_{1,t} = 0$ for all t and $(\boldsymbol{B}, \boldsymbol{T})$ that jointly solve the following equations for all t

$$T_{2t+1} = B_{2t}, \quad T_{2t} + \beta B_{2t} = \bar{Z}, \quad B_{2t+1} = 0 \tag{16}$$

and

$$u'\left((1-\bar{\tau})\bar{l} + 2\beta B_{2t} - \frac{1}{1+\gamma}\bar{l}^{1+\gamma}\right) = u'(T_{2t+1} - 2B_{2t}). \tag{17}$$

Agent 2's budget constraint and (2) imply that $B_{2t} < 0$ for all t. Thus, the government issues debt in even periods. The government repays this debt in odd periods by levying (negative) lump sum transfers. Agent 2 holds government debt to smooth marginal utility intertemporally. We denote an optimum equilibrium with natural debt limits and these transfer and debt sequence as $\left(\left\{\boldsymbol{c}_i^{nat}, \boldsymbol{l}_i^{nat}, \boldsymbol{b}_i^{nat}\right\}_i, \boldsymbol{B}^{nat}, \boldsymbol{q}^{nat}, \boldsymbol{\tau}^{nat}, \boldsymbol{T}^{nat}\right)$.

Now consider the economy in which private debt constraints are not enforced, so that agents' debts must satisfy

$$b_{i,t} \geq 0 \text{ for all } i, t. \tag{18}$$

Observe that $\left(\left\{\boldsymbol{c}_i^{nat}, \boldsymbol{l}_i^{nat}, \boldsymbol{b}_i^{nat}\right\}_i, \boldsymbol{B}^{nat}, \boldsymbol{q}^{nat}, \boldsymbol{\tau}^{nat}, \boldsymbol{T}^{nat}\right)$ still satisfies agents' budget constraints under debt limits (18), so it is also an equilibrium in the economy without private borrowing. But now we can construct an equilibrium with higher welfare.

Thus, it is possible to show that $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ is part of an equilibrium with ad hoc limits (18) if and only if budget constraints (7) holds for both agents (with $\theta_{1,t} = 0$ for all t), feasibility (2), market clearing (3), and borrowing constraints (18) are satisfied, and the following equations also hold:

$$l_{2,t}^{\gamma} = (1-\tau_t)\theta_{2,t}, \tag{19a}$$

$$q_t u'\left(c_{2,t} - \frac{1}{1+\gamma} l_{2,t}^{1+\gamma}\right) \geq \beta u'\left(c_{2,t+1} - \frac{1}{1+\gamma} l_{2,t+1}^{1+\gamma}\right), \tag{19b}$$

$$\left[q_t u'\left(c_{2,t} - \frac{1}{1+\gamma} l_{2,t}^{1+\gamma}\right) - \beta u'\left(c_{2,t+1} - \frac{1}{1+\gamma} l_{2,t+1}^{1+\gamma}\right)\right] b_{2,t} = 0, \tag{19c}$$

$$q_t \geq \beta, \tag{19d}$$

$$[q_t - \beta] b_{1,t} = 0. \tag{19e}$$

Eq. (19a) is the optimality condition for labor of agent 2; Eqs. (19b)–(19e) are optimality conditions for savings that hold with inequality only if the agent's assets are zero, and with equality otherwise.

A key observation about these conditions is that there exist equilibrium q_t that are higher than the discount factor β when the assets chosen by agent 1 are zero. We show in the online appendix 2 that for any $\varrho \geq \beta$ we can construct an

[9] For instance, see Woodford (1990), Aiyagari and McGrattan (1998), and Azzimonti et al. (2014). Some commentators observed that these breakdowns of Ricardian equivalence implicitly require that it is easier to extract a dollar from an agent in taxes than in debt service. Our analysis indicates that it is an optimal choice for the government to choose arrangements that produce that outcome even if the same technology is available for enforcing both types of payments.

equilibrium in which $\tau_t = \bar{\tau}$ and $b_{1,t} = 0$ for all t and an (inverse of) gross interest rate sequence $\boldsymbol{q}(\varrho) = (\varrho, \beta, \varrho, \beta, \ldots)$. This equilibrium is supported by transfer and debt sequences $(\boldsymbol{T}(\varrho), \boldsymbol{B}(\varrho))$ that satisfies

$$T_{2t+1}(\varrho) = B_{2t}(\varrho), \quad T_{2t}(\varrho) + \varrho B_{2t}(\varrho) = \bar{Z}, \quad B_{2t+1}(\varrho) = 0, \tag{20}$$

which generalizes (16). Differentiate to obtain

$$\frac{\partial}{\partial \varrho} \sum_t \beta^t T_t(\varrho) \bigg|_{\varrho=\beta} = -\sum_t \beta^{2t+1} B_{2t}^{nat} > 0. \tag{21}$$

As welfare is simply $\Sigma_t \beta^t T_t(\varrho)$, it follows that, for ϱ close to β, lowering equilibrium interest rates (increasing ϱ) improves welfare. Since $\varrho = \beta$ corresponds to welfare in the optimum equilibrium with natural debt limits, this also proves that welfare with ad hoc limits is strictly higher.

Example 1 illustrates what determines an optimal quantity of debt. If the government issues debt in equilibrium, it is generally better off if interest payments on that debt are lower. In the economy with natural debt limits, equilibrium interest rates are determined implicitly by a competition between the government and agent 1 to supply savings ("liquidity") to agent 2. Even though in that equilibrium agent 1 does not supply liquidity, he would, by issuing private risk-less debt whenever the interest rate drops below the inverse of his rate of time preference, namely, β^{-1}. When private debt contracts are unenforceable, agent 1 cannot issue riskless debt, so the government becomes a monopoly supplier of liquidity to agent 2. The government can use its monopoly power to extract additional surplus from agent 2 by issuing debt at a lower interest rate.[10]

Results of Woodford (1990) and Aiyagari and McGrattan (1998) are often interpreted as justifying a beneficial role for government debt by its increasing the supply of savings instruments. Our analysis instead suggests that the government should *decrease* the aggregate supply of liquidity by limiting the enforcement of private debt contracts and using market power thereby acquired to extract monopoly rents from providing liquidity.

While an optimal continuation level of government debt is determined in the equilibria we have been analyzing, the initial level of government debt is irrelevant for welfare in the same sense as in Proposition 1.

Proposition 4. *Proposition 1 holds in an economy with ad hoc debt limits. If $\boldsymbol{B}'$ is the optimal path of debt given $(\{b'_{i,-1}\}_i, B'_{-1})$, then $\boldsymbol{B}'$ is also the optimal path of debt given $(\{b''_{i,-1}\}_i, B''_{-1})$ if $b'_{i,-1} - b''_{i,-1}$ is independent of i.*

Proof. Suppose $(\boldsymbol{\tau}', \boldsymbol{T}')$ are the optimal taxes in the economy with initial assets $(\{b'_{i,-1}\}_i, B'_{-1})$. Define a sequence $\boldsymbol{T}''$ by $T''_0 = T'_0 + b'_{I,-1} - b''_{I,-1}$ and $T''_t = T'_t$ for all $t > 0$. Following the same steps as in the proof of Lemma 2 we can verify that $(\boldsymbol{\tau}', \boldsymbol{T}'')$ are optimal taxes in the economy with initial assets $(\{b''_{i,-1}\}_i, B''_{-1})$. □

To understand why the initial level of government debt is welfare-irrelevant, note that the welfare gains in Example 1 are obtained from the government's ability to influence prices of future debt. The value of legacy debt with which the government enters period 0 was set in the past and is not affected by future policies. Thus, the initial debt level plays a role no different from that in Section 3. Note that Proposition 4 shows not only that welfare but also that the optimal debt path is independent of the level of initial government debt B_{-1}, though they generally do depend on how initial assets are distributed across agents. Thus, transitions to an optimal debt level take exactly one period, independently of the initial debt.

As a final remark, Lemma 2 and Proposition 1 - 4 continue to hold when we allow for idiosyncratic income risk. More details for economies with idiosyncratic risk are provided in the online appendix 3.

5. Informationally-constrained optimal taxes

The analysis of previous sections follows the Ramsey tradition by a priori restricting the tax-transfer system to take a particular form, in our case the affine tax system (4). An alternative approach is to put explicit constraints on the government's information and then to derive optimal government policies that respect them. This approach originated in the work of Mirrlees (1971) and was introduced to macro by Golosov et al. (2003) and Werning (2007). In this section, we investigate the role of debt and taxes when government actions are restricted only by such informational frictions.

An *informationally-constrained optimum* is a sequence $\{\mathbf{c}_i, \mathbf{l}_i\}_i$ that maximizes (6) subject to feasibility (2) and constraints that specify the government's information about agents. *Informationally-constrained taxes* are tax functions that use observable variables as their arguments; *optimal informationally-constrained taxes* implement an informationally-constrained optimum as a competitive equilibrium.

Since Mirrlees (1971), a standard assumption is that the government does not observe an individual's labor supply $l_{i,t}$ or productivity $\theta_{i,t}$ but that it does observe labor earnings $y_{i,t}$. We maintain this assumption throughout this section. The

[10] This example illustrates a more general principle. In an economy with a natural debt limit, observe that the resource flow between agents 1 and 2 is determinate, but that the level of borrowing is not – in order to let agent 2 smooth consumption, either agent 1 or the government can borrow from agent 2. Lower interest rates in a period t benefit the agent who experiences the net resource inflow in that period, whether he borrows himself or receives this inflow through transfers. Lowering interest rates is desirable when the government favors such agents. Bassetto (2014), Niepelt (2004), and Yared (2012) apply this principle in other contexts.

role of public and private debt depends critically on whether the government observes individuals' assets and consumptions. If agents' assets are observable, public or private debt plays no interesting role: *any* sequence ($\boldsymbol{B}$, $\{\boldsymbol{b}_i\}_i$) that satisfies feasibility (3) can be supported in an optimal competitive equilibrium for the following simple reason. Let $\{\mathbf{c}_i^{ob}, \mathbf{l}_i^{ob}\}_i$ be an informationally-contained optimum with observable assets and let $\boldsymbol{y}_i^{ob}$ be defined by $y_{i,t}^{ob} \equiv \theta_{i,t} l_{i,t}^{ob}$. The government can implement $\{\mathbf{c}_i^{ob}, \mathbf{l}_i^{ob}\}_i$ by offering a menu of I tax schedules of the form $\{\mathcal{T}_t(y_t, b_{t-1}, b_{-1}, i)\}_t$ and letting agents permanently self-select into one of them in period $t = -1$.[11]

The problem becomes more interesting when agents' assets are unobservable. Assume that interactions in asset markets are anonymous and that agents and the government can issue and buy debt, but that it is impossible for the government to ascertain an individual agent's asset holdings. This assumption also requires that individual consumption is not observable.

The informationally-constrained optimum can be characterized by invoking the Revelation principle and setting up a mechanism design problem.[12] We now define an informationally-constrained optimal allocation with unobservable assets associated with a mechanism design problem that determines labor income $\{\mathbf{y}_i\}_i$ and payments $\{\mathbf{x}_i\}_i$ as well as a debt sequence $\mathbf{B}$. A reporting strategy is a function r: $I \to I$. A mechanism $\{\boldsymbol{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ is feasible if there exists an allocation $\{\mathbf{c}_i, \mathbf{l}_i\}_i$, asset choices $\{\mathbf{b}_i\}_i$, a reporting strategy r and bond prices $\mathbf{q}$ such that each agent i chooses $\{\mathbf{c}_i, \mathbf{l}_i\}_i$, $\mathbf{b}_i$, $r(i)$ to maximize (1) subject to the budget constraint

$$c_{i,t} + q_t b_{i,t} = x_{r(i),t} + b_{i,t-1}, \tag{22}$$

with $b_{i,t}$ satisfying either natural or ad hoc debt limits. Prices $\mathbf{q}$ are such that debt market clearing (3) and feasibility

$$\sum_i n_i c_{i,t} = \sum_i n_i y_{r(i),t} \tag{23}$$

are satisfied. A feasible mechanism $\{\boldsymbol{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ is incentive compatible if the associated reporting strategy $r(i) = i$. An informationally constrained optimum is an incentive compatible mechanism $\{\boldsymbol{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ such that the associated allocation $\{\mathbf{c}_i, \mathbf{l}_i\}_i$ maximizes (6).

The ability of agents to trade assets anonymously lowers welfare, which implies that the government would find it optimal to minimize enforcement of private debt contracts. Given that, we first analyze this economy when private borrowing is subject to the ad hoc limit (15). Then we discuss how our conclusions would change if debt enforcement on private markets were perfect.

Consider any incentive compatible mechanism $\{\boldsymbol{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ and let $\{\mathbf{b}_i, \mathbf{c}_i\}_i$ be the associated optimal asset and consumption choices and let $\mathbf{q}$ be bond prices. A necessary condition for incentive compatibility is

$$\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t U^i\left(c_{i,t}, \frac{y_{i,t}}{\theta_{i,t}}\right) \geq \mathbb{E}_{-1}\left[U^i\left(c_{j,0} + b_{i,-1} - b_{j,-1}, \frac{y_{j,0}}{\theta_{i,0}}\right) + \sum_{t=1}^{\infty} \beta^t U^i\left(c_{j,t}, \frac{y_{j,t}}{\theta_{i,t}}\right)\right], \tag{24}$$

for all pairs i, j. The left side is the utility of agent i when he receives allocation $\{\boldsymbol{x}_i, \mathbf{y}_i\}$. This should be at least as high as utility from claiming a bundle $(\boldsymbol{x}_j, \boldsymbol{y}_j)$ and choosing asset profile $\boldsymbol{b}_j$ on the anonymous market at the same prices $\mathbf{q}$. The payoff from that choice is the right side of constraint (24). In principle, agent i can further increase his utility from bundle $(\boldsymbol{x}_j, \boldsymbol{y}_j)$ if he chooses some other asset profile $\boldsymbol{b}'$; but as we show below, if trading is subject to ad hoc debt limits, an optimally chosen debt sequence $\boldsymbol{B}$ prevents such retrading.

Let $\{\mathbf{c}_i^{adhoc}, \mathbf{y}_i^{adhoc}\}_i$ be a maximizer of the objective function (6) subject to feasibility (3) and incentive constraint (24). Let $B_t = \underline{b}$ for all t and choose any $\boldsymbol{q}$ that satisfies

$$q_t \geq \beta \frac{\mathbb{E}_t U_c^i\left(c_{j,t+1}^{adhoc}, \frac{y_{j,t+1}^{adhoc}}{\theta_{i,t+1}}\right)}{U_c^i\left(c_{j,t}^{adhoc}, \frac{y_{j,t}^{adhoc}}{\theta_{i,t}}\right)} \text{ for } t > 0, \text{ all } i, j,$$

$$q_0 \geq \beta \frac{\mathbb{E}_0 U_c^i\left(c_{j,1}^{adhoc}, \frac{y_{j,1}^{adhoc}}{\theta_{i,1}}\right)}{U_c^i\left(c_{j,0}^{adhoc} + b_{i,-1} - b_{j,-1}, \frac{y_{j,0}^{adhoc}}{\theta_{i,0}}\right)} \text{ for all } i, j. \tag{25}$$

Choose sequence $\{\mathbf{x}_i^{adhoc}\}_i$ such that

$$c_{i,t}^{adhoc} - q_t \underline{b} = x_{i,t}^{adhoc} + \mathring{b}_{i,t}, \tag{26}$$

where $\mathring{b}_{i,t} = -\underline{b}$ for $t > 0$ and $\mathring{b}_{i,t} = b_{i,-1}$ for $t = 0$.

For an agent of i type who claims sequence $\boldsymbol{x}_j^{adhoc}$ and faces debt prices $\boldsymbol{q}$, it is optimal to borrow up to the maximum debt limit $\underline{b}$ and therefore obtain the after-tax consumption allocation $c_{j,0}^{adhoc} + b_{i,-1} - b_{j,-1}$ and $\{c_{j,t}^{adhoc}\}_{t>0}$. Constraint

[11] It is easy to implement an optimal allocation with smooth tax functions. See, for example, Kocherlakota (2005), Werning (2009), and Grochulski and Kocherlakota (2010). The conclusion that neither public nor private assets are pinned down continues to hold in those implementations.

[12] See Golosov and Tsyvinski (2007) for details.

(24) ensures that the optimal report is $r(i) = i$ for all i, verifying that $\{\mathbf{c}_i^{adhoc}, \mathbf{y}_i^{adhoc}\}_i$ is indeed an informationally-constrained optimum. This optimum can be implemented by a sequence of tax functions of the form $\{\mathcal{T}_t(y_t, i)\}_{i,t}$.

Observe that government debt $\boldsymbol{B}$ plays the same role here as it did in Section 4. When agents face ad hoc borrowing constraints the government can affect interest rates by choosing the level of its debt. As in Section 4, the government exploits monopoly power on asset markets and lowers interest rates. The size of the borrowing constraint $\underline{b}$ is irrelevant for welfare because the government covers its interest expenses by adjusting the stream of transfers to agents without affecting final allocations.

Like our discussion in Section 4, it is crucial for this result that private debt contracts are enforced imperfectly. If agents can trade on anonymous markets subject only to a natural debt limit, the government loses its ability to influence interest rates through $\boldsymbol{B}$, so the Ricardian equivalence result of Proposition 2 reemerges. Since Eq. (25) would hold with equality in an equilibrium with natural debt limits, welfare would be lower.

The role of the initial debt level and initial asset inequality also mirrors that described in Propositions 1 and 4. The absolute level of government debt B_{-1} per se does not affect welfare in the constrained optimum, but asset inequality does, as can be seen from the right side of (24).

We summarize our analysis in the following proposition.

Proposition 5. *The initial level of debt B_{-1} does not affect welfare with optimal informationally-constrained taxes, but the level of initial asset inequality $b_{i,-1} - b_{I,-1}$ generally does. Necessary conditions for the optimal path of government debt $\mathbf{B}$ to be determinate are anonymous asset trades and ad hoc borrowing constraints. Welfare is higher in the economy with ad hoc debt limits than in the economy with natural debt limits.*

It is straightforward to generalize these results to economies with idiosyncratic shocks, richer asset markets, and capital.[13]

6. Concluding remarks

A principal message of this paper is that without exogenous restrictions on transfers, the level of government debt doesn't matter. What matters is how ownership of government debt is distributed. Depending on society's attitudes toward unequal distributions of consumption and work, the cross-section distribution of government debt across assets can matter very much. This means that in order to interpret empirical correlations between output growth rates and ratios of government debt to GDP as in Reinhart and Rogoff (2010), we would want to know much more about how the distributions of net assets across people have varied across countries and how they have interacted with risks to interest rates and to the underlying sources of unequal productivities across people. An optimal path of government debt is determined when agents' abilities to borrow are restricted because that allows prospective public debt issues to affect interest rates.

We restricted our analysis to economies in which the government commits to future policies. A promising topic for research is the role of debt when a government cannot commit. As our discussion in Section 4 suggests, imperfect commitment can impose additional restrictions on transfers and debt that are feasible in equilibrium. We leave this extension to future work.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jmoneco.2017.09.007.

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[13] Under mild technical assumptions (e.g., assumption 1 in Kocherlakota (2005)) one can implement informationally-constrained optimal allocations using tax schemes that depend on past histories of individual incomes. Bassetto and Kocherlakota (2004) show that allowing tax functions that are sufficiently flexible in terms of history dependence makes the path of government debt irrelevant. Gonzalez-Eiras and Niepelt (2015) extend analysis of Ricardian equivalence to political economy by studying conditions under which different policy regimes (institutions) are politico-economically equivalent in the sense that both regimes give rise to politico-economic equilibria with identical allocations.

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