Social Insurance, Information Revelation, and Lack of Commitment

Online appendix

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Appendix A: Proofs for Section 2

1 Proof of Corollary 2

For $0 \leq \varsigma \leq 1$, we denote with $U(\varsigma)$ the on-the-equilibrium value defined as

$$U(\varsigma) \equiv \max_{u_1,u_2,l} \varsigma u_1 + (1-\varsigma)u_2, \tag{A.1}$$

subject to (u_1, l), $(u_1, 0)$, $(u_2, 0) \in dom(C)$ and

$$\varsigma p\{C(u_1, l) - l\} + \varsigma(1-p)C(u_1, 0) + (1-\varsigma)C(u_2, 0) = 0.$$

Similarly, we let $W(\varsigma)$ be the off-the-equilibrium value, which is defined as

$$W(\varsigma) \equiv \max_{u_1,u_2,l} \varsigma p u_1 + (1-\varsigma p)u_2, \tag{A.2}$$

subject to (u_1, l), $(u_2, 0) \in dom(C)$ and

$$\varsigma p\{C(u_1, l) - l\} + (1-\varsigma p)C(u_2, 0) = 0.$$

Therefore, $U^{fb} \equiv W(1)$ and $U^{sb} \equiv U(1)$ and $\bar{\Upsilon} = W(1) - U(1)$.

We begin by proving that $\bar{\Upsilon} > 0$. More specifically, we prove the more general claim that $W(\varsigma) > U(\varsigma)$, for all $\varsigma > 0$. For all ς, $U(\varsigma)$ equals the value of the following auxiliary problem:

$$U(\varsigma) = \max_{u_1,u_2,u_3,l} \varsigma p u_1 + \varsigma(1-p)u_2 + (1-\varsigma)u_3$$

subject to (u_1, l), $(u_2, 0)$, $(u_3, 0) \in dom(C)$,

$$\varsigma p\{C(u_1, l) - l\} + \varsigma(1-p)C(u_2, 0) + (1-\varsigma)C(u_3, 0) \leq 0,$$

and

$$u_1 = u_2.$$

Note that, if the last constraint does not bind, the value of the auxiliary problem equals $W(\varsigma)$ (for $\varsigma < 1$, by strict convexity of C we have $u_2^* = u_3^*$). We want to show that the last constraint binds for all $\varsigma > 0$. Suppose it does not. The problem is convex, thus, its solution is characterized by a saddle point Lagrangian:

$$\mathcal{L} = \varsigma p u_1 + (1 - \varsigma p)u_2 - \zeta[\varsigma p\{C(u_1, l) - l\} + (1 - \varsigma p)C(u_2, 0)],$$

where ζ is the multiplier on the resource constraint. The resource constraint must be binding, thus, $\zeta > 0$. Lemma 1 in the paper then implies $l^* > 0$. We need to consider several cases. Suppose first that (u_1^*, l^*) is at the lower bound of the domain of C (that is, $u_1^* = U(0, l^*)$). Since utility is strictly decreasing in l, we must have $u_1^* < u_2^*$, which violates the constraint. The same conclusion is true if both (u_1^*, l^*) and $(u_2^*, 0)$ are at the upper bound of the domain. Finally, in any other case u_1^* and u_2^* must satisfy the first-order conditions $1 \geq \zeta C_1(u_1^*, l^*)$ and $1 \leq \zeta C_1(u_2^*, 0)$, respectively, with equality if the allocation is interior. However, since $C_{12} > 0$, we have $C_1(u_2^*, 0) \geq 1/\zeta \geq C_1(u_1^*, l^*) > C_1(u_1^*, 0)$, which once again implies $u_1^* < u_2^*$. We conclude that the constraint $u_1 = u_2$ must be binding and, therefore, $U(\varsigma) < W(\varsigma)$, for all $\varsigma > 0$. In particular, $\bar{\Upsilon} > 0$.

We now prove the preliminary result that $U(\varsigma)$ is strictly increasing for $\Upsilon \in (0, \bar{\Upsilon})$. Problem (A.1) is convex, thus, its solution is characterized by a saddle point Lagrangian:

$$\mathcal{L} = \varsigma u_1 + (1 - \varsigma)u_2 - \zeta[\varsigma p\{C(u_1, l) - l\} + \varsigma(1 - p)C(u_1, 0) + (1 - \varsigma)C(u_2, 0)],$$

where ζ is the multiplier on the resource constraint. For any $\varsigma \in (0, 1)$, the envelope theorem gives

$$U'(\varsigma) = \{u_1^* - \zeta[p\{C(u_1^*, l^*) - l^*\} + (1 - p)C(u_1^*, 0)]\} - \{u_2^* - \zeta C(u_2^*, 0)\}.$$

We want to show that $U'(\varsigma) > 0$. Since the resource constraint must bind, $\zeta > 0$. Let $f(u_1, l) \equiv u_1 - \zeta[p\{C(u_1, l) - l\} + (1 - p)C(u_1, 0)]$. Clearly, $f(u_2, 0) = u_2 - \zeta C(u_2, 0)$. It is immediate to see that f is strictly concave and the pair (u_1^*, l^*) maximizes $f(u_1, l)$. Inada conditions imply $l^* > 0$. By strict concavity of f, we conclude that $f(u_1^*, l^*) > f(u_2^*, 0)$.

Now suppose $\Upsilon < \bar{\Upsilon}$. We must have $\varsigma^*(\Upsilon) < 1$, otherwise, we would obtain the contradiction $\Upsilon \geq W(1) - U(1) = \bar{\Upsilon}$. Since $\varsigma^*(\Upsilon) < 1$, the sustainability constraint must bind at Υ, thus, $U(\varsigma^*(\Upsilon)) = W(\varsigma^*(\Upsilon)) - \Upsilon$. The latter, together with $U(0) = W(0)$, also implies that $\varsigma^*(\Upsilon) > 0$. Since $0 < \varsigma^*(\Upsilon) < 1$ problem (6) in the paper must have two solutions, $\sigma^* = 1$ and $\sigma^* = 0$.

To prove the converse result, it is enough to show that the sustainability constraint does not bind when $\Upsilon \geq \bar{\Upsilon}$. The latter follows immediately from strict monotonicity of U.

Finally, we show that $\varsigma^*(\Upsilon)$ is strictly increasing for all $\Upsilon \leq \bar{\Upsilon}$. Take any pair Υ, $\hat{\Upsilon}$ such that $\Upsilon < \hat{\Upsilon} \leq \bar{\Upsilon}$ and suppose $\varsigma^*(\hat{\Upsilon}) \leq \varsigma^*(\Upsilon)$. Since $\Upsilon < \bar{\Upsilon}$, previous arguments imply $\varsigma^*(\Upsilon) < 1$, thus, the sustainability constraint must be binding at Υ. Then $U(\varsigma^*(\Upsilon)) = W(\varsigma^*(\Upsilon)) - \Upsilon > W(\varsigma^*(\Upsilon)) - \hat{\Upsilon}$. Thus, sustainability constraint is slack at $\varsigma^*(\Upsilon) \geq \varsigma^*(\hat{\Upsilon})$. Since $U(\varsigma)$ is strictly increasing, the latter contradicts optimality of $\varsigma^*(\hat{\Upsilon})$. Therefore, $\varsigma^*(\hat{\Upsilon}) > \varsigma^*(\Upsilon)$ for all $\Upsilon < \hat{\Upsilon} \leq \bar{\Upsilon}$.

2 Proofs for Section 2.1.1

We extend the model of Section 2 to allow for agent heterogeneity. More specfically, agents differ for some payoff-irrelevant publicly-observed characteristic ξ which, wuthout loss of generality, is uniformly distributed on $[0,1]$. In addition, we introduce sunspot variables as in Section 2. In particular, we consider the following three-stage game:

0. The government announces functions $\{c_H(z,\xi), c_L(z,\xi), l(z,\xi)\}$, for all $z \in [0,1]$ and $\xi \in [0,1]$.

1. Each agent learns about his job opportunity and draws a random publicly-observed signal z. Signals z are drawn from a uniform distribution on $[0,1]$, independently for each agent. An agent who has a job opportunity chooses the probability $\sigma(z,\xi)$ with which he reveals this information by reporting H; he reports L otherwise. A jobless agent reports L with probability 1.

2. The government chooses allocations of consumption and labor $\{\tilde{c}_H(z,\xi), \tilde{l}(z,\xi)\}$ and consumption $\{\tilde{c}_L(z,\xi)\}$ for agents who report H and L, respectively. Bundles must be feasible, that is, total consumption cannot exceed total output. The government incurs a utility cost $\Upsilon > 0$ if allocations differ from those announced in stage 0 for any positive measure of agents.

Utility of the agents and the associated function C satisfy the same properties as in the paper. In particular, utility function satisfies Assumption 1 and, as a result, Lemma 1 holds. In addition, we define the set X of incentive-compatible allocations and the set $\mathcal{C}$ of allocations in the domain of C exactly as in the main paper.

Given a collection of measures $\{\psi_\xi\}_\xi$ on X, the highest payoff that a deviating government can achieve is still given by $\tilde{W}(\{\psi_\xi\}_\xi)$:

$$\tilde{W}\left(\{\psi_\xi\}_\xi\right) \equiv \max_{\{\tilde{u}_H(x),\tilde{u}_L(x),\tilde{l}(x)\}_{x\in X}} \int\int \left[p\sigma(x)\,\tilde{u}_H(x) + (1-p\sigma(x))\,\tilde{u}_L(x)\right]\psi_\xi(dx)\,d\xi - \Upsilon, \tag{A.3}$$

subject to

$$\int\int \left[p\sigma(x)\left(C\left(\tilde{u}_H(x), \tilde{l}(x)\right) - \tilde{l}(x)\right) + (1-p\sigma(x))\,C\left(\tilde{u}_L(x),0\right)\right]\psi_\xi(dx)\,d\xi \le 0$$

and $(\tilde{u}_H(\cdot), \tilde{u}_L(\cdot), \tilde{l}(\cdot)) \in L^1(X, \mathcal{C}, \psi)$.

In particular, note that upon deviation the government maximizes a welfare function in which all agents have equal weights.

Finally, to make notation compact, we define functions g and f exactly as in the paper.

A *Perfect Bayesian Equilibrium (PBE)* is a collection of measures $\{\psi_\xi\}_\xi$ on X such that allocations are feasible, $\int\int f d\psi_\xi d\xi \le 0$, and sustainable, $\int g d\psi_\xi d\xi \ge \tilde{W}\left(\{\psi_\xi\}_\xi\right)$. A collection of measures $\{\psi^*_\xi\}_\xi$ on X is a *best PBE* if it is a PBE and there is no other PBE that gives a weakly higher utility to all agents and a strictly higher utility to a positive mass of agents.

Any best PBE is a PBE that maximzes a weighted average of agents' expected utility, i.e. that is a solution to

$$\max_{\{\psi_\xi\}_\xi} \int \hat{\alpha}(\xi) \int g d\psi_\xi d\xi, \tag{A.4}$$

subject to

$$\begin{aligned} \int\int f d\psi_\xi d\xi &\leq 0, \\ \int\int g d\psi_\xi d\xi &\geq \tilde{W}\left(\{\psi_\xi\}_\xi\right), \end{aligned}$$

for some sequence of Pareto weights $\{\hat{\alpha}(\xi)\}_\xi$. Without loss of generality, we focus on non-decreasing sequences of Pareto weights.

The analysis follows the same steps as in Section 2 of the paper. First, Lemma 2 and Corollary 1 in Section 2 extend almost immediately to the case with heterogenous agents. Thus, we can replace $\tilde{W}\left(\{\psi_\xi\}_\xi\right)$ with $\int\int W(x) d\psi_\xi d\xi$, where $W(x)$ is defined exactly as in Lemma 2. Second, the maximization problem (A.4) can be rewritten in the Lagrangian form as

$$\max_{\{\psi_\xi\}_\xi} \int \left[\hat{\alpha}(\xi) g - \zeta f - \chi W\right] d\psi_\xi d\xi, \tag{A.5}$$

for some multipliers $\zeta > 0$ and $\chi \geq 0$. Finally, linearity of (A.5) allows us to solve the overall problem in two steps. First, given a Pareto weight $\alpha = \hat{\alpha}(\xi)$ and a reporting probability σ, we find the allocations $(u_{H,\alpha,\sigma}, u_{L,\alpha,\sigma}, l_{\alpha,\sigma})$ that solve

$$\kappa\left(\alpha,\sigma\right) \equiv \max_{u_H,u_L,l} \alpha(p\sigma u_H + (1-p\sigma)\,u_L) - \zeta\left[p\sigma\left(C\left(u_H,l\right)-l\right) + (1-p\sigma)\,C\left(u_L,0\right)\right], \tag{A.6}$$

subject to $u_H \geq u_L$, $(1-\sigma)\left[u_H - u_L\right] = 0$ and $(u_H, u_L, l) \in \mathcal{C}$. Second, any optimal reporting strategy σ_α^* can be found by solving

$$\max_{\sigma\in[0,1]} \kappa\left(\alpha,\sigma\right) - \chi W\left(\sigma\right). \tag{A.7}$$

The following proposition, which is the analogue of Proposition 1 in the paper, characterizes value functions and optimal reporting strategies.

Proposition A.1 *For all $\alpha > 0$, $\kappa(\alpha, \cdot)$ is strictly increasing, convex and not linear. W is strictly increasing and linear. Therefore, the optimal σ_α^* satisfies $\sigma_\alpha^* \in \{0,1\}$. When both $\sigma_\alpha^* = 0$ and $\sigma_\alpha^* = 1$ are optimal, corresponding allocations are unique and satisfy $u_{L,\alpha,0} > u_{L,\alpha,1} = u_{H,\alpha,1}$.*

Proof. The only difference between this proposition and its analogue in the paper is that here the Pareto weight α can be any strictly positive number, whereas in the paper it was restricted to be 1. However, none of the arguments in the proof of Proposition 1 relies on this restriction, therefore, the same arguments apply here. ■

Lemma A.1 *The solution to problem (A.7) takes one of the following forms:*
(i) there is a threshold $\bar{\alpha}$ (which might be ∞) such that $\sigma^*_\alpha = 1$ for $\alpha < \bar{\alpha}$, $\sigma^*_\alpha = \{0,1\}$ for $\alpha = \bar{\alpha}$, and $\sigma^*_\alpha = 0$ for $\alpha > \bar{\alpha}$;
(ii) there is a threshold $\bar{\alpha}$ such that $\sigma^*_\alpha = \{0,1\}$ for $\alpha \leq \bar{\alpha}$, and $\sigma^*_\alpha = 0$ for $\alpha > \bar{\alpha}$.
Furthermore, if utility is unbounded below, only case (i) is possible.

Proof. Let $h(\alpha) \equiv (\kappa(\alpha,1) - \chi W(1)) - (\kappa(\alpha,0) - \chi W(0))$. We show that h is non-increasing and strictly decreasing for sufficiently high α. h is differentiable and, by the envelope theorem, its derivative is $h'(\alpha) = u_{\alpha,1} - u_{\alpha,0}$, where $u_{\alpha,1}$ and $u_{\alpha,0}$ are the solution to the maximization problem (A.6) for $\sigma = 1$ and $\sigma = 0$, respectively. We prove that $u_{\alpha,1} - u_{\alpha,0} \leq 0$ for all α, with a strict inequality if $(u_{\alpha,1}, 0)$ is interior.

First, observe that we cannot have $h(\alpha) < 0$ for all α, otherwise $\sigma^*_\alpha = 0$ would be the unique solution for all α and the sustainability constraint would be slack, leading to a contradiction. Therefore, we must have $h(\alpha) \geq 0$ for some α. Second, using the same arguments in the proof of Proposition 1 in the paper, we can exclude both *(i)* the case in which $(u_{\alpha,1}, 0)$ is at the upper bound of $dom(C)$ for some α, and *(ii)* the case in which $(u_{\alpha,1}, 0)$ is at the lower bound of $dom(C)$ for all α. Thus, there exists some $\hat{\alpha}$ such that $(u_{\hat{\alpha},1}, 0)$ is interior.

Now, suppose that $(u_{\alpha,1}, 0)$ is at the lower bound of its domain, then obviously $u_{\alpha,1} \leq u_{\alpha,0}$. If, instead, $(u_{\alpha,1}, 0)$ is interior, then it must satisfy the first-order condition

$$\alpha - \zeta\{pC_1(u_{\alpha,1}, l_{\alpha,1}) + (1-p)C_1(u_{\alpha,1}, 0)\} = 0. \tag{A.8}$$

If $u_{\alpha,1} \geq u_{\alpha,0}$, then we would have $C_1(u_{\alpha,0}, 0) \leq C_1(u_{\alpha,1}, 0)$ and $C_1(u_{\alpha,0}, 0) < C_1(u_{\alpha,1}, l_{\alpha,1})$, which, together with the fact that $u_{\alpha,0}$ must satisfy $\alpha \leq \zeta C_1(u_{\alpha,0}, 0)$, contradict equation (A.8). Thus, $u_{\alpha,1} < u_{\alpha,0}$.

We now prove that, if $(u_{\alpha,1}, 0)$ is interior, then so is $(u_{\tilde{\alpha},1}, 0)$ for all $\tilde{\alpha} > \alpha$. To see this, suppose on the contrary that $(u_{\tilde{\alpha},1}, 0)$ is at the lower bound of $dom(C)$. Then, $u_{\tilde{\alpha},1} < u_{\alpha,1}$ and $C_1(u_{\tilde{\alpha},1}, 0) < C_1(u_{\alpha,1}, 0)$. Also, $l_{\alpha,1}$ satisfies the first-order condition $C_2(u_{\alpha,1}, l_{\alpha,1}) = 1$ and, since $C_{11}C_{22} - C_{12}^2 > 0$ from the proof of Lemma 1 in the paper, we have $C_1(u_{\tilde{\alpha},1}, l_{\tilde{\alpha},1}) \leq C_1(u_{\alpha,1}, l_{\alpha,1})$. Since $(u_{\alpha,1}, 0)$ is interior, it must satisfy equation (A.8). The latter conditions therefore imply

$$\tilde{\alpha} - \zeta\{pC_1(u_{\tilde{\alpha},1}, l_{\tilde{\alpha},1}) + (1-p)C_1(u_{\tilde{\alpha},1}, 0)\} > 0,$$

which contradicts optimality of $u_{\tilde{\alpha},1}$. Thus, $(u_{\tilde{\alpha},1}, 0)$ must be interior.

The arguments above prove that $h(\alpha) \geq 0$ for some α, h is non-increasing and becomes strictly decreasing for sufficiently high α. In addition, h may fail to be strictly decreasing only at the points $(u_{\alpha,1}, 0)$ at the lower bound of $dom(C)$. The statement of the lemma then follows directly from these properties of h. ■

We are now ready to present the main result of this section. The next proposition shows that, for any sequence of Pareto weights, there is always a PBE such that the governmet conditions allocations on agent characteristics alone, without using sunspots. What is more, allocations in such PBE are very simple: agents with characteristic ξ below a certain threshold reveal full information while all other agents reveal no information.

Proposition A.2 *For any (non-decreasing) sequence of Pareto weights $\{\hat{\alpha}(\xi)\}_\xi$, there is a solution to (A.4) and a threshold $\bar{\xi} \in [0,1]$ such that all agents with $\xi \leq \bar{\xi}$ report truthfully while agents with $\xi > \bar{\xi}$ reveal no information.*

Proof. Take any non-decreasing sequence of Pareto weights $\{\hat{\alpha}(\xi)\}_\xi$ and an associated solution $\{\psi^*_\xi\}_\xi$ to (A.4). Consider the case in which utility is unbounded below. From Lemma A.1, there is at most one Pareto weight $\bar{\alpha}$ such that the government is indifferent between $\sigma = 0$ and $\sigma = 1$. In addition, in the proof of Lemma A.1, we prove that $\kappa(\cdot, 1) - \kappa(\cdot, 0)$ is non-increasing. Thus, the statement of the proposition is immediately verified if the set $\Xi \equiv \{\xi \in [0,1] : \hat{\alpha}(\xi) = \bar{\alpha}\}$ has probability zero. Suppose instead that Ξ has a positive probability. Since Pareto weights are non-decreasing, Ξ must be an interval, i.e. $\Xi = [\xi_1, \xi_2]$, for some $\xi_1 < \xi_2$. In addition, all agents with $\xi \in \Xi$ receive either $(u_{\bar{\alpha},1}, u_{\bar{\alpha},1}, l_{\bar{\alpha},1})$ or $(u_{\bar{\alpha},0}, u_{\bar{\alpha},0}, 0)$, possibly as a function of sunspots. Let Ψ be the total mass of agents with $\xi \in \Xi$ receiving the former allocation.

Let $\bar{x}_1 \equiv (u_{\bar{\alpha},1}, u_{\bar{\alpha},1}, l_{\bar{\alpha},1}, 1)$ and $\bar{x}_2 \equiv (u_{\bar{\alpha},0}, u_{\bar{\alpha},0}, 0, 0)$ and consider the alternative collection of measures $\{\psi^{**}_\xi\}_\xi$ such that $\psi^{**}_\xi = \psi^*_\xi$, for all $\xi \notin \Xi$, $\psi^{**}_\xi(\bar{x}_1) = 1$, for all $\xi \in [\xi_1, \xi_1 + \Psi]$, and $\psi^{**}_\xi(x_2) = 1$ for all $\xi \in (\xi_1 + \Psi, \xi_2]$. By construction, $\{\psi^{**}_\xi\}_\xi$ leave total consumption and total labor unchanged, thus, the resource constraint is satisfied. In addition, the mass of agents reporting with $\sigma = 1$ is also unchanged, thus, the value of deviation for the government is also unchanged. Finally, since $\{\psi^{**}_\xi\}_\xi$ and $\{\psi^*_\xi\}_\xi$ differ only for values of ξ such that $\xi \in \Xi$ and, at these values, the planner is indifferent between the bundles $\bar{x}_1$ and $\bar{x}_2$, they must yield the same value of (A.4).

If utility is bounded below, from Lemma A.1 we need to consider two cases. For case *(i)*, the arguments above apply without any change. For case *(ii)*, we let $\Xi \equiv \{\xi \in [0,1] : \hat{\alpha}(\xi) \leq \bar{\alpha}\}$ (which must be an interval, i.e. $\Xi = [0, \hat{\xi}]$, for some $\hat{\xi} > 0$) and follow the same arguments as above. ■

Decentralization. It is easy to decentralize the equilibrium described in Proposition A.2. Optimal allocations can be achieved by a two-tier insurance system, where one tier provides relatively poor benefits, while the other tier has more generous benefits but access to them is limited. Finally, employed agents pay lump-sum taxes.

More specifically, all agents qualify for the "unemployment benefit" b^{UI}_ξ. However, only agents whose characteristic is above a threshold $\bar{\xi}$ qualify for the more generous "disability benefit" b^{DI}_ξ. Employed agents forgo their benefits and pay a lump-sum tax τ_ξ. Note that the agent's characteristic determines not only access to the more generous insurance tier, but also the actual value of benefits and taxes.

Formally, an agent with characteristic ξ who receives a job opportunity solves

$$\max_{c,l} U(c, l), \tag{A.9}$$

subject to the budget constraint

$$c \leq \mathcal{I}(l = 0) b^{UI}_\xi + \mathcal{I}(l = 0)\mathcal{I}(\xi > \bar{\xi})(b^{DI}_\xi - b^{UI}_\xi) + wl - \mathcal{I}(l > 0)\tau_\xi,$$

where $\mathcal{I}(\cdot)$ is the indicator function. We use c_ξ and l_ξ to denote optimal choices. An agent without a job opportunity receives utility $U(b^{UI}_\xi + \mathcal{I}(\xi > \bar{\xi})(b^{DI}_\xi - b^{UI}_\xi), 0)$.

The production side of the economy consists of a competitive firm which hires labor to produce the consumption good according to the technology $y = l$. We normalize the price of the final good to 1.

Finally, taxes and benefits must satisfy the government's budget constraint. Since only agents with $\xi \leq \bar{\xi}$ have an incentice to work and, thus, pay taxes, we have:

$$\int_0^1 \mathcal{I}(l_\xi > 0)\tau_\xi d\xi = \int_0^{\bar{\xi}} \mathcal{I}(l_\xi = 0) b_\xi^{UI} d\xi + \int_{\bar{\xi}}^1 \mathcal{I}(l_\xi = 0) b_\xi^{DI} d\xi. \tag{A.10}$$

Definition A.1 (competitive equilibrium) *A competitive equilibrium is a collection of agent decisions, $\{(c_\xi, l_\xi)\}_\xi$, a wage, w, and a welfare system, $\{(\tau_\xi, b_\xi^{UI}, b_\xi^{DI})\}_\xi$, such that agent decisions solve (A.9), labor market clears, and the government budget constraint (A.10) holds.*

Proposition A.3 (decentralization) *Any best PBE described in Proposition A.2 can be decentralized through a competitive equilibrium.*

Proof. First, observe that constant returns to scale in production imply $w = 1$. Now, take any best PBE described in Proposition A.2 and associated $(u^*_{H,\alpha,1}, u^*_{L,\alpha,1}, u^*_{L,\alpha,0}, l^*_{\alpha,1})$, where $\alpha = \hat{\alpha}(\xi)$ for all $\xi \in [0,1]$. Let $b_\xi^{DI} \equiv C(u^*_{L,\xi,0}, 0)$, $b^{UI} \equiv C(u^*_{L,\xi,1}, 0)$, and $\tau \equiv l^*_{\xi,1} - C(u^*_{H,\xi,1}, l^*_{\xi,1})$. Note that $b_\xi^{DI} > b_\xi^{UI}$ since $u^*_{L,\xi,0} > u^*_{L,\xi,1} = u^*_{H,\xi,1}$. Using the budget constraint with $l > 0$, optimal choice of labor satisfies the first-order condition

$$U_c(l_\xi - \tau_\xi, l_\xi) + U_l(l_\xi - \tau_\xi, l_\xi) = 0,$$

which, using the definition of C and Lemma 1 in the paper, becomes

$$C_2(u_\xi, l_\xi) = 1,$$

where $u_\xi \equiv U(l_\xi - \tau_\xi, l_\xi)$. Using the definition of τ_ξ, $u_\xi = U(l_\xi - l^*_{\xi,1} + C(u^*_{H,\xi,1}, l^*_{\xi,1}), l_\xi)$. Since $C_2(u^*_{H,\xi,1}, l^*_{\xi,1}) = 1$ from Proposition A.1, the latter implies $l_\xi = l^*_{\xi,1}$ and $u_\xi = u^*_{H,\xi,1}$, for all $\xi \leq \bar{\xi}$. If, instead, the agent does not work, he receives benefits b_ξ^{UI} which by construction deliver utility $u^*_{L,\xi,1}$. Since $u^*_{H,\xi,1} = u^*_{L,\xi,1}$, the agent will work if given the opportunity.

Consider now the problem of an agent with $\xi > \bar{\xi}$ and a job opportunity. If such agent works, he will again receive $u^*_{H,\xi,1}$. Instead, if he does not work, he receives the more generous benefits b_ξ^{DI}, which by construction deliver utility $u^*_{L,\xi,0}$. Since $u^*_{H,\xi,1} < u^*_{L,\xi,0}$, an agent with $\xi > \bar{\xi}$ will never work.

Finally, we need to make sure that the government budget constraint (A.10) is satisfied. This follows immediately by noting that, with the values of τ_ξ, b_ξ^{UI}, and b_ξ^{DI} we have just constructed, equation (A.10) becomes identical to the economy's resource constraint, which must be satisfied in any PBE. ■

3 Proofs for Section 2.1.2

We assume that $\Upsilon(\cdot)$ is non-negative, convex and differentiable with $\Upsilon(0) = 0$. We prove that the insights of Proposition 1 in the main text continue to hold under this more general commitment cost. We first extend Lemma 2. For any given PBE ψ^*, there exists a scalar $\lambda^* \geq 0$ that defines a function $W(x)$ given by

$$W(x) \equiv \max_{(\tilde{u}_H(\cdot), \tilde{u}_L(\cdot), \tilde{l}(\cdot)) \in L^1(X, \mathcal{C}, \psi)} \begin{bmatrix} p\sigma(x)\{\tilde{u}_H(x) - \Upsilon(u_H(x) - \tilde{u}_H(x))\} \\ +(1 - p\sigma(x))\{\tilde{u}_L(x) - \Upsilon(u_L(x) - \tilde{u}_L(x))\} \end{bmatrix}$$
$$-\lambda^* \left[p\sigma(x)\left(C\left(\tilde{u}_H(x), \tilde{l}(x)\right) - \tilde{l}(x)\right) + (1 - p\sigma(x)) C(\tilde{u}_L(x), 0)\right].$$

The arguments in Lemma 2 in the main text prove that $\tilde{W}(\psi) \leq \int W d\psi$, with equality at $\psi = \psi^*$. Function $W(x)$ is linear in $\sigma(x)$ and can thus be rewritten as

$$W(x) = p\sigma(x) F_H(u_H(x)) + (1 - p\sigma(x)) F_L(u_L(x)),$$

where

$$\begin{aligned} F_H(u) &\equiv \max_{(\tilde{u}_H, \tilde{l}) \in dom(C)} \left\{ \tilde{u}_H - \Upsilon(u - \tilde{u}_H) - \lambda^* \left(C\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l} \right) \right\}, \\ F_L(u) &\equiv \max_{(\tilde{u}_L, 0) \in dom(C)} \left\{ \tilde{u}_L - \Upsilon(u - \tilde{u}_L) - \lambda^* C(\tilde{u}_L, 0) \right\}. \end{aligned}$$

Standard arguments prove that F_H, F_L are differentiable. Optimal allocations will then be a solution to

$$\begin{aligned} V(\sigma) \equiv \max_{u_H, u_L, l} \quad & p\sigma [u_H - \zeta (C(u_H, l) - l) - \chi F_H(u_H)] \\ & + (1 - p\sigma) [u_L - \zeta C(u_L, 0) - \chi F_L(u_L)], \end{aligned}$$

subject to $u_H \geq u_L$, $(1-\sigma)(u_H - u_L) = 0$ and $(u_H, u_L, l) \in \mathcal{C}$. Notice that $V(\sigma)$ summarizes both the benefits – captured by $\kappa(\sigma)$ in the benchmark case – and the costs – captured by $\chi W(\sigma)$ in the benchmark case – of information revelation.

Proposition A.4 (general commitment costs) *Suppose $\Upsilon(\cdot)$ is non-negative, convex and differentiable with $\Upsilon(0) = 0$. If the incentive constraint is binding at the optimum, then V is convex and not linear. Therefore, the optimal σ^* satisfies $\sigma^* \in \{0, 1\}$. When both $\sigma^* = 0$ and $\sigma^* = 1$ are optimal, corresponding allocations are unique and satisfy $u_{L,0} > u_{L,1} = u_{H,1}$.*

Proof. We focus on the case in which the sustainability constraint binds, i.e. $\chi > 0$, otherwise the proof is identical to the one of Proposition 1 in the main text.

Convexity of V follows from the same arguments in Proposition 1. We prove that V is not linear. We first consider the case in which $u_{H,\sigma} = u_{L,\sigma} \equiv u_\sigma$ is such that $(u_\sigma, 0)$ is in the interior of $dom(C)$. Let μ_σ be the Lagrange multiplier on the incentive constraint, the optimal allocation $(u_{H,\sigma}, u_{L,\sigma}, l_\sigma)$ must satisfy the first-order conditions (Luenberger (1969), Theorem 1, p. 249)

$$\begin{aligned} C_2(u_{H,\sigma}, l_\sigma) - 1 &= 0, \\ 1 - \zeta C_1(u_{H,\sigma}, l_\sigma) - \chi F_{H,1}(u_{H,\sigma}) + \mu_\sigma &= 0, \\ 1 - \zeta C_1(u_{L,\sigma}, 0) - \chi F_{L,1}(u_{L,\sigma}) - \mu_\sigma &= 0. \end{aligned} \tag{A.11}$$

Since the incentive constraint binds at the optimum, $\mu_{\tilde{\sigma}} \neq 0$ for some $\tilde{\sigma}$. Suppose V is linear. Let $(u_{H,1}, u_{L,1}, l_1)$ be a maximizer of $V(1)$. Suppose $(u_{H,1}, u_{L,1}, l_1)$ is not a maximizer of $V(\hat{\sigma})$ for some $\hat{\sigma} < 1$, then $V(\hat{\sigma}) > V(1) + (\hat{\sigma} - 1) V'(1)$, which contradicts linearity. Thus, $(u_{H,1}, u_{L,1}, l_1)$ must be a maximizer of $V(\sigma)$ for all σ, which is true only if $u_{H,1} = u_{L,1} \equiv u_1$. Thus, (u_1, l_1) must satisfy the first-order condition

$$p\sigma [1 - \zeta C_1(u_1, l_1) - \chi F_{H,1}(u_1)] + (1 - p\sigma) [1 - \zeta C_1(u_1, 0) - \chi F_{L,1}(u_1)] = 0.$$

Since the latter must hold for all σ, it implies

$$\begin{aligned} 1 - \zeta C_1(u_1, l_1) - \chi F_{H,1}(u_1) &= 0, \\ 1 - \zeta C_1(u_1, 0) - \chi F_{L,1}(u_1) &= 0, \end{aligned}$$

which contradicts $\mu_{\tilde{\sigma}} \neq 0$. Therefore, V cannot be linear.

We now consider the case in which $u_{H,\sigma} = u_{L,\sigma} \equiv u_\sigma$ is such that $(u_\sigma, 0)$ is at the boundary of *dom* (C). The only relevant case is the lower bound which, without loss of generality, we suppose to be equal to 0. If $u_{\hat{\sigma}} > 0$ for at least one optimal $\hat{\sigma}$, then the arguments above can be applied in a neighborhood of $\hat{\sigma}$. We now rule out the case with $u_\sigma = 0$ for all optimal σ. Suppose that $u_\sigma = 0$ for all optimal σ. The arguments in the proof of Proposition 1 in the main text show that the latter can be true only if $\zeta = 0$. Let $(\tilde{u}_H^*, \tilde{l}^*)$ and $\tilde{u}_L^*$ be the maximizers of $F_H(0)$ and $F_L(0)$, respectively. They satisfy the first-order conditions

$$\begin{aligned} 1 - \lambda^* C_1 \left(\tilde{u}_H^*, \tilde{l}^*\right) + \Upsilon_1 \left(-\tilde{u}_H^*\right) &\leq 0, \\ 1 - \lambda^* C_1 \left(\tilde{u}_L^*, 0\right) + \Upsilon_1 \left(-\tilde{u}_L^*\right) &\leq 0. \end{aligned}$$

We cannot have $\tilde{u}_H^* = \tilde{u}_L^* = 0$, otherwise $\lambda^* = 0$ and the government could achieve a higher payoff by choosing $u'_\sigma = \tilde{u}_H'^* = \tilde{u}_L'^* = \varepsilon$ for all σ and $\varepsilon > 0$. For ε small enough, this alternative allocation would be feasible and satisfy the sustainability constraint. Therefore, either $\tilde{u}_H^* > 0$ or $u_L^* > 0$ or both. Convexity of Υ, $C_{12} > 0$ and $\tilde{l}^* > 0$ imply $\tilde{u}_L^* > \tilde{u}_H^* \geq 0$ and, hence, the second first-order condition must hold with equality. Also, since the sustainability constraint holds with equality, we have $p(\tilde{u}_H^* - \Upsilon(-\tilde{u}_H^*)) + (1-p)(\tilde{u}_L^* - \Upsilon(-\tilde{u}_L^*)) = 0$. The properties of Υ, together with $u_L^* > 0$, imply $\tilde{u}_L^* \leq \Upsilon(-\tilde{u}_L^*)$. Using convexity, differentiability and $\Upsilon(0) = 0$, the latter in turn yields $1 \leq -\Upsilon_1(-\tilde{u}_L^*)$, which contradicts the first-order condition for $\tilde{u}_L^*$. Therefore, we cannot have $u_\sigma = 0$ for all optimal σ.

Finally, suppose that both $\sigma^* = 0$ and $\sigma^* = 1$, are optimal, we prove that $u_{L,0} > u_{H,1} = u_{L,1}$. Suppose instead that $u_{H,1} = u_{L,1} \geq u_{L,0}$. Then the alternative allocation $\hat{u}_{H,1} = u_{H,1}$, $\hat{u}_{L,1} = u_{L,0}$, $\hat{l}_1 = l_1$, satisfies the incentive constraint and delivers a payoff

$$\begin{aligned} & p\left[u_{H,1} - \zeta\left(C\left(u_{H,1}, l_1\right) - l_1\right) - \chi F_H\left(u_{H,1}\right)\right] + (1-p)\left[u_{L,0} - \zeta C\left(u_{L,0}, 0\right) - \chi F_L\left(u_{L,0}\right)\right] \\ > \; & p\left[u_{H,1} - \zeta\left(C\left(u_{H,1}, l_1\right) - l_1\right) - \chi F_H\left(u_{H,1}\right)\right] + (1-p)\left[u_{L,1} - \zeta C\left(u_{L,1}, 0\right) - \chi F_L\left(u_{L,1}\right)\right] = V(1), \end{aligned}$$

where the inequality comes from the fact that $u_{L,0}$ maximizes $V(0)$ and that $\mu_1 > 0$. The latter contradicts the fact that $(u_{H,1}, u_{L,1}, l_1)$ is a maximizer of $V(1)$. ■

4 Proofs for Section 2.1.3

We now extend the model of Section 2 to allow for moral hazard.

We consider the following three-stage game.

0. The government announces functions $\{c_H(z), c_L(z), l(z)\}$, for all $z \in [0,1]$, of consumption if the agent finds a job, consumption if the agent does not find a job, and labor.

1. Each agent draws a random publicly-observed signal z. Signals z are drawn from a uniform distribution on $[0,1]$, independently for each agent. An agent exerts effort with probability $\sigma(z)$. If the agent finds a job, he reveals this information by reporting H; he reports L otherwise. A jobless agent can only report L.

2. The government chooses allocations of consumption and labor $\{\tilde{c}_H(z), \tilde{c}_L(z), \tilde{l}(z)\}$ for agents who find a job or not, respectively. Bundles must be feasible, that is, total consumption cannot exceed total output. The government incurs a utility cost $\Upsilon > 0$, if allocations differ from those announced in stage 0, for any positive measure of agents.

Notice that it is never optimal for an agent who finds a job to report L. If such agent reported L, he would obtain the same consumption as an agent who did not exert effort but would pay the effort cost. It is thus without loss of generality not to let agents randomize over reports in stage 1.

Let $U(C^e(u,l),l) - e \equiv u$ and $U(C^{ne}(u),0) \equiv u$. The function C^e satisfies all the properties of Lemma 1.

Best PBEs. Let u_H be the utility of an agent who finds a job and u_L the utility of an agent who exerts effort but does not find a job. An agent who does not exert effort saves on the disutility cost and, thus, receives $u_L + e$. Let X be the space of $x \equiv (u_H, u_L, l, \sigma)$ such that (i) (u_H, l) and $(u_L, 0)$ lie in the domain of C, (ii) $\sigma \in [0,1]$, and (iii) (u_H, u_L, σ) satisfies the incentive constraint

$$\begin{aligned} pu_H + (1-p)\,u_L &\geq u_L + e, \\ (1-\sigma)\left[pu_H + (1-p)\,u_L - (u_L + e)\right] &= 0. \end{aligned}$$

Notice that X might be empty when the effort cost e is high enough. We thus make the following assumption.

Assumption A.1 *For all σ there is a subset of (u_L, l) with a non-empty interior such that $(u_H, u_L, l, \sigma) \in X$ for some u_H.*

Let ψ a distribution over X and let

$$g(x) \equiv \sigma p u_H + \sigma(1-p)\,u_L + (1-\sigma)(u_L + e)$$

and

$$f(x) \equiv \sigma p\left\{C^e(u_H, l) - l\right\} + \sigma(1-p)\,C^e(u_L, 0) + (1-\sigma)\,C^{ne}(u_L + e).$$

If the government decides to break its promises in stage 2, the best payoff $\tilde{W}(\psi)$ it can achieve is given by

$$\tilde{W}(\psi) \equiv \max_{\tilde{u}_H, \tilde{u}_L, \tilde{l}} \int \left[\sigma(x)\,p\tilde{u}_H + \sigma(x)(1-p)\,\tilde{u}_L + (1-\sigma(x))(\tilde{u}_L + e)\right] d\psi - \Upsilon,$$

subject to $(\tilde{u}_H, \tilde{l}), (\tilde{u}_L, 0) \in dom(C^e)$ and

$$\int \left[\sigma(x)\,p\left(C^e\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l}\right) + \sigma(x)(1-p)\,C^e(\tilde{u}_L, 0) + (1-\sigma(x))\,C^{ne}(\tilde{u}_L + e)\right] d\psi \leq 0.$$

A best PBE is then a solution to

$$\max_{\psi: \int f d\psi \leq 0, \int g d\psi \geq \tilde{W}(\psi)} \int g d\psi. \tag{A.12}$$

Modified problem. As in Section 2 we can replace the sustainability constraint in problem (A.12) with a simpler constraint. Formally, given some best PBE ψ^*, we use Lagrangian methods (Luenberger (1969), Theorem 1, p. 224) to show that there exists a scalar $\lambda^* \geq 0$ and a function

$$\begin{aligned} W(x) \equiv \; & \max_{\tilde{u}_H,\tilde{u}_L,\tilde{l}} \sigma(x) p\tilde{u}_H + \sigma(x)(1-p)\tilde{u}_L + (1-\sigma(x))(\tilde{u}_L + e) \\ & -\lambda^* \left[\sigma(x) p \left\{C^e\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l}\right\} + \sigma(x)(1-p) C^e(\tilde{u}_L, 0) + (1-\sigma(x)) C^{ne}(\tilde{u}_L + e)\right] - \Upsilon, \end{aligned}$$

subject to $(\tilde{u}_H, \tilde{l}), (\tilde{u}_L, 0) \in dom(C^e)$, such that

$$\tilde{W}(\psi) \leq \int W d\psi,$$

for all ψ, with equality at $\psi = \psi^*$. The proof is identical to that of Lemma 2 in the main text. Also, it is immediate to verify that $W(x)$ depends only on $\sigma(x)$ and is independent of $(u_H(x), u_L(x), l(x))$.

We can then define the modified problem

$$\max_{\psi: \int f d\psi \leq 0, \int (g-W) d\psi \geq 0} \int g d\psi, \tag{A.13}$$

such that any PBE ψ^* is a solution to (A.13). Conversely, any ψ^{**} that solves (A.13) is a best PBE.

Characterization. Problem (A.13) is convex, thus, we can set up the Lagrangian

$$\mathcal{L} = \max_{\psi} \int [g - \zeta f - \chi W] d\psi,$$

for some multipliers $\zeta > 0$ and $\chi \geq 0$. By linearity, ψ^* must assign positive measure only to those x^* that maximize the term inside the integral. Such x^* can be found using a two-step procedure. First, we can find the optimal optimal allocations $(u_{H,\sigma}, u_{L,\sigma}, l_\sigma)$ for any given reporting strategy σ. These allocations are solutions to

$$\begin{aligned} \kappa(\sigma) = \max_{u_H,u_L,l} \quad & \sigma p u_H + \sigma(1-p) u_L + (1-\sigma)(u_L + e) \\ & -\zeta \left[\sigma p \{C^e(u_H, l) - l\} + \sigma(1-p) C^e(u_L, 0) + (1-\sigma) C^{ne}(u_L + e)\right], \end{aligned}$$

subject to $(u_H, l), (u_L, 0) \in dom(C^e)$,

$$\begin{aligned} p u_H + (1-p) u_L &\geq u_L + e, \\ (1-\sigma)\left[p u_H + (1-p) u_L - (u_L + e)\right] &= 0. \end{aligned}$$

Second, any optimal reporting strategy σ^* is a solution to

$$\max_{\sigma} \kappa(\sigma) - \chi W(\sigma),$$

where, with a slight abuse of notation, we wrote $W(\sigma)$ instead of $W(x)$.

It is immediate to see that all the steps in the proof of Proposition 1 in the main text extend to the setting with moral hazard. Therefore, also in this setting we have the following proposition.

Proposition A.5 (moral hazard) *Suppose that Assumption A.1 is satisfied. Then,* κ *is strictly increasing, convex and not linear.* W *is strictly increasing and linear. Therefore, the optimal* σ^* *satisfies* $\sigma^* \in \{0,1\}$*. When both* $\sigma^* = 0$ *and* $\sigma^* = 1$ *are optimal, corresponding allocations are unique and satisfy* $u_{L,0} > u_{L,1} = u_{H,1}$.

Proof. From their definitions, it follows immediately that $C^{ne}(u+e) = C^{e}(u,0)$ for all u. We first prove that the incentive constraint holds with equality. Suppose it does not, which can happen only when $\sigma = 1$. Then we can rewrite the problem as

$$\kappa(1) = \max_{u_H, u_L, l} \; pu_H + (1-p)\,u_L - \zeta\left[p\left\{C^e(u_H, l) - l\right\} + (1-p)\,C^e(u_L, 0)\right],$$

subject to $(u_H, l), (u_L, 0) \in dom(C^e)$. The first-order conditions give $u_H^* < u_L^*$, which violate the IC constraint. As a result, the incentive constraint binds and

$$u_H = u_L + \frac{1}{p}e.$$

The problem is then

$$\kappa(\sigma) = \max_{u_L, l} \; u_L + e - \zeta\left[\sigma p\left\{C^e\left(\frac{1}{p}e + u_L, l\right) - l\right\} + (1-\sigma p)\,C^e(u_L, 0)\right],$$

$(\frac{1}{p}e + u_L, l), (u_L, 0) \in dom(C^e)$. Therefore, the same arguments in Proposition 1 go through. In particular, $\kappa(\sigma)$ is convex and not linear, hence, since $W(\sigma)$ is linear, $\sigma^* \in \{0,1\}$. ■

5 Proofs for Section 2.1.4

We begin with the definition of the optimization problems of agents and firms. An agent with characteristic ξ who receives a job opportunity solves

$$\max_{c,l} U(c,l), \tag{A.14}$$

subject to the budget constraint

$$c \le \mathcal{I}(l=0)b^{UI} + \mathcal{I}(l=0)\mathcal{I}(\xi > \bar{\xi})(b^{DI} - b^{UI}) + wl - \mathcal{I}(l>0)\tau.$$

We use c_ξ and l_ξ to denote optimal choices. An agent without a job opportunity receives utility $U(b^{UI} + \mathcal{I}(\xi > \bar{\xi})(b^{DI} - b^{UI}), 0)$.

The production side of the economy consists of a representative firm which takes prices as given and hires labor to produce the consumption good with technology $y = l$. We normalize the price of the final good to 1.

Finally, taxes and benefits must satisfy the government's budget constraint. Since only agents with $\xi \le \bar{\xi}$ have an incentice to work and, thus, pay taxes, we have:

$$\tau \Pr(l_\xi > 0) = b^{UI} \Pr(l_\xi = 0 | \xi \le \bar{\xi}) \Pr(\xi \le \bar{\xi}) + b^{DI} \Pr(l_\xi = 0 | \xi > \bar{\xi}) \Pr(\xi > \bar{\xi}). \tag{A.15}$$

Definition A.2 (competitive equilibrium) *A competitive equilibrium is a collection of agent decisions,* (c_ξ, l_ξ)*, a wage,* w*, and a welfare system,* (τ, b^{UI}, b^{DI})*, such that agent decisions solve (A.14), labor market clears, and the government budget constraint (A.15) holds.*

Proposition A.6 (decentralization) *The best PBE characterized in Proposition 1 can be decentralized through a competitive equilibrium.*

Proof. First, observe that constant returns to scale in production imply $w = 1$. Now, let $(u^*_{H,1}, u^*_{L,1}, u^*_{L,0}, l^*_1, \varsigma^*)$ be the objects defined in Proposition 1, where ς^* is the fraction of agents with job opportunities who receive allocation $u^*_{H,1}$. Let $b^{DI} \equiv C(u^*_{L,0}, 0)$, $b^{UI} \equiv C(u^*_{L,1}, 0)$, and $\tau \equiv l^*_1 - C(u^*_{H,1}, l^*_1)$. Note that $b^{DI} > b^{UI}$ since $u^*_{L,0} > u^*_{L,1}$. Define the threhsold $\bar{\xi}$ as the solution to $\varsigma^* = p \Pr(\xi \leq \bar{\xi})$. Consider first the problem of an agent with $\xi \leq \bar{\xi}$ and a job opportunity. Using the budget constraint with $l > 0$, optimal choice of labor satisfies the first-order condition

$$U_c(l_\xi - \tau, l_\xi) + U_l(l_\xi - \tau, l_\xi) = 0,$$

which, using the definition of C and Lemma 1 in the paper, becomes

$$C_2(u_\xi, l_\xi) = 1,$$

where $u_\xi \equiv U(l_\xi - \tau, l_\xi)$. Using the definition of τ, $u_\xi = U(l_\xi - l^*_1 + C(u^*_{H,1}, l^*_1), l_\xi)$. Since $C_2(u^*_{H,1}, l^*_1) = 1$ from the proof of Proposition 1, the latter implies $l_\xi = l^*_1$ and $u_\xi = u^*_{H,1}$, for all $\xi \leq \bar{\xi}$. If, instead, the agent does not work, he receives benefits b^{UI} which by construction deliver utility $u^*_{L,1}$. Since $u^*_{H,1} = u^*_{L,1}$, the agent will work if given the opportunity.

Consider now the problem of an agent with $\xi > \bar{\xi}$ and a job opportunity. If such agent works, he will again receive $u^*_{H,1}$. Instead, if he does not work, he receives the more generous benefits b^{DI}, which by construction deliver utility $u^*_{L,0}$. Since $u^*_{H,1} < u^*_{L,0}$, an agent with $\xi > \bar{\xi}$ will never work.

Finally, we need to make sure that the government budget constraint (A.15) is satisfied. This follows immediately by noting that, with the values of τ, b^{UI}, and b^{DI} above, equation (A.15) becomes

$$p \Pr(\xi \leq \bar{\xi})(l^*_1 - C(u^*_{H,1}, l^*_1)) = (1-p) \Pr(\xi \leq \bar{\xi}) C(u^*_{L,1}, 0) + \Pr(\xi > \bar{\xi}) C(u^*_{L,0}, 0)$$

or, using $\varsigma^* = p \Pr(\xi \leq \bar{\xi})$,

$$p\varsigma^*(C(u^*_{H,1}, l^*_1) - l^*_1) + (1-p)\varsigma^* C(u^*_{L,1}, 0) + (1 - \varsigma^*) C(u^*_{L,0}, 0) = 0.$$

The latter coincides with the economy's resource constraint which is satisfied in the equilibrium of Proposition 1. ■

Appendix B: Proofs for Section 3

1 Preliminaries

We lay out the details of the dynamic game of Section 3.

Each period t is divided in two stages. In stage 1, each agent observes the realization of his sunspot variable z_t and his current type $\theta_t \in \{\theta_H, \theta_L\}$, where we use $\theta_t = \theta_H$ to denote an agent who received a job offer in period t and $\theta_t = \theta_L$ to denote an agent who did not receive an offer in period t. He then sends a report $m_t \in \{H, L\}$ to the government. Reports are a function of current and past realizations of the agent's types $\theta^t \equiv (\theta_0, ..., \theta_t)$, current and past realizations of the agent's sunspot variables $z^t \equiv (z_0, ..., z_t)$, agent's past reports $m^{t-1} \equiv (m_0, ..., m_{t-1})$, and the aggregate history G^{t-1}, which we describe below.

Let $\breve{h}^t \equiv \left(m^{t-1}, z^t\right)$ and $h^t \equiv \left(m^t, z^t\right)$ be the histories of agent's reports and realizations of the idiosyncratic sunspot variable, before and after he submits the current period's report m_t, respectively. Let $\breve{H}^t$ and H^t be the spaces of all such histories. A reporting strategy $\boldsymbol{\sigma}_t$ induces a probability distribution over $\{H, L\}$ denoted by $\boldsymbol{\sigma}_t(\cdot|\breve{h}^t, \theta^t)$. To simplify notation we have omitted from $\boldsymbol{\sigma}_t$ the explicit dependence on the aggregate history G^{t-1}. We assume that the law of the large numbers holds and the aggregate distribution of histories h^t, denoted by μ_t, is given by[1]

$$\mu_t\left(h^t\right) = \mu_{t-1}\left(h^{t-1}\right) \Pr\left(z_t\right) \sum_{\theta^t \in \{\theta_H, \theta_L\}^t} \pi_t\left(\theta^t\right) \boldsymbol{\sigma}_t\left(m_t | \left(h^{t-1}, z_t\right), \theta^t\right).$$

The triple H^t, its Borel sigma algebra, and μ_t is a probability space.

In stage 2, the government observes the past aggregate history G^{t-1} and the current distribution μ_t, and chooses allocations. Allocations are measurable functions $\mathbf{u}_t : H^t \to [\underline{U}, \bar{U})$, where $\underline{U}$ and $\bar{U}$ are, respectively, the greatest lower bound and the least upper bound of U, and $\mathbf{l}_t : H^t \to \mathbb{R}_+$ that satisfy $\mathbf{l}_t((\breve{h}^t, \theta_L)) = 0$. Allocations must lie in the domain of function C. To make notation compact, we let $\mathbf{C}$ be the set of sequences $\{\mathbf{u}_t, \mathbf{l}_t\}_t$ such that $(\mathbf{u}_t((\breve{h}^t, \theta_H)), \mathbf{u}_t((\breve{h}^t, \theta_L)), \mathbf{l}_t((\breve{h}^t, \theta_H)) \in \mathcal{C}$ for all $\breve{h}^t$ and t. The feasibility constraint is

$$\mathbb{E}_\mu\left[C\left(\mathbf{u}_t, \mathbf{l}_t\right) - \mathbf{l}_t\right] \le 0, \quad \{\mathbf{u}_t, \mathbf{l}_t\}_t \in \mathbf{C} \text{ a.s.}, \tag{B.1}$$

for all t.[2] Again, to simplify notation we have omitted from $\mathbf{u}_t$ and $\mathbf{l}_t$ the explicit dependence on the aggregate history G^{t-1}.

The aggregate history G^t includes the history of distributions $\{\mu_s\}_{s \le t}$ and the history of allocations chosen by the government, $\{\mathbf{u}_s, \mathbf{l}_s\}_{s \le t}$. Finally, at time t, the full history of the game consists of the aggregate history and of the private history of each agent.

[1] Strictly speaking, since z_t is a continuous variable, μ_t is defined as follows. Let $\mu_{-1} = \pi_{-1}$. Any Borel set A^t of H^t can be represented as a product $A^t = A^{t-1} \times \{0, 1\} \times B_z$, where A^{t-1} is a Borel set of H^{t-1} and B_z is some Borel set of Z. Then μ_t is defined as

$$\mu_t\left(A^t\right) = \mu_{t-1}\left(A^{t-1}\right) \Pr\left(z_t \in B_z\right) \sum_{\theta^t} \pi_t\left(\theta^t\right) \boldsymbol{\sigma}_t\left(m_t | A^{t-1}, B_z, \theta^t\right).$$

[2] Given a distribution μ_t and a measurable function f_t, we use the shorthand notation $\mathbb{E}_\mu\left[f_t\right] = \int f_t d\mu_t$. Similarly, given a reporting strategy $\boldsymbol{\sigma}$ and a sequence of measurable functions $\{f_t\}_t$, we let $\mathbb{E}_{\boldsymbol{\sigma}}\left[\sum_{t=0}^{\infty} \beta^t f_t\right]$ denote the discounted expected sum computed using the distributions $\{\mu_t\}_t$ induced by $\boldsymbol{\sigma}$. The "a.s." requirement is with respect to such measure.

The definition of aggregate history formalizes our assumption that agents are atomistic. Since only the *distribution* of agent histories is observable, any event to which μ_t assigns zero probability will not affect the aggregate history and, therefore, will not be observed. In particular, the reporting strategy of any individual agent does not affect the aggregate history in the game.

Perfect Bayesian equilibrium. A PBE consists of strategies of agents and the government and posterior beliefs such that, at each history of the game, each player chooses his best response given his posterior beliefs formulated using Bayes' rule. A best PBE is a PBE such that there is no other PBE that gives higher utility to a set of agents of measure 1, and strictly higher utility to a positive measure of agents.

2 Best Equilibrium

A convenient property of PBEs is the one-stage deviation principle (see Theorem 4.2, p. 110 in Fudenberg and Tirole (1991)). The next lemma shows that this principle applies to our environment.

Lemma B.1 *The strategy profile $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ is part of a PBE if and only if there is no history Γ^t at which a player i (i.e. either the government or any individual agent) has an alternative strategy, which differs from the equilibrium strategy only at Γ^t and which delivers a higher payoff to player i, conditional on Γ^t being reached.*

Proof. The "only if" part of the lemma follows directly from the definition of PBEs. We prove the "if" part. Also, we focus on the case in which an agent deviates; the case in which the government deviates is analogous. Suppose that *(i)* $\boldsymbol{\sigma}$ satisfies the "one-stage deviation" property, that is, there is no other strategy that differs from $\boldsymbol{\sigma}$ only at one history and delivers a higher payoff to the agent; *(ii)* there is a reporting strategy $\tilde{\boldsymbol{\sigma}}$ and a history $(\breve{h}^t, \theta^t)$ at which $\tilde{\boldsymbol{\sigma}}_t$ is preferred to $\boldsymbol{\sigma}_t$. Formally, if we let $V\left(\tilde{\boldsymbol{\sigma}}|\breve{h}^t, \theta^t\right) \equiv \mathbb{E}_{\tilde{\boldsymbol{\sigma}}}\left[\sum_{s=t}^{\infty} \beta^{s-t}\mathbf{u}_s \middle| \breve{h}^t, \theta^t\right]$, then condition *(ii)* states that

$$V(\tilde{\boldsymbol{\sigma}}|\breve{h}^t, \theta^t) > V(\boldsymbol{\sigma}|\breve{h}^t, \theta^t).$$

Notice that this condition implies that $\boldsymbol{\sigma}$ cannot be part of a PBE. Suppose first that there is a time $T < \infty$ such that $\boldsymbol{\sigma}_\tau = \tilde{\boldsymbol{\sigma}}_\tau$, for any $(\breve{h}^\tau, \theta^\tau)$ with $\tau \geq T+1$. The one-stage deviation property implies that

$$V(\boldsymbol{\sigma}|\breve{h}^T, \theta^T) \geq V(\tilde{\boldsymbol{\sigma}}|\breve{h}^T, \theta^T).$$

Consider an alternative strategy $\hat{\boldsymbol{\sigma}}$ which coincides with $\tilde{\boldsymbol{\sigma}}$ for all histories $(\breve{h}^\tau, \theta^\tau)$ with $t \leq \tau \leq T-1$ and coincides with $\boldsymbol{\sigma}$ elsewhere. The previous inequalities imply

$$V(\hat{\boldsymbol{\sigma}}|\breve{h}^t, \theta^t) > V(\boldsymbol{\sigma}|\breve{h}^t, \theta^t).$$

However, $\hat{\boldsymbol{\sigma}}$ differs from $\boldsymbol{\sigma}$ only for $\tau \leq T-1$. Repeating this process, we eventually find a reporting strategy that differs from $\boldsymbol{\sigma}$ only at time t and yields a higher payoff to the agent, thus, contradicting condition *(i)*.

Suppose now $T = \infty$. Let

$$\epsilon \equiv V(\tilde{\boldsymbol{\sigma}}|\breve{h}^t, \theta^t) - V(\boldsymbol{\sigma}|\breve{h}^t, \theta^t) > 0.$$

Let $\hat{\boldsymbol{\sigma}}$ be the strategy that coincides with $\tilde{\boldsymbol{\sigma}}$ for all histories $(\breve{h}^s, \theta^s)$, $s \leq \tau$, and coincides with $\boldsymbol{\sigma}$ for all other histories. Since utility is bounded, we can choose τ high enough that

$$V(\tilde{\boldsymbol{\sigma}}|\breve{h}^t, \theta^t) - V(\hat{\boldsymbol{\sigma}}|\breve{h}^t, \theta^t) \leq \frac{\epsilon}{2}.$$

Thus, $\hat{\boldsymbol{\sigma}}$ delivers a higher payoff to the agent at history $(\breve{h}^t, \theta^t)$ and differs from $\boldsymbol{\sigma}$ only for finitely many periods. We can then follow the same steps as in the case with $T < \infty$ to reach a contradiction. ■

Following standard arguments, equilibrium strategies are supported by a threat to revert to a PBE that gives the government the lowest utility, which we call a worst PBE. Next lemma constructs such an equilibrium.

Lemma B.2 *In a worst PBE, the strategy profile* $(\boldsymbol{\sigma}^w, \mathbf{u}^w, \mathbf{l}^w)$ *is such that* $\boldsymbol{\sigma}_t^w(L|\breve{h}^t, \theta^t) = 1$ *and* $(\mathbf{u}_t^w, \mathbf{l}_t^w) \in \arg\max_{\{\mathbf{u}_t, \mathbf{l}_t\}_t \text{ s.t. (B.1)}} \mathbb{E}_\mu[\mathbf{u}_t]$*, for all histories and distributions* μ_t.

Proof. By Lemma B.1, it is enough to prove that single deviations are not profitable. Given $\boldsymbol{\sigma}^w$, after observing a distribution of histories μ_t, the highest payoff the government can achieve is given by the allocation described in the statement of the lemma. The problem is convex, therefore, there exists a Lagrange multiplier $\lambda > 0$ such that the optimal allocation $(\mathbf{u}_t^w, \mathbf{l}_t^w)$ satisfies the first-order conditions

$$1 - \lambda C_1\left(\mathbf{u}_t^w(\breve{h}^t, H), \mathbf{l}_t^w(\breve{h}^t, H)\right) \leq 0,$$

$$1 - \lambda C_1\left(\mathbf{u}_t^w(\breve{h}^t, L), 0\right) \leq 0,$$

and

$$C_2\left(\mathbf{u}_t^w(\breve{h}^t, H), \mathbf{l}_t^w(\breve{h}^t, H)\right) - 1 = 0,$$

where the last condition holds with equality by Lemma 1 in the main text. First, observe that we cannot have $\mathbf{u}_t^w(\breve{h}^t, m_t) = 0$, for all $\breve{h}^t$ and m_t. Otherwise, since the last condition implies $\mathbf{l}_t^w(\breve{h}^t, H) > 0$, the resource constraint would be slack, which contradicts optimality. Second, by Lemma 1 in the main text, $C_1(u, l) > C_1(u, 0)$, for all u and $l > 0$. Therefore, $\mathbf{u}_t^w(\breve{h}^t, H) < \mathbf{u}_t^w(\breve{h}^t, L)$, for all $\breve{h}^t$. Given $(\mathbf{u}^w, \mathbf{l}^w)$, an agent of type θ_H is strictly better off by reporting L, thus, $\boldsymbol{\sigma}_t^w(L|\breve{h}^t, \theta^t) = 1$, for all $\breve{h}^t$ and θ^t. Therefore, $(\boldsymbol{\sigma}^w, \mathbf{u}^w, \mathbf{l}^w)$ is a PBE.

Note that, since all agents report L with probability 1, feasibility implies $\mathbf{l}_t^w(h^t) = 0$ and $\mathbf{u}_t^w(h^t) = 0$, for all h^t on the equilibrium path. Therefore, the government's payoff is 0. Since the allocation $(\mathbf{u}^w, \mathbf{l}^w)$ is feasible for any other reporting strategy of the agents, the government's payoff must be at least 0 in any PBE. Therefore, the constructed equilibrium is a worst PBE. ■

We now turn to best PBEs. Let $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ a strategy profile and let $\{\mu_t\}_t$ be the induced distributions over histories of reports. At any time t, the full history of the game is given by the public history and by the private history of each agent. In particular, let $\{G^t\}_t$ be the sequence of public histories *on the equilibrium path.* Note that, since there is no aggregate uncertainty, $\{G^t\}_t$ is deterministic.

We first specify the strategies in the continuation game following a detectable deviation, that is, a deviation that affects the aggregate history and is thus observed by everybody. We consider non-detectable deviations below. Given our assumptions, any deviation by the government or by a positive mass of agents will affect the public history and, hence, will be detectable. Standard arguments imply that, to sustain the best equilibrium, it is necessary to punish deviations in the harshest possible way. We thus assume that, in the continuation game starting from any $G'^t \neq G^t$, $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ coincides with $(\boldsymbol{\sigma}^w, \mathbf{u}^w, \mathbf{l}^w)$, which is defined in Lemma B.2. Note that, by Lemma B.2, neither the government nor the agents have an incentive to deviate in the history where $(\boldsymbol{\sigma}^w, \mathbf{u}^w, \mathbf{l}^w)$ is played.

We now study the behavior on the equilibrium path. By Lemma B.1, it is enough to ensure that single deviations are not profitable. Consider the government first. Deviations by the government are detectable, hence, they trigger a reversion to a worst PBE where, by Lemma B.2, the government achieves a payoff of 0. The highest payoff that the government can achieve by deviating in period t is thus

$$\tilde{W}_t(\mu_t) = \max_{\{\mathbf{u}_t, \mathbf{l}_t\}_t \text{ s.t. (B.1)}} \mathbb{E}_\mu[\mathbf{u}_t]. \tag{B.2}$$

As a consequence, the government will not deviate if the following best-response constraint—which we refer to as the "sustainability constraint"—is satisfied:

$$\mathbb{E}_{\boldsymbol{\sigma}} \sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_s \geq \tilde{W}_t(\mu_t) \text{ for all } t. \tag{B.3}$$

Consider now the agents. A single agent's report does not affect the aggregate history, hence, by Lemma B.1, we can write the incentive constraint of each agent as

$$\boldsymbol{\sigma}_t\left(\cdot|\breve{h}^t, \theta^t\right) \in \arg\max_{\sigma} \sum_{m_t} \sigma(m_t) \left(\mathbf{u}_t\left(h^t\right) + \beta \mathbb{E}_{\boldsymbol{\sigma}}\left[\sum_{s=0}^{\infty} \beta^s \mathbf{u}_{s+t+1} \middle| h^t, \theta^t\right]\right), \tag{B.4}$$

for all $\breve{h}^t, \theta^t$, with $\theta_t = \theta_H$.

We are left to define strategies following a non-detectable deviation, that is, a deviation that is observed only by the agent who deviates. The aggregate history is not affected by such deviations, hence, there cannot be a reversion to the worst equilibrium. We thus assume that, following this type of deviation, the reporting strategy $\boldsymbol{\sigma}_t$ will still satisfy (B.4).

So far, we have characterized equilibrium allocations and reporting strategies, however, we are interested in best PBEs. Since the government is utilitarian, it is immediate to see that, in a best PBE, the government's payoff must be maximized. Formally, a best PBE is a solution to the following problem:

$$\max_{\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l}} \mathbb{E}_{\boldsymbol{\sigma}} \sum_{t=0}^{\infty} \beta^t \mathbf{u}_t, \tag{B.5}$$

subject to (B.1), (B.3) and (B.4).

3 Recursive Problem

We start the analysis by simplifying strategies and allocations. The following lemma is a key intermediate step for our recursive characterization of best PBEs. It shows that all the information required to characterize the agents' behavior after any period t can be summarized in a variable w that captures the agent's expected continuation payoff in period t along the equilibrium path.

Lemma B.3 *Any best PBE is payoff equivalent to a PBE in which $\boldsymbol{\sigma}_t$ is independent of θ^{t-1} and for which the following property holds: if there is some $w \in \mathbb{R}$ and histories h'^t, h''^t such that*

$$w = \mathbb{E}_{\boldsymbol{\sigma}}\left[\sum_{s=t}^{\infty} \beta^{s-t}\mathbf{u}_{s+1}\middle| h'^t\right] = \mathbb{E}_{\boldsymbol{\sigma}}\left[\sum_{s=t}^{\infty} \beta^{s-t}\mathbf{u}_{s+1}\middle| h''^t\right],$$

then $\boldsymbol{\sigma}_T(m|\breve{h}'^T, \theta_T) = \boldsymbol{\sigma}_T(m|\breve{h}''^T, \theta_T)$, $\mathbf{u}_T(\breve{h}'^T, m_T) = \mathbf{u}_T(\breve{h}''^T, m_T)$, $\mathbf{l}_T(\breve{h}'^T, m_T) = \mathbf{l}_T(\breve{h}''^T, m_T)$, for all $T > t$ where $\breve{h}'^T = (h'^t, z_{t+1}, m_{t+1}, ..., z_T)$, $\breve{h}''^T = (h''^t, z_{t+1}, m_{t+1}, ..., z_T)$, for some $(z_{t+1}, m_{t+1}, ..., z_T)$ and m_T.

Proof. For any $\breve{h}^t \in \breve{H}^t$, define strategy $\boldsymbol{\sigma}'$ by

$$\boldsymbol{\sigma}'_t(\cdot|\breve{h}^t, (\hat{\theta}^{t-1}, \theta_t)) = \sum_{\theta^{t-1}} \boldsymbol{\sigma}_t(\cdot|\breve{h}^t, (\theta^{t-1}, \theta_t))\Pr(\theta^{t-1}),$$

for all $\hat{\theta}^{t-1}$. By construction, $\boldsymbol{\sigma}'_t(\cdot|\breve{h}^t, (\hat{\theta}^{t-1}, \theta_t)) = \boldsymbol{\sigma}'_t(\cdot|\breve{h}^t, (\tilde{\theta}^{t-1}, \theta_t))$ for all $\tilde{\theta}^{t-1}, \hat{\theta}^{t-1}$. Since any agent with a history $(\breve{h}^t, (\tilde{\theta}^{t-1}, \theta_t))$ can replicate the strategy of the agent with a history $(\breve{h}^t, (\hat{\theta}^{t-1}, \theta_t))$ and achieve the same payoff as that agent, and $\boldsymbol{\sigma}_t(\cdot|\breve{h}^t, (\hat{\theta}^{t-1}, \theta_t))$ is the optimal choice of the agent with history $(\breve{h}^t, (\hat{\theta}^{t-1}, \theta_t))$, the new strategy $\boldsymbol{\sigma}'$ satisfies the agents' best response constraint (B.4). The strategy $\boldsymbol{\sigma}'$ induces distributions $\{\mu'_t\}_t$ which satisfy $\mu'_t = \mu_t$ for all aggregate histories, hence, the feasibility constraint (B.1) is still satisfied if agents play $\boldsymbol{\sigma}'$. Finally, $\boldsymbol{\sigma}'_t(\hat{\theta}_L|\breve{h}^t, (\hat{\theta}^{t-1}, \theta_L)) = 1$, for all histories.

For simplicity, we assume that $\mu_t(h'^t), \mu_t(h''^t) > 0$. Let $\alpha = \mu_t(h'^t)/(\mu_t(h'^t) + \mu_t(h''^t))$ and define $\phi' : [0, \alpha] \to [0, 1]$ by $\phi'(z) = z/\alpha$ and $\phi'' : [\alpha, 1] \to [0, 1]$ by $\phi''(z) = (z - \alpha)/(1 - \alpha)$. Define a new reporting strategy and allocations $(\boldsymbol{\sigma}', \mathbf{u}', \mathbf{l}')$, for all $T \geq 1$, $h^t \in \{h'^t, h''^t\}$, and θ^{t+T}, as

$$\begin{aligned}
\mathbf{u}'_{t+T}((h^t, z_{t+1}, m_{t+1}, ..., m_{t+T})) &= \mathbf{u}^*_{t+T}((h'^t, \phi'(z_{t+1}), m_{t+1}, ..., m_{t+T})),\\
\mathbf{l}'_{t+T}((h^t, z_{t+1}, m_{t+1}, ..., m_{t+T})) &= \mathbf{l}^*_{t+T}((h'^t, \phi'(z_{t+1}), m_{t+1}, ..., m_{t+T})),\\
\boldsymbol{\sigma}'_{t+T}(\cdot|(h^t, z_{t+1}, m_{t+1}, ..., z_{t+T}), \theta^{t+T}) &= \boldsymbol{\sigma}^*_{t+T}(\cdot|(h'^t, \phi'(z_{t+1}), m_{t+1}, ..., z_{t+T}), \theta^{t+T}),
\end{aligned}$$

if $z_{t+1} \leq \alpha$, and

$$\begin{aligned}
\mathbf{u}'_{t+T}((h^t, z_{t+1}, m_{t+1}, ..., m_{t+T})) &= \mathbf{u}^*_{t+T}((h''^t, \phi''(z_{t+1}), m_{t+1}, ..., m_{t+T})),\\
\mathbf{l}'_{t+T}((h^t, z_{t+1}, m_{t+1}, ..., m_{t+T})) &= \mathbf{l}^*_{t+T}((h''^t, \phi''(z_{t+1}), m_{t+1}, ..., m_{t+T})),\\
\boldsymbol{\sigma}'_{t+T}(\cdot|(h^t, z_{t+1}, m_{t+1}, ..., z_{t+T}), \theta^{t+T}) &= \boldsymbol{\sigma}^*_{t+T}(\cdot|(h''^t, \phi''(z_{t+1}), m_{t+1}, ..., z_{t+T}), \theta^{t+T}),
\end{aligned}$$

if $z_{t+1} > \alpha$, and $\mathbf{u}'_s = \mathbf{u}_s$, $\mathbf{l}'_s = \mathbf{l}_s$, $\boldsymbol{\sigma}'_s = \boldsymbol{\sigma}_s$ for all other histories and periods s. Agents with histories h'^t, h''^t could have replicated each other strategies after period t, so they must be indifferent between them. The strategy $\boldsymbol{\sigma}'$ gives them the same utility for all histories following $\left\{h'^t, h''^t\right\}$ leaving all other histories unchanged, thus, it is incentive compatible, i.e. satisfies (B.4). The strategy profile $\boldsymbol{\sigma}'$ induces $\{\mu_t'\}_t$, which assigns the same probability to any realization of $\mathbf{u}$ and $\mathbf{l}$ as $\{\mu_t\}_t$, thus, the feasibility constraint (B.1) is satisfied. Therefore, $(\boldsymbol{\sigma}', \mathbf{u}', \mathbf{l}')$ is a PBE which is payoff equivalent to $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$. ■

Lemma B.3 allows us to greatly simplify the dependence of allocations and reporting strategies on the past. More specifically, without loss of generality, we can focus on strategy profiles $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ such that *(i)* the reporting strategy $\boldsymbol{\sigma}_t$ is independent of θ^{t-1} and, in addition, *(ii)* the expected payoff to an agent at any history h^t, $w_t \equiv \mathbb{E}_{\boldsymbol{\sigma}}\left[\sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_{s+1} \middle| h^t\right]$, is a sufficient statistic for the information contained in h^t.

We can use these results to rewrite problem (B.5) as follows. First, instead of maximizing over allocations and reporting strategies that depend on the random variable z_t, we can equivalently choose probability distributions over bundles of utility and reporting probability. Second, we can use Lemma B.3 and replace each agent's history with a single number, namely, the agent's expected payoff at that particular history.

Formally, the government chooses sequences of probability measures $\{\psi_{t,w}\}_{t,w}$ over the space X defined in the main text, which satisfy the promise-keeping constraint (11) in the main text. Any such sequence will in turn generate a sequence of probability measures $\{\pi_{t-1}\}_t$ over continuation values defined by the recursion (12). We can then rewrite the resource constraint (B.1) as equation (13). Similarly, with a slight abuse of notation, the value at time t of the government's best deviation (B.2) is given by the function $\tilde{W}\left(\left\{\psi_{t,w}\right\}_w, \pi_{t-1}\right)$ defined in the main text. As a result, the sustainability constraint (B.3) becomes constraint (14). Putting everything together, best PBEs can be found as a solution to problem (15).

The next lemma, which is the analogue of Lemma 2 in the main text, shows how to construct a bound on the function $\tilde{W}$, which will be key to obtain a recursive representation of problem (15).

Lemma B.4 *Let $\{\psi_{t,w}^*\}$ be a best PBE. Then, there exists a sequence of non-negative numbers $\{\lambda_t^*\}$ that defines functions $\{W_t(\cdot)\}$ given by*

$$W_t(x) \equiv \max_{(\tilde{u}_H, \tilde{u}_L, \tilde{l}) \in \mathcal{C}} [p\sigma(x)\tilde{u}_H + (1-p\sigma(x))\tilde{u}_L] + \lambda_t^*[p\sigma(x)\left\{C\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l}\right\} + (1-p\sigma(x))C(\tilde{u}_L, 0)],$$

such that

$$\int\int W_t(x)\, d\psi_{t,w} d\pi_{t-1} \geq \tilde{W}\left(\{\psi_{t,w}\}, \pi_{t-1}\right)$$

for any feasible $\{\psi_{t,w}\}$, and

$$\int\int W_t(x)\, d\psi_t^* d\pi_{t-1} = \tilde{W}\left(\{\psi_{t,w}^*\}, \pi_{t-1}\right).$$

Furthermore, $W_t(x)$ is linear in $\sigma(x)$.

Proof. Problem (B.2) is convex, thus, its solution is characterized by the saddle point of the Langrangian (Luenberger (1969), Theorem 1, p. 224)

$$\begin{aligned} &\tilde{W}\left(\{\psi_{t,w}^*\}, \pi_{t-1}\right) \\ = &\min_{\lambda_t} \max_{(\tilde{u}_H, \tilde{u}_L, \tilde{l}) \in \mathcal{C}} \int\int \left\{ \begin{array}{c} p\sigma(x)\tilde{u}_H + (1-p\sigma(x))\tilde{u}_L \\ -\lambda_t[p\sigma(x)\left(C\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l}\right) + (1-p\sigma(x))C(\tilde{u}_L, 0)] \end{array} \right\} d\psi_{t,w}^* d\pi_{t-1}. \end{aligned}$$

The proof then follows exactly the same steps as the proof of Lemma 2 and Corollary 1 in the main text. ■

Following the same arguments in Section 2 in the main text, by Lemma B.4, we can define a "modified problem" by replacing the sustainability constraint (14) with the alternative constraint (16) in problem (15). Then, any solution $\{\psi_{t,w}\}_{t,w}$ to this modified problem is a best PBE. We apply the techniques developed in Farhi and Werning (2007) on this modified problem. We first let $\beta^t\zeta_t^*$ and $\beta^t\chi_t^*$ be the Langrange multipliers on constraints (13) and (16). Straightforward arguments prove that the Lagrangian of the modified problem is given by $\mathcal{L}$ in the main text. The linear structure of $\mathcal{L}$ implies that we can solve the modified problem by focusing on the value functions $\{K_t\}_t$ defined by equation (17). These value functions satisfy a useful recursion.

Lemma B.5 *The functions $\{K_t\}_t$ satisfy the recursion*

$$K_t(\tilde{w}) = \max_{\psi_{0,\tilde{w}}\ s.t.\ (11)} \int \left[g - \zeta_t f - \chi_t W_t + \hat{\beta}_{t+1} K_{t+1}\right] d\psi_{t,\tilde{w}},$$

for all t and $\tilde{w} \in [0, \frac{\bar{U}}{1-\beta})$, where $\hat{\beta}_{t+1} \equiv \bar{\beta}_{t+1}/\bar{\beta}_t$ and where $K_{t+1}(x)$ is a shorthand notation for $p\sigma K_{t+1}(w_H) + (1-p\sigma) K_{t+1}(w_L)$.

Proof. We prove the statement for $t = 0$, the arguments are exactly the same for all other time periods. By definition,

$$K_0(\tilde{w}) = \frac{1}{\bar{\beta}_0} \max_{\{\psi_{s,w},\pi_{s-1}\}_{s,w}} \sum_{s=0}^{\infty} \bar{\beta}_s \int\int [g - \zeta_s f - \chi_s W_s]\, d\psi_{s,w} d\pi_{s-1},$$

subject to (11) and (12) in the main text, with $\pi_{-1}(w) = 1 - \mathcal{I}_{w\leq\tilde{w}}$. We rewrite the objective in $K_0(\tilde{w})$ as

$$\begin{aligned}
& \bar{\beta}_0 \int [g - \zeta_0 f - \chi_0 W_0]\, d\psi_{0,\tilde{w}} + \sum_{s=1}^{\infty} \bar{\beta}_s \int\int [g - \zeta_s f - \chi_s W_s]\, d\psi_{s,w} d\pi_{s-1} \\
= \; & \bar{\beta}_0 \int [g - \zeta_0 f - \chi_0 W_0]\, d\psi_{0,\tilde{w}} + \bar{\beta}_1 \sum_{s=1}^{\infty} \frac{\bar{\beta}_s}{\bar{\beta}_1} \int\int [g - \zeta_s f - \chi_s W_s]\, d\psi_{s,w} d\pi_{s-1}.
\end{aligned}$$

Using (12),

$$\begin{aligned}
K_0(\tilde{w}) \; = \; & \max_{\{\psi_{s,w},\pi_{s-1}\}_{s,w} \text{ s.t. } (11),\,(12)} \int [g - \zeta_0 f - \chi_0 W_0]\, d\psi_{0,\tilde{w}} \\
& + \frac{\bar{\beta}_1}{\bar{\beta}_0} \sum_{s=1}^{\infty} \frac{\bar{\beta}_s}{\bar{\beta}_1} \int [g - \zeta_s f - \chi_s W_s]\, d\psi_{s,w} d\pi_{s-1} \\
= \; & \max_{\psi_{0,\tilde{w}} \text{ s.t. } (11)} \int [g - \zeta_0 f - \chi_0 W_0]\, d\psi_{0,\tilde{w}} \\
& + \max_{\{\psi_{s,w},\pi_{s-1}\}_{s\geq 1,w} \text{ s.t. } (11),\,(12)} \frac{\bar{\beta}_1}{\bar{\beta}_0} \sum_{s=1}^{\infty} \frac{\bar{\beta}_s}{\bar{\beta}_1} \int\int [g - \zeta_s f - \chi_s W_s]\, d\psi_{s,w} d\pi_{s-1} \\
= \; & \max_{\psi_{0,\tilde{w}} \text{ s.t. } (11)} \int\int \left\{g - \zeta_0 f - \chi_0 W_0 + \hat{\beta}_1 \left[p\sigma K_{t+1}(w_H) + (1-p\sigma) K_{t+1}(w_L)\right]\right\} d\psi_{0,\tilde{w}},
\end{aligned}$$

where $\hat{\beta}_1 \equiv \bar{\beta}_1/\bar{\beta}_0$. ■

4 Proofs for Section 3.2

Proof of Lemma 3 (extra details). We show in detail how to apply the results in Topkis (2011). First, note that the set $dom(C)$ consists of all pairs (u, l) such that $\{u \geq 0, u \leq U(\infty, l), l \geq 0\}$. Similarly, the set $dom(K_{t+1})$ consists of utility levels w such that $0 \leq w < \bar{U}/(1-\beta)$. Now, let $x = (u, -l)$, $y = (v, -\Delta)$ and let $X = \mathbb{R}^2$, $Y = \mathbb{R}^2$. Thus, X and Y with the usual ordering are lattices. Let $S \subset \mathbb{R}^4$ be the subset of $X \times Y$ defined as

$$S = \left\{ (x, y) : \begin{array}{c} x_1 \geq 0,\ x_1 \leq U(\infty, -x_2),\ -x_2 \geq 0,\ y_1 \geq 0,\ y_1 < \bar{U}/(1-\beta), \\ (y_1 - x_1)/\beta \geq 0,\ (y_1 - x_1)/\beta < \bar{U}/(1-\beta),\ -x_2 \leq -y_2,\ -y_2 \geq 0 \end{array} \right\}.$$

Finally, let

$$g_t(y) = \max_{x \in S_y} f_t(x, y),$$

where

$$f_t(x, y) = x_1 - \zeta_t (C(x_1, -x_2) + x_2) + \hat{\beta}_{t+1} K_{t+1}\left(\frac{y_1 - x_1}{\beta}\right).$$

We first prove that S is a sublattice. We can represent S with the following real-valued functions: $g_1(x_1) = x_1$, $g_2(x_2) = -x_2$, $g_3(y_1) = \mathcal{I}\left(0 \leq y_1 < \bar{U}/(1-\beta)\right) - 1$, $g_4(y_2) = -y_2$, and $h_1(x_2, x_1) = U(\infty, -x_2) - x_1$, $h_2(y_1, x_1) = (y_1 - x_1)/\beta$, $h_3(x_1, y_1) = \mathcal{I}(\bar{U}/(1-\beta) - (y_1 - x_1)/\beta > 0) - 1$, $h_4(x_2, y_2) = x_2 - y_2$. Each function h_i is bimonotone, i.e. it is bivariate, increasing in the first variable, and decreasing in the second one (Topkis (2011), pag. 23). By Example 2.2.7(a) in Topkis (2011), S is a sublattice of $\mathbb{R}^4$.

We now prove that f_t is supermodular on S. First, S is a subset of $\mathbb{R}^4$ and each $\mathbb{R}$, with the usual ordering, is a chain. Also,

$$\frac{\partial^2}{\partial x_1 \partial x_2} f_t(x, y) = \zeta_t C_{12}(x_1, -x_2) > 0$$

and, for $x_1' > x_1$,

$$\frac{\partial}{\partial y_1} f_t\left((x_1', x_2), y\right) - \frac{\partial}{\partial y_1} f_t\left((x_1, x_2), y\right) = \frac{1}{\beta}\hat{\beta}_{t+1}\left[K_{t+1}'\left(\frac{y_1 - x_1'}{\beta}\right) - K_{t+1}'\left(\frac{y_1 - x_1}{\beta}\right)\right] \geq 0,$$

by concavity of K_{t+1}. The other cross-partial derivatives are trivially 0. Therefore, f_t has increasing differences on S and, by Theorem 2.6.3, is supermodular on S.

It is then immediate to apply Theorem 2.7.6. The only assumption we need to verify is that $g_t(y)$ is finite on the projection of S on Y. This is immediate. Therefore, $g_t(y)$ is supermodular, hence, it has increasing differences in (y_1, y_2).

With a slight abuse of notation, we use $g_t(v, \Delta)$ to denote $g_t((v, -\Delta))$. Let $G_t(v, \Delta) \equiv g_t(v, \Delta) - g_t(v, 0)$. G_t is positive, $G_t(\cdot, \Delta)$ is non-increasing, for each Δ because of the results above. Finally, $G_t(v, \cdot)$ is non-decreasing, for each v, because the constraint set becomes larger as Δ increases. Consider a sequence $\Delta_n \to \infty$ as $n \to \infty$. For each v, $G_t(v, \Delta_n)$ is monotone, thus, it has a limit $\bar{G}_t(v) \equiv \lim_{n\to\infty} G_t(v, \Delta_n)$. Take now $v_1 < v_2$. Then, for each Δ_n, $G_t(v_1, \Delta_n) \geq G_t(v_2, \Delta_n)$. Taking the limit as $n \to \infty$, we have $\bar{G}_t(v_1) \geq \bar{G}_t(v_2)$. Therefore, $\bar{G}_t$ is also non-increasing. ■

Appendix C: Proofs for Section 4

1 Preliminaries

In this appendix we present the details of the dynamic economy in Section 4.

We use a superscript "n" to denote agents born at time n and a superscript "o" for the initial old. We let $J = \{o\} \cup \{0, 1, 2, ...\}$. At time 0, there is a measure $1 - \delta$ of "initial old" who are indexed by (v, ι), where v is entitlement to lifetime utility and $\iota \in \{0, 1\}$ denotes whether the agent is unemployed or employed at time 0, respectively. Let π^o_{-1} denote the distribution of (v, ι).

Each period t is divided in two stages. We use $\iota_t = 1$ if the agent is employed at time t and $\iota_t = 0$ otherwise. In stage 1, an agent born at time $n \leq t$ observes his employment status $\iota_t \in \{0, 1\}$, the realization of his sunspot variable z_t, and his current type $\theta_t \in \{\theta_H, \theta_L\}$. He then sends a report $m_t \in \{H, L\}$ to the government. Since the employment status is observable, an employed agent can only report $m_t = H$. Reports are a function of current and past realizations of the agent's types $\theta^{n,t} \equiv (\theta_n, ..., \theta_t)$, the agent's current and past employment statuses $\iota^{n,t-1} \equiv (\iota_n, ..., \iota_t)$, current and past realizations of the agent's sunspot variables $z^{n,t} \equiv (z_n, ..., z_t)$, the agent's past reports $m^{n,t-1} \equiv (m_n, ..., m_{t-1})$, and the aggregate history G^{t-1}, which we describe below. We similarly define $\theta^{o,t}$, $z^{o,t}$, and $m^{o,t}$, $t \geq 0$, for the initial old.

For an agent born at time n, let $\breve{h}^{n,t} \equiv (\iota_n, z_n, m_n, ..., \iota_{t-1}, z_{t-1}, m_{t-1}, \iota_t, z_t)$ and $h^{n,t} \equiv (\iota_n, z_n, m_n, ..., \iota_t, z_t, m_t)$, $t \geq n$, be the histories of the agent's reports and realizations of the idiosyncratic sunspot variable, before and after he submits the current period's report m_t, respectively. Similarly, let $\breve{h}^{o,t} \equiv (v, \iota, z_0, m_0, ..., \iota_{t-1}, z_{t-1}, m_{t-1}, \iota_t, z_t)$ and $h^{o,t} \equiv (v, \iota, z_0, m_0, ..., \iota_t, z_t, m_t)$, $t \geq 0$, be the time-t histories of the initial old. Let $\breve{H}^{j,t}$ and $H^{j,t}$, $j \in J$, be the spaces of all such histories. A reporting strategy $\boldsymbol{\sigma}^j_t$ induces a probability distribution over $\{H, L\}$ denoted by $\boldsymbol{\sigma}^j_t(\cdot|\breve{h}^{j,t}, \theta^{j,t})$. To simplify notation we have omitted explicit dependence on the aggregate history G^{t-1}. We assume that the law of the large numbers holds and the aggregate distribution of histories $h^{j,t}$, $j \in J$, denoted by μ^j_t, is given by $\mu^o_{-1} = \pi^o_{-1}$, $\mu^n_{n-1} = 1$, and

$$\mu^j_t \left(h^{j,t}\right) = (1 - \delta) \, \mu^j_{t-1}(h^{j,t-1}) \Pr(z_t) \sum_{\theta^{j,t} \in \{\theta_H, \theta_L\}^t} \Pr\left(\theta^{j,t}\right) \boldsymbol{\sigma}^j_t(m_t|\breve{h}^{j,t}, \theta^{j,t}) \times \begin{cases} 1, \text{ if } m_{t-1} = L, \\ q, \text{ if } m_{t-1} = H, \end{cases}$$

for $\iota_t = 0$,

$$\mu^j_t(h^{j,t}) = (1 - \delta) \, \mu^j_{t-1}(h^{j,t-1}) \Pr(z_t) \times \begin{cases} 0, \text{ if } m_{t-1} = L, \\ 1 - q, \text{ if } m_{t-1} = H, \end{cases}$$

for $\iota_t = 1$ and $m_t = H$, and $\mu^j_t(h^{j,t}) = 0$, for $\iota_t = 1$ and $m_t = L$.[3] The triple $H^{j,t}$, its Borel sigma algebra, and μ^j_t is a probability space, for all $j \in J$. Similarly for the initial old. To ease notation, we let $\mu_t \equiv \left(\mu^o_t, \mu^1_t, ..., \mu^n_t\right)$.

In stage 2, the government observes the past aggregate history G^{t-1} and the current distributions μ_t, and chooses allocations. Allocations are measurable functions $(\mathbf{u}^j_t, \mathbf{l}^j_t) : H^{j,t} \rightarrow$

[3] Formally, since z_t is a continuous variable, the distribution μ_t should be defined over any Borel set A^t of H^t, which can be represented as a product $A^t = A^{t-1} \times \{0, 1\} \times B_z \times \{0, 1\}$, where A^{t-1} is a Borel set of H^{t-1} and B_z is some Borel set of Z.

$[\underline{U}, \bar{U}) \times \mathbb{R}_+$, $j \in J$, that satisfy $\mathbf{l}_t^j((\check{h}^{j,t}, \theta_L)) = 0$. As in the model of Section 3, allocations must lie in the domain of function C. We define set $\mathbf{C}$ as in Appendix B. The feasibility constraint is

$$\sum_{j \in J} \omega_{t,j} \mathbb{E}_\mu^j C\left(\mathbf{u}_t^j, \mathbf{l}_t^j\right) \leq \sum_{j \in J} \omega_{t,j} \mathbb{E}_\mu^j \mathbf{l}_t^j, \quad \{\mathbf{u}_t^j, \mathbf{l}_t^j\}_t \in \mathbf{C}, \text{ for all } j \in J, \text{ a.s.,} \tag{C.1}$$

for all t.[4] Note that, to make notation more compact, we have defined $\omega_{t,o} \equiv 1 - \delta$, $\omega_{t,n} \equiv \delta$, for $n \leq t$, and $\omega_{t,n} \equiv 0$, for $n > t$, and we have, once again, omitted the explicit dependence on the aggregate history G^{t-1}. Also, given a sequence $\{\mathbf{x}_t^j\}_{t,j}$, we use the shorthand notation: $\mathbf{x}_t \equiv \{\mathbf{x}_t^j\}_j$, $\mathbf{x}^j \equiv \{\mathbf{x}_t^j\}_t$, and $\mathbf{x} \equiv \{\mathbf{x}_t^j\}_{t,j}$.

The *aggregate* history G^t includes the history of distributions $\{\mu_s\}_{s \leq t}$ and the history of allocations chosen by the government, $\{(\mathbf{u}_s, \mathbf{l}_s)\}_{s \leq t}$. Finally, at time t, the *full* history of the game consists of the aggregate history and of the private history of each agent.

The definition of aggregate history formalizes our assumption that agents are atomistic. Since only the *distribution* of agent histories is observable, any event to which μ_t assigns zero probability will not affect the aggregate history and, therefore, will not be observed. In particular, the reporting strategy of any individual agent does not affect the aggregate history in the game.

Perfect Bayesian equilibrium. A PBE consists of strategies of agents and the government and posterior beliefs such that, at each history of the game, each player chooses his best response given his posterior beliefs formulated using Bayes' rule. A best PBE is a PBE such that there is *(i)* no other PBE that gives higher utility to a set of agents of measure 1, and strictly higher utility to a positive measure of agents; *(ii)* each initial old individual receives lifetime expected utility at least equal to his entitlement v.

Without loss of generality, we assume that an initial old with entitlement v receives a lifetime expected utility exactly equal to v.

2 Best Equilibrium

A convenient property of PBEs is the one-stage deviation principle (see Theorem 4.2, p. 110 in Fudenberg and Tirole (1991)). The next lemma shows that this principle applies to our environment.

Lemma C.1 *The strategy profile $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ is part of a PBE if and only if there is no history Γ^t at which a player i (i.e. either the government or any individual agent) has an alternative strategy, which differs from the equilibrium strategy only at Γ^t and which delivers a higher payoff to player i, conditional on Γ^t being reached.*

Proof. The "only if" part of the lemma follows directly from the definition of PBEs. We prove the "if" part. Also, we focus on the case in which an agent deviates; the case in which the government deviates is analogous. Consider an agent who is an "initial old",

[4] Given a distribution μ_t^j, $j \in J$, it is immediate to define the expectation $\mathbb{E}_\mu^j[f_t]$ for any measurable function $f_t : H^{j,t} \to \mathbb{R}$. Also, given reporting strategies $\{\boldsymbol{\sigma}_t^j\}_t$ and a sequence of measurable functions $\{f_t\}_t$, we let $\mathbb{E}_{\boldsymbol{\sigma}}^j\left[\sum_{t=0}^{\infty} \beta^t f_t\right]$ denote the discounted expected sum computed using the sequence of distributions $\{\mu_t^j\}_t$ induced by $\{\boldsymbol{\sigma}_t^j\}_t$. The "a.s." requirement is with respect to such measure.

the proof is identical if the agent is born at any $t \geq 0$. Suppose that *(i)* $\boldsymbol{\sigma}^o$ satisfies the "one-stage deviation" property, that is, there is no other strategy that differs from $\boldsymbol{\sigma}^o$ only at one history and delivers a higher payoff to the agent; *(ii)* there is a reporting strategy $\tilde{\boldsymbol{\sigma}}^o$ and a history $(\breve{h}^{o,t}, \theta^{o,t})$ at which $\tilde{\boldsymbol{\sigma}}_t^o$ is preferred to $\boldsymbol{\sigma}_t^o$. Formally, if we let $V\left(\boldsymbol{\sigma}^o | \breve{h}^{o,t}, \theta^t\right) \equiv \mathbb{E}_{\boldsymbol{\sigma}}^o \left[\sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_s \middle| \breve{h}^{o,t}, \theta^{o,t}\right]$, then condition *(ii)* states that

$$V\left(\tilde{\boldsymbol{\sigma}}^o | \breve{h}^{o,t}, \theta^{o,t}\right) > V\left(\boldsymbol{\sigma}^o | \breve{h}^{o,t}, \theta^{o,t}\right).$$

Notice that this condition implies that $\boldsymbol{\sigma}^o$ cannot be part of a PBE. Suppose first that there is a time $T < \infty$ such that $\boldsymbol{\sigma}_\tau^o = \tilde{\boldsymbol{\sigma}}_\tau^o$, for any $(\breve{h}^{o,\tau}, \theta^{o,\tau})$ with $\tau \geq T+1$. The one-stage deviation property implies that

$$V\left(\boldsymbol{\sigma}^o | \breve{h}^{o,T}, \theta^T\right) \geq V\left(\tilde{\boldsymbol{\sigma}}^o | \breve{h}^{o,T}, \theta^T\right).$$

Consider an alternative strategy $\hat{\boldsymbol{\sigma}}^o$ which coincides with $\tilde{\boldsymbol{\sigma}}$ for all histories $(\breve{h}^{o,\tau}, \theta^{o,\tau})$ with $t \leq \tau \leq T-1$ and coincides with $\boldsymbol{\sigma}^o$ elsewhere. The previous inequalities imply

$$V\left(\hat{\boldsymbol{\sigma}}^o | \breve{h}^{o,t}, \theta^{o,t}\right) > V\left(\boldsymbol{\sigma}^o | \breve{h}^{o,t}, \theta^{o,t}\right).$$

However, $\hat{\boldsymbol{\sigma}}^o$ differs from $\boldsymbol{\sigma}^o$ only for $\tau \leq T-1$. Repeating this process, we eventually find a reporting strategy that differs from $\boldsymbol{\sigma}^o$ only at time t and yields a higher payoff to the agent, thus, contradicting condition *(i)*.

Suppose now $T = \infty$. Let

$$\epsilon \equiv V\left(\tilde{\boldsymbol{\sigma}}^o | \breve{h}^{o,t}, \theta^{o,t}\right) - V\left(\boldsymbol{\sigma}^o | \breve{h}^{o,t}, \theta^{o,t}\right) > 0.$$

Let $\hat{\boldsymbol{\sigma}}^o$ be the strategy that coincides with $\tilde{\boldsymbol{\sigma}}^o$ for all histories $(\breve{h}^{o,s}, \theta^{o,s})$, $s \leq \tau$, and coincides with $\boldsymbol{\sigma}^o$ for all other histories. Since utility is bounded, we can choose τ high enough that

$$V\left(\tilde{\boldsymbol{\sigma}}^o | \breve{h}^{o,t}, \theta^{o,t}\right) - V\left(\hat{\boldsymbol{\sigma}}^o | \breve{h}^{o,t}, \theta^{o,t}\right) \leq \frac{\epsilon}{2}.$$

Thus, $\hat{\boldsymbol{\sigma}}^o$ delivers a higher payoff to the agent at history $(\breve{h}^{o,t}, \theta^{o,t})$ and differs from $\boldsymbol{\sigma}^o$ only for finitely many periods. We can then follow the same steps as in the case with $T < \infty$ to reach a contradiction. ■

Following standard arguments, equilibrium strategies are supported by a threat to revert to a PBE that gives the lowest utility to the government, which we call a worst PBE.

When agents were not dying and the economy was populated only by "initial old" (as in Section 3 in the main text), a worst equilibrium was simply given by the repetition of the static Nash equilibrium where every agent reported to be jobless. Things are substantially more complicated when agents are born and die. The reason is as follows. When the government maximizes the utility of the people who are alive, his objective is not time consistent: some of the people whose utility will be maximized in the next period are not around in the current period. As a result, there might be an equilibrium which delivers a lower payoff than the simple repetition of the static Nash equilibrium.

As an example, suppose we want to find an equilibrium that delivers the lowest utility to the agents alive at t. One way to achieve this is to let the government at $t+1$ punish the individuals who were also alive at t, by redistributing towards those who are born at $t+1$ and, thus, were not yet alive at t. Clearly, this redistribution is costly for the perspective of the government at $t+1$ who is utilitarian. However, we can reward the government at $t+1$ by having agents at $t+2$ play a better equilibrium than the static Nash equilibrium. As a result, we can make the punishment of people alive at t to be incentive compatible for the government.

It turns out that, although finding a worst equilibrium may be difficult, the value of any such equilibrium must satisfy a crucial property, which is all we need in our analysis.

Lemma C.2 *The value of a worst equilibrium at any history depends only on the fraction of people employed at that history.*

Proof. Take any time t and two full histories of the game Γ_t and $\hat{\Gamma}_t$. Suppose that the fraction of people who are employed in the two histories is the same and equals N_t. Finally, suppose that $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ and $(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{u}}, \hat{\mathbf{l}})$ are the worst equilibria in the continuation games starting from Γ_t and $\hat{\Gamma}_t$, respectively. For $j \in J$, let $v_t^j\left(h^{j,t}, \theta^{j,t}\right) \equiv \mathbb{E}_{\boldsymbol{\sigma}}^j\left[\sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_s \middle| h^{j,t}, \theta^{j,t}\right]$ (resp. $\hat{v}_t^j\left(h^{j,t}, \theta^{j,t}\right)$) be the lifetime utility of an agent at history $h^{j,t}$ if $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ (resp. $(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{u}}, \hat{\mathbf{l}})$) is played. Finally, let μ_t and $\hat{\mu}_t$ be the distributions over agents' histories at Γ_t and $\hat{\Gamma}_t$, respectively. Suppose the government achieves a higher payoff at $\hat{\Gamma}_t$ than at Γ_t:

$$\sum_{j \in J} \omega_{t,j} \mathbb{E}_{\mu}^j v_t^j > \sum_{j \in J} \omega_{t,j} \mathbb{E}_{\hat{\mu}}^j \hat{v}_t^j.$$

We want to prove that *(i)* $(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{u}}, \hat{\mathbf{l}})$ is also an equilibrium in the continuation game starting from Γ_t and that *(ii)* it achieves a lower payoff than $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$.

At both histories, we can divide the agents into two groups, those with a job and those without one. We then match each agent in each group starting from Γ_t with an agent in the corresponding group starting from $\hat{\Gamma}_t$. This is possible as the groups following the two histories have the same size by assumption. Notice that the matching does not take into account when agents are born. However, since preferences are independent of the time when agents are born, if $(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{u}}, \hat{\mathbf{l}})$ is played an agent receives utility $\hat{v}_s^j\left(h^{j,s}, \theta^{j,s}\right)$ at history $\left(h^{j,s}, \theta^{j,s}\right)$, $s \geq t$, $j \in J$, independently of whether he was born at time j or at any other time. It is then immediate to verify that $(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{u}}, \hat{\mathbf{l}})$ is an equilibrium also in the subgame starting from Γ_t and that $(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{u}}, \hat{\mathbf{l}})$ delivers a strictly lower payoff to the government at history Γ_t, contradicting the fact that $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ is a worst equilibrium. ∎

We now turn to best PBEs. Let $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ be a strategy profile and let $\{\mu_t\}_t$ be the induced distributions over histories of reports. At any time t, the full history of the game is given by the public history and by the private history of each agent. In particular, let $\{G^t\}_t$ be the sequence of public histories *on the equilibrium path.* Note that, since there is no aggregate uncertainty, $\{G^t\}_t$ is deterministic.

We first specify the strategies in the continuation game following a detectable deviation, that is, a deviation that affects the aggregate history and is thus observed by everybody. We consider non-detectable deviations below. Given our assumptions, any deviation by the government or by a positive mass of agents will affect the public history and, hence, will be detectable. Standard arguments imply that, to sustain the best equilibrium, it is necessary to punish deviations in the harshest possible way. We thus assume that, in the continuation game starting from any $G'^t \neq G^t$, $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ coincides with a worst PBE. Notice that, since detectable deviations are punished with reversion to a worst equilibrium, once such reversion occurs, neither the government nor the agents have an incentive to deviate.

We now study the behavior on the equilibrium path. By Lemma B.1, it is enough to ensure that single deviations are not profitable. Consider the government first. Deviations by the government are detectable by construction, hence, they will trigger a reversion to a worst PBE. By Lemma C.2, the value of any such PBE will depend only on the fraction of people employed. Let $N_t^{\boldsymbol{\sigma}}$ denote such fraction. The notation emphasizes that the fraction of people employed depends on the agents' reporting strategies. As a result, a deviation by the government will, in general, affect the fraction of people employed. Finally, let $\underline{V}\left(N_t^{\boldsymbol{\sigma}}\right)$ be the value of any worst PBE. The highest payoff that the government can achieve by deviating in period t is then

$$\tilde{W}_t = \max_{\{\mathbf{u}_t, \mathbf{l}_t\}_t \text{ s.t. (C.1)}} \sum_{j \in J} \omega_{t,j} \mathbb{E}_{\mu}^j \mathbf{u}_t^j + \beta \underline{V}(N_{t+1}^{\boldsymbol{\sigma}}). \tag{C.2}$$

As a consequence, the government will not deviate if the following best-response constraint—which we refer to as the "sustainability constraint"—is satisfied:

$$\sum_{j \in J} \omega_{t,j} \mathbb{E}_{\boldsymbol{\sigma}}^j \sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_s^j \geq \tilde{W}_t, \text{ for all } t. \tag{C.3}$$

Consider now the agents. A single agent's report does not affect the aggregate history, hence, by Lemma C.1, we can write the incentive constraint of each agent as

$$\boldsymbol{\sigma}_t^j\left(\cdot|\breve{h}^{j,t}, \theta^{j,t}\right) \in \arg\max_{\sigma} \sum_{m_t \in \{0,1\}} \sigma\left(m_t\right) \left(\mathbf{u}_t^j\left(h^{j,t}\right) + \beta \mathbb{E}_{\boldsymbol{\sigma}}^j \left[\sum_{s=0}^{\infty} \beta^s \mathbf{u}_{t+1+s}^j \middle| h^{j,t}, \theta^{j,t}\right]\right), \tag{C.4}$$

for all $\breve{h}^{j,t}$, $j \in J$, $\theta^{j,t}$, with $\theta_t = \theta_H$ and $h^{j,t} = (\breve{h}^{j,t}, m_t)$.

Finally, in a best PBE, an initial old with entitlement v must receive a lifetime expected utility of exactly v:

$$\mathbb{E}_{\boldsymbol{\sigma}}^o \left[\sum_{t=0}^{\infty} \beta^t \mathbf{u}_t^o \middle| v\right] = v. \tag{C.5}$$

We are left to define strategies following a non-detectable deviation, that is, a deviation that is observed only by the agent who deviates. The aggregate history is not affected by such deviations, hence, there cannot be a reversion to the worst equilibrium. We thus assume that, following this type of deviation, the reporting strategies $\boldsymbol{\sigma}_t^j$ will still satisfy (C.4).

So far, we have characterized equilibrium allocations and reporting strategies, however, we are interested in best PBEs. We trace out the Pareto frontiers of the game by giving different weights to different generations. Formally, let $\{\varphi_j\}$, with $\varphi_o = 1$ and $\varphi_n \geq 0$, $n \geq 0$, be a sequence of Pareto weights, a best PBE is a solution to the following problem:

$$\max_{\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l}} \sum_{j \in J} \varphi_j \mathbb{E}_{\boldsymbol{\sigma}}^j \sum_{t=0}^{\infty} \omega_{t,j} \beta^{t - n\mathcal{I}_{j=n}} \mathbf{u}_t^j, \tag{C.6}$$

subject to (C.1), (C.3), (C.4) and (C.5).

3 Recursive Problem

We start the analysis by simplifying strategies and allocations. The following lemma is a key intermediate step for our recursive characterization of best PBEs. It shows that all the information required to characterize the agents' behavior after any period t can be summarized with a variable w that captures the agent's expected continuation payoff in period t along the equilibrium path and with the indicator ι of the agent's employment status.

Lemma C.3 *Any best PBE is payoff equivalent to a PBE in which $\boldsymbol{\sigma}_t^j$, $j \in J$, is independent of $\theta^{j,t-1}$ and for which the following property holds: if for some $j \in J$ there is $w \in \mathbb{R}$ and histories $\left(h'^{j,t-1}, \iota_t'\right)$, $\left(h''^{j,t-1}, \iota_t''\right)$ such that $\iota_t' = \iota_t''$ and*

$$w = \mathbb{E}_{\boldsymbol{\sigma}}^j \left[\sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_s^j \middle| h'^{j,t-1} \right] = \mathbb{E}_{\boldsymbol{\sigma}}^j \left[\sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_s^j \middle| h''^{j,t-1} \right],$$

then $\boldsymbol{\sigma}_T^j(m|\breve{h}'^{j,T}, \theta_T) = \boldsymbol{\sigma}_T^j(m|\breve{h}''^{j,T}, \theta_T)$, $\mathbf{u}_T^j(\breve{h}'^{j,T}, m_T) = \mathbf{u}_T^j(\breve{h}''^{j,T}, m_T)$, $\mathbf{l}_T^j(\breve{h}'^{j,T}, m_T) = \mathbf{l}_T^j(\breve{h}''^{j,T}, m_T)$, for all $T \geq t$, where $\breve{h}'^{j,T} = \left(h'^{j,t-1}, \iota_t', z_t, m_t, ..., \iota_T, z_T\right)$, $\breve{h}''^{j,T} = \left(h''^{j,t-1}, \iota_t'', z_t, m_t, ..., \iota_T, z_T\right)$, for some $(z_t, m_t, ..., \iota_T, z_T)$ and m_T.

Proof. For any $\breve{h}^{j,t} \in \breve{H}^{j,t}$, $j \in J$, $t \geq 0$, define strategy $\boldsymbol{\sigma}'$ by

$$\boldsymbol{\sigma}_t'(\cdot|\breve{h}^{j,t}, (\hat{\theta}^{j,t-1}, \theta_t)) = \sum_{\theta^{j,t-1}} \boldsymbol{\sigma}_t^j(\cdot|\breve{h}^{j,t}, (\theta^{j,t-1}, \theta_t)) \Pr(\theta^{j,t-1}),$$

for all $\hat{\theta}^{j,t-1}$. By construction, $\boldsymbol{\sigma}_t'(\cdot|\breve{h}^{j,t}, (\hat{\theta}^{j,t-1}, \theta_t)) = \boldsymbol{\sigma}_t'(\cdot|\breve{h}^{j,t}, (\tilde{\theta}^{j,t-1}, \theta_t))$ for all $\tilde{\theta}^{j,t-1}, \hat{\theta}^{j,t-1}$. Since any agent with a history $(\breve{h}^{j,t}, (\tilde{\theta}^{j,t-1}, \theta_t))$ can replicate the strategy of the agent with a history $(\breve{h}^{j,t}, (\hat{\theta}^{j,t-1}, \theta_t))$ and achieve the same payoff as that agent, and $\boldsymbol{\sigma}_t^j(\cdot|\breve{h}^{j,t}, (\hat{\theta}^{j,t-1}, \theta_t))$ is the optimal choice of the agent with history $(\breve{h}^{j,t}, (\hat{\theta}^{j,t-1}, \theta_t))$, the new strategy $\boldsymbol{\sigma}'$ satisfies the agents' best response constraint (C.4). The strategy $\boldsymbol{\sigma}'$ induces distributions $\{\mu_t'\}_t$ which satisfy $\mu_t' = \mu_t^j$ for all aggregate histories, hence, the feasibility constraint (C.1) is still satisfied if agents play $\boldsymbol{\sigma}'$. Finally, $\boldsymbol{\sigma}_t'(0|\breve{h}^{j,t}, (\hat{\theta}^{j,t-1}, 0)) = 1$, for all histories, and $\boldsymbol{\sigma}_t'(1|\breve{h}^{j,t}, \hat{\theta}^{j,t})) = 1$ for all $\breve{h}^{j,t}$ with $\iota_t = 1$.

For simplicity, we assume that $\mu_t^j\left(h'^{j,t-1}\right), \mu_t^j\left(h''^{j,t-1}\right) > 0$. Let $\alpha = \mu_t^j\left(h'^{j,t-1}\right) / (\mu_t^j\left(h'^{j,t-1}\right) + \mu_t^j\left(h''^{j,t-1}\right))$ and define $\phi' : [0, \alpha] \to [0, 1]$ by $\phi'(z) = z/\alpha$ and $\phi'' : [\alpha, 1] \to [0, 1]$ by $\phi''(z) = (z - \alpha) / (1 - \alpha)$. Define a new reporting strategy and allocations $(\boldsymbol{\sigma}', \mathbf{u}', \mathbf{l}')$, for all $T \geq 0$, $h^{j,t-1} \in \left\{h'^{j,t-1}, h''^{j,t-1}\right\}$, $\iota_t = \iota_t' = \iota_t''$, and $\theta^{j,t+T}$, as

$$\begin{aligned}
\mathbf{u}'^j_{t+T}((h^{j,t-1}, \iota_t, z_t, m_t, ..., m_{t+T})) &= \mathbf{u}^j_{t+T}((h'^{j,t-1}, \iota_t, \phi'(z_t), m_t, ..., m_{t+T})), \\
\mathbf{l}'^j_{t+T}((h^{j,t-1}, \iota_t, z_t, m_t, ..., m_{t+T})) &= \mathbf{l}^j_{t+T}((h'^{j,t-1}, \iota_t, \phi'(z_t), m_t, ..., m_{t+T})), \\
\boldsymbol{\sigma}'^j_{t+T}(\cdot|(h^{j,t-1}, \iota_t, z_t, m_t, ..., z_{t+T}), \theta^{j,t+T}) &= \boldsymbol{\sigma}^j_{t+T}(\cdot|(h'^{j,t-1}, \iota_t, \phi'(z_t), m_t, ..., z_{t+T}), \theta^{j,t+T}),
\end{aligned}$$

if $z_t \leq \alpha$ and

$$\begin{aligned}
\mathbf{u}'^j_{t+T}((h^{j,t-1}, \iota_t, z_t, m_t, ..., m_{t+T})) &= \mathbf{u}^j_{t+T}((h''^{j,t-1}, \iota_t, \phi''(z_t), m_t, ..., m_{t+T})), \\
\mathbf{l}'^j_{t+T}((h^{j,t-1}, \iota_t, z_t, m_t, ..., m_{t+T})) &= \mathbf{l}^j_{t+T}((h''^{j,t-1}, \iota_t, \phi''(z_t), m_t, ..., m_{t+T})), \\
\boldsymbol{\sigma}'^j_{t+T}(\cdot|(h^{j,t-1}, \iota_t, z_t, m_t, ..., z_{t+T}), \theta^{j,t+T}) &= \boldsymbol{\sigma}^j_{t+T}(\cdot|(h''^{j,t-1}, \iota_t, \phi''(z_t), m_t, ..., z_{t+T}), \theta^{j,t+T}),
\end{aligned}$$

if $z_t > \alpha$ and $\mathbf{u}_s'^k = \mathbf{u}_s^k$, $\mathbf{l}_s'^k = \mathbf{l}_s^k$, $\boldsymbol{\sigma}_s'^k = \boldsymbol{\sigma}_s^k$ for all other histories, periods s, and $k \in J$. Agents with histories $h'^{j,t-1}, h''^{j,t-1}$ and $\iota_t' = \iota_t''$ could have replicated each other strategies after period t, so they must be indifferent between them. The strategy $\boldsymbol{\sigma}'^j$ gives them the same utility for all histories following $\{h'^{j,t-1}, h''^{j,t-1}\}$ and $\iota_t' = \iota_t''$ leaving all other histories unchanged, thus, it is incentive compatible, i.e. satisfies (C.4). The strategy $\boldsymbol{\sigma}'^j$ induces $\{\mu_t'^j\}_t$, which assigns the same probability to any realization of $\mathbf{u}_t^j$ and $\mathbf{l}_t^j$ as $\{\mu_t^j\}_t$, thus, the feasibility constraint (C.1) is satisfied. Therefore, $(\boldsymbol{\sigma}', \mathbf{u}', \mathbf{l}')$ is a PBE which is payoff equivalent to $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$. ■

Using Lemma C.3, we can greatly simplify the dependence of allocations and reporting strategies on the past. More specifically, without loss of generality, we can focus on strategy profiles $(\boldsymbol{\sigma}, \mathbf{u}, \mathbf{l})$ such that *(i)* the reporting strategy is independent of $\theta^{j,t-1}$ and, in addition, *(ii)* the expected payoff to an agent at any history $h^{j,t-1}$, $w_{t-1} \equiv \mathbb{E}_{\boldsymbol{\sigma}}^j \left[\sum_{s=t}^{\infty} \beta^{s-t} \mathbf{u}_s^j \middle| h^{j,t-1} \right]$, together with his employment status following that history, ι_t, are a sufficient statistic for the information contained in $\left(h^{j,t-1}, \iota_t\right)$.

We can use these results to rewrite problem (C.6) as follows. First, instead of maximizing over allocations and reporting strategies that depend on the random variable z_t, we can equivalently choose probability distributions over bundles of utility and reporting probability. Second, we can use Lemma C.3 and replace each agent's history with a single number, namely, the agent's continuation utility – his expected payoff – at that particular history.

The continuation utility of unemployed agents is bounded below by 0 since any such agent can always claim to be jobless and receive a period utility of at least 0. When jobs are persistent, since the agent's job status is observable, the continuation utility of an employed agent is bounded below by $\underline{U}/(1 - \tilde{\beta}(1-q))$, where $\tilde{\beta} \equiv (1-\delta)\beta$. For any n, let $\psi_{t,w}^{n,0}$, $t \geq n$, $w \in [0, \frac{\bar{U}}{1-\tilde{\beta}})$, be a probability distribution over the set

$$X^0 = \left\{ \left(u_H, u_L, l, \sigma, w_H^0, w_H^1, w_L\right) : \begin{array}{c} u_H + \tilde{\beta}\left[(1-q)\, w_H^1 + q w_H^0\right] \geq u_L + \beta(1-\delta)\, w_L, \\ (1-\sigma)\left[u_H + \tilde{\beta}\left[(1-q)\, w_H^1 + q w_H^0\right] - u_L - \beta(1-\delta)\, w_L\right] = 0 \end{array} \right\}$$

and let $\psi_{t,w}^{n,1}$, with $t \geq n$, $w \in [\frac{\underline{U}}{1-\tilde{\beta}(1-q)}, \frac{\bar{U}}{1-\tilde{\beta}})$, be a probability distribution over the set $X^1 = \{\left(u_H, l, w_H^0, w_H^1\right)\}$. We define $\psi_{t,w}^{o,0}$ and $\psi_{t,w}^{o,1}$, $t \geq 0$, analogously. Also, starting from some $\tilde{w} \in [0, \frac{\bar{U}}{1-\tilde{\beta}})$, let π_t^n, $t \geq n$, be the distribution defined by the following recursion:

$$\begin{aligned} \pi_n^n(w,0) &= (1-\delta)\int \left[(1-p\sigma)\,\mathcal{I}_{w_L \leq w} + qp\sigma \mathcal{I}_{w_H^0 \leq w}\right] d\psi_{t,\tilde{w}}^{n,0}, \qquad \text{(C.7)} \\ \pi_n^n(w,1) &= (1-\delta)(1-q)\int p\sigma \mathcal{I}_{w_H^1 \leq w}\, d\psi_{t,\tilde{w}}^{n,0}, \\ \pi_t^n(w,0) &= (1-\delta)\int\int \left[(1-p\sigma)\,\mathcal{I}_{w_L \leq w} + qp\sigma \mathcal{I}_{w_H^0 \leq w}\right] d\psi_{t,\tilde{w}}^{n,0} \pi_{t-1}^n(d\tilde{w},0) + \\ &\quad (1-\delta)\, q \int\int \mathcal{I}_{w_H^0 \leq w}\, d\psi_{t,\tilde{w}}^{n,1} \pi_{t-1}^n(d\tilde{w},1), \\ \pi_t^n(w,1) &= (1-\delta)(1-q)\int\int p\sigma \mathcal{I}_{w_H^1 \leq w}\, d\psi_{t,\tilde{w}}^{n,0} \pi_{t-1}^n(d\tilde{w},0) + \\ &\quad (1-\delta)(1-q)\int\int \mathcal{I}_{w_H^1 \leq w}\, d\psi_{t,\tilde{w}}^{n,1} \pi_{t-1}^n(d\tilde{w},1). \end{aligned}$$

We define π^o_t, $t \geq 0$, analogously, starting from the initial distribution π^o_{-1}. Finally, to make notation more compact, let

$$\begin{pmatrix} g^0\left(x^0\right) \\ f^0\left(x^0\right) \\ b^0\left(x^0\right) \end{pmatrix} \equiv \begin{pmatrix} p\sigma u_H + (1-p\sigma)\, u_L \\ p\sigma\left(C\left(u_H, l\right) - l\right) + (1-p\sigma)\, C\left(u_L, 0\right) \\ p\sigma \end{pmatrix}$$

and

$$\begin{pmatrix} g^1\left(x^1\right) \\ f^1\left(x^1\right) \\ b^1\left(x^1\right) \end{pmatrix} \equiv \begin{pmatrix} u_H \\ C\left(u_H, l\right) - l \\ 1 \end{pmatrix}$$

for $x^0 \in X^0$ and $x^1 \in X^1$.

We can then rewrite constraint (C.1) as

$$\sum_{j\in J} \omega_{t,j} \int\int f^\iota\left(x^\iota\right) d\psi^{j,\iota}_{t,w} \pi^j_{t-1} \leq 0 \text{ for all } t. \tag{C.8}$$

Similarly, with a slight abuse of notation, we can rewrite the value of the government's best deviation (C.2) as

$$\begin{aligned} \tilde{W}(\{\psi^{j,\iota}_{t,w}\}_{w,j,\iota}, \{\pi^j_{t-1}\}_j) = \max_{\tilde{u}_H, \tilde{u}_L, \tilde{l}, \hat{u}_H, \hat{l}} \quad & \sum_{j\in J} \omega_{t,j} \int\int \left[p\sigma\left(x^0\right)\tilde{u}_H + \left(1 - p\sigma\left(x^0\right)\right)\tilde{u}_L\right] d\psi^{j,0}_{t,w} \pi^j_{t-1}\left(dw, 0\right) + \\ & \sum_{j\in J} \omega_{t,j} \int\int \hat{u}_H d\psi^{j,1}_{t,w} \pi^j_{t-1}\left(dw, 1\right) + \beta \underline{V}\left(N^{\boldsymbol{\sigma}}_{t+1}\right), \end{aligned} \tag{C.9}$$

subject to $(\tilde{u}_H, \tilde{u}_L, \tilde{l}) \in \mathcal{C}$, $(\hat{u}_H, \hat{l}) \in dom(C)$ and

$$\begin{aligned} & \sum_{j\in J} \omega_{t,j} \int\int \left[p\sigma\left(x^0\right)\left(C\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l}\right) + \left(1 - p\sigma\left(x^0\right)\right) C\left(\tilde{u}_L, 0\right)\right] d\psi^{j,0}_{t,w} \pi^j_{t-1}\left(dw, 0\right) + \\ & \sum_{j\in J} \omega_{t,j} \int\int \left(C\left(\hat{u}_H, \hat{l}\right) - \hat{l}\right) d\psi^{j,1}_{t,w} \pi^j_{t-1}\left(dw, 1\right) \leq 0. \end{aligned}$$

Therefore, problem (C.6) is equivalent to

$$\max_{\{\psi^{j,\iota}_{t,w}, \pi^j_{t-1}\}_{j,\iota,t,w}} \sum_{j\in J} \varphi_j \sum_{t=0}^{\infty} \omega_{t,j} \beta^{t-n\mathcal{I}_{j=n}} \int\int g^\iota\left(x^\iota\right) d\psi^{j,\iota}_{t,w} d\pi^j_{t-1}, \tag{C.10}$$

subject to (C.7), (C.8),

$$\sum_{j\in J} \omega_{t,j} \sum_{s=t}^{\infty} \beta^{s-t} \int\int g^\iota\left(x^\iota\right) d\psi^{j,\iota}_{s,w} d\pi^j_{s-1} \geq \tilde{W}(\{\psi^{j,\iota}_{t,w}\}_{w,j,\iota}, \{\pi^j_{t-1}\}_j) \text{ for all } t \tag{C.11}$$

and the "promise-keeping" constraints

$$\begin{aligned} w &= \int \left\{p\sigma\left(u_H + \tilde{\beta}\left[\left(1-q\right) w^1_H + q w^0_H\right]\right) + (1-p\sigma)\left(u_L + \tilde{\beta} w_L\right)\right\} d\psi^{j,0}_{t,w}, \quad \text{(C.12)} \\ w &= \int \left(u_H + \tilde{\beta}\left[\left(1-q\right) w^1_H + q w^0_H\right]\right) d\psi^{j,1}_{t,w}, \end{aligned}$$

for all t, w, j.

We now construct an uppoer bound on $\tilde{W}$ which will be key to obtain a recursive representation of problem (C.10).

Lemma C.4 *Let $\{\psi_{t,w}^{j,\iota *}\}$ be a best PBE. The value of the best deviation $\tilde{W}$ at any history depends only on the fraction of people employed at that history.*

Proof. By Lemma C.2, the value of a worst equilibrium is a function only of the fraction of people employed at any given history. Following the same steps in the proof of that lemma, it is immediate to show that the value of the best deviation, given by (C.9), has the same property. ■

Lemma C.4 allows us to replace the sustainability constraint (C.11) with the following constraints:

$$\begin{aligned} \sum_{j\in J}\omega_{t,j}\sum_{s=t}^{\infty}\beta^{s-t}\int\int g^{\iota}\left(x^{\iota}\right)d\psi_{s,w}^{j,\iota}d\pi_{s-1}^{j} &\geq \tilde{W}\left(N_t\right), \\ \sum_{j\in J}\omega_{t,j}\int\int b^{\iota}\left(x^{\iota}\right)d\psi_{t,w}^{j,\iota}d\pi_{t-1}^{j} &\leq N_t, \end{aligned} \tag{C.13}$$

for all t, where, by Lemma C.4 and with a slight abuse of notation, we used $\tilde{W}(N_t)$ to denote the value of the best deviation. Then, any solution to problem (C.10) where (C.11) is replaced with (C.13)—which we refer to as "the modified problem"—is a best PBE.

The modified problem can be written recursively using the techniques developed in Farhi and Werning (2007). More specifically, Since the modified problem is convex, we can define the Lagrangian (we omit the arguments of the functions to make notation more compact)

$$\mathcal{L}=\max_{\{\psi_{t,w}^{j,\iota},\pi_{t-1}^{j}\}_{j,\iota,t,w}}\sum_{t=0}^{\infty}\sum_{j\in J}\bar{\beta}_{t,j}\int\int\left[g^{\iota}-\zeta_{t,j}f^{\iota}-\chi_{t,j}b^{\iota}\right]d\psi_{t,w}^{j,\iota}\pi_{t-1}^{j},$$

subject to (C.7) and (C.12), where $\bar{\beta}_{t,j}=\omega_{t,j}\beta^{t}\left(\varphi_j+\sum_{s=n\mathcal{I}_{j=n}}^{t}\mu_s^*\right)$, $\zeta_{t,j}=\beta^t\zeta_t^*/\bar{\beta}_{t,j}$, and $\chi_{t,j}=\beta^t\chi_t^*/\bar{\beta}_{t,j}$, $j\in J$, for some multipliers $\{\beta^t\zeta_t^*\}_t$ and $\{\beta^t\mu_t^*,\beta^t\chi_t^*\}_t$ associated to the constraints (C.8) and (C.13), respectively. We must have $\chi_t\geq 0$, $\zeta_t>0$, and $\bar{\beta}_{t,j}/\bar{\beta}_{t-1,j}\geq\beta$, with equality if and only if $\mu_t^*=0$.

By standard arguments (see Theorem 1, p. 217 in Luenberger (1969)), any solution to the modified problem, and therefore any best PBE, must be a solution to this Lagrangian problem. The converse does not need to be true, as there might be some solutions to the Lagrangian problem that do not satisfy constraints (C.8) and (C.13). However, any solution to the Lagrangian problem that also satisfies (C.8) and (C.13) is a PBE.

The problem above takes as given the initial distribution π_{-1}^{o} of entitlements and employment statuses of the initial old. Also, for any $t>0$, the distributions $\{\pi_{t-1}^{j}\}_j$ are a sufficient statistic for the remaining problem. We now focus on *steady states* of the economy, which are defined as collections of distributions and Lagrange multipliers $\{\psi_{t,w}^{j,\iota},\pi_{t-1}^{j},\bar{\beta}_{t,j},\zeta_{t,j},\chi_{t,j}\}$, which *(i)* solve the Lagrangian above, *(ii)* satisfy (C.8) and (C.13), and are such that *(iii)* $\psi_{t,w}^{j,\iota}$ is independent of t and j, the distributions $\sum_j \omega_{t,j}\pi_{t-1}^j$ are independent of t, and $\bar{\beta}_{t,j}/\bar{\beta}_{t-1,j} = \bar{\beta}$, $\zeta_{t,j} = \bar{\beta}^t\zeta$, $\chi_{t,j} = \bar{\beta}^t\chi$, for some $\zeta > 0$, $\chi \geq 0$, $\bar{\beta} \geq \beta$.

Suppose the economy is in a steady state and define the value function of an agent who is currently unemployed ($\iota = 0$) or employed ($\iota = 1$) as

$$K(\tilde{w},\iota) \equiv \max_{\{\psi_w^\iota,\pi_{t-1}\}_{\iota,w,t}} \int \left[g^\iota - \zeta f^\iota - \chi b^\iota\right] d\psi_{\tilde{w}}^\iota + \sum_{t=1}^{\infty}\bar{\beta}^t \int\int \left[g^{\hat{\imath}} - \zeta f^{\hat{\imath}} - \chi b^{\hat{\imath}}\right] d\psi_w^{\hat{\imath}} d\pi_{t-1} \tag{C.14}$$

subject to (C.7) and (C.12). Clearly, $\mathcal{L} = \max_w \int \bar{\beta}_{0,o} K(\tilde{w},0)\, d\pi_{-1}^o + \bar{\beta}_{0,0} K(w,1)$.

Lemma C.5 *The functions $K(\cdot,\iota)$, $\iota \in \{0,1\}$, satisfy the recursion*

$$K(\tilde{w},\iota) = \max_{\psi_{\tilde{w}}^\iota \text{ s.t. (C.12)}} \int \left[g^\iota - \zeta f^\iota - \chi b^\iota + \hat{\beta}\bar{K}^\iota\right] d\psi_{\tilde{w}}^\iota,$$

for all $\tilde{w} \in [0, \frac{\bar{U}}{1-\bar{\beta}})$, when $\iota = 0$, and $\tilde{w} \in [\frac{U}{1-\bar{\beta}(1-q)}, \frac{\bar{U}}{1-\bar{\beta}})$, when $\iota = 1$, where $\hat{\beta} \equiv \bar{\beta}(1-\delta)$ and where $\bar{K}^0(x^0) \equiv p\sigma\left[(1-q)K(w_H^1,1) + qK(w_H^0,0)\right] + (1-p\sigma)K(w_L,0)$ and $\bar{K}^1(x^1) \equiv qK(w_H^0,0) + (1-q)K(w_H^1,1)$, respectively.

Proof. We prove the statement for $K(\tilde{w},0)$, the same arguments apply to $K(\tilde{w},1)$. We can rewrite (C.14), with $\iota = 0$, as follows:

$$\begin{aligned}
&\max_{\{\psi_w^\iota,\pi_{t-1}\}_{\iota,w,t} \text{ s.t. (C.7), (C.12)}} \int\int \left[g^0 - \zeta f^0 - \chi b^0\right] d\psi_{\tilde{w}}^0 + \\
&\qquad \int \sum_{t=1}^{\infty}\bar{\beta}^t \int\int \left[g^{\hat{\imath}} - \zeta f^{\hat{\imath}} - \chi b^{\hat{\imath}}\right] d\psi_w^{\hat{\imath}} d\pi_{t-1} \\
&= \max_{\psi_{\tilde{w}}^0 \text{ s.t. (C.12)}} \int \left[g^0 - \zeta f^0 - \chi b^0\right] d\psi_{\tilde{w}}^0 + \\
&\qquad \max_{\{\psi_w^\iota,\pi_{t-1}\}_{\iota,w,t\geq 1} \text{ s.t. (C.7), (C.12)}} \int \sum_{t=1}^{\infty}\bar{\beta}^t \int\int \left[g^{\hat{\imath}} - \zeta f^{\hat{\imath}} - \chi b^{\hat{\imath}}\right] d\psi_w^{\tilde{\imath}} d\pi_{t-1} \\
&= \max_{\psi_{\tilde{w}}^0 \text{ s.t. (C.12)}} \int \left[g^0 - \zeta f^0 - \chi b^0\right] d\psi_{\tilde{w}}^0 + \\
&\qquad \bar{\beta} \max_{\{\psi_w^\iota,\pi_{t-1}\}_{\iota,w,t\geq 0} \text{ s.t. (C.7), (C.12)}} \int \sum_{t=0}^{\infty}\bar{\beta}^t \int\int \left[g^{\hat{\imath}} - \zeta f^{\hat{\imath}} - \chi b^{\hat{\imath}}\right] d\psi_w^{\tilde{\imath}} d\pi_t.
\end{aligned}$$

Finally, using (C.7) and the definition of $K(w,\iota)$, the latter can be rewritten as

$$\max_{\psi_{\tilde{w}}^0 \text{ s.t. (C.12)}} \int \left[g^0 - \zeta f^0 - \chi b^0 + \hat{\beta}\bar{K}^0\right] d\psi_{\tilde{w}}^0.$$

■

4 Proofs for Section 4.1

In this section we prove Proposition 3 in the main text, which we report here for convenience.

Proposition C.1 *The best equilibrium payoff in the steady state can be achieved with reporting strategies σ_w^* such that $\sigma_w^*(z) \in \{0,1\}$ for all z, and information revelation is decreasing in w, i.e.* $\Pr(\sigma_w^* = 1)$ *is decreasing in w.*

We prove this proposition in three steps: (1) we define a value function $\kappa(v, \sigma, 0)$ for the unemployed agent; (2) we guess that K is submodular and show that the steady state payoff can be achieved with $\sigma_w^* \in \{0,1\}$ with probability 1 and that information revelation is decreasing in w; (3) we verify that K is submodular.

In the i.i.d. case, step (2) followed directly from $C_{12} > 0$. When jobs are persistent, however, we also need to prove that the value function (C.14) is submodular, that is, $K_1(w,1) \leq K_1(w,0)$ for all $w > 0$.

Step (1)

Consider the problem of the unemployed agent. As in Section 3 in the main text, we break this problem in two parts. First, for given σ, we choose allocations $\left(u_H, u_L, w_H^1, w_H^0, w_L\right)$ so as to maximize

$$\begin{aligned}\kappa(v,\sigma,0) = \max_{u_H,u_L,l,w_H^1,w_H^0,w_L} \quad & p\sigma\left[u_H - \zeta\left(C(u_H,l) - l\right) + \hat{\beta}\left((1-q)K\left(w_H^1,1\right) + qK\left(w_H^0,0\right)\right)\right] + \\ & (1-p\sigma)\left[u_L - \zeta C(u_L,0) + \hat{\beta}K(w_L,0)\right], \end{aligned} \tag{C.15}$$

subject to $(u_H, u_L, l) \in \mathcal{C}$, $w_H^0, w_L \in dom(K(\cdot,0))$, $w_H^1 \in dom(K(\cdot,1))$ and

$$\begin{aligned} \{u_H + \tilde{\beta}\left[(1-q)w_H^1 + qw_H^0\right]\} - \{u_L + \tilde{\beta}w_L\} &\geq 0, \\ (1-\sigma)\left[\{u_H + \tilde{\beta}\left[(1-q)w_H^1 + qw_H^0\right] - \{u_L + \tilde{\beta}w_L\}\right] &= 0, \\ p\sigma\{u_H + \tilde{\beta}\left[(1-q)w_H^1 + qw_H^0\right]\} + (1-p\sigma)\{u_L + \tilde{\beta}w_L\} &= v. \end{aligned}$$

Then, we find the optimal reporting strategy:

$$k(v,0) = \max_{\sigma} \kappa(v,\sigma,0) - \chi p\sigma.$$

Finally, the value function $K(\cdot,0)$ is given by the convex envelope of k: $K(\cdot,0) = co(k(\cdot,0))$.

For the employed agent, we only need to choose allocations, thus,

$$\kappa(v,1) = \max_{u_H,l,w_H^1,w_H^0} u_H - \zeta\left(C(u_H,l) - l\right) + \hat{\beta}\left((1-q)K\left(w_H^1,1\right) + qK\left(w_H^0,0\right)\right), \tag{C.16}$$

subject to $(u_H, l) \in dom(C)$, $w_H^0 \in dom(K(\cdot,0))$, $w_H^1 \in dom(K(\cdot,1))$ and

$$u_H + \tilde{\beta}\left[(1-q)w_H^1 + qw_H^0\right] = v,$$

therefore,

$$K(v,1) = \kappa(v,1) - \chi.$$

Step (2)

We prove the result with a series of intermediate lemmas. In what follows, at $w = 0$, $K_1(w, 0)$ denotes the left derivative of $K(w, 0)$.

Lemma C.6 *The functions* $\{K(\cdot, \iota)\}_\iota$ *are concave and continuously differentiable, with* $\lim_{v \to \bar{U}/(1-\tilde{\beta})} K_1(v, \iota) = -\infty$, $\iota \in \{0, 1\}$.

Proof. Concavity of $K(\cdot, \iota)$, $\iota \in \{0, 1\}$, follows from standard arguments on the sequence problems (C.14). Also, the proof that $K(v, \iota)$, $\iota \in \{0, 1\}$, is continuously differentiable is analogous to the proof of Lemma 5 in the main text.

We focus on $K(v, 0)$, the case with $K(v, 1)$ is analogous. Let $m \equiv \max_{(u,l) \in dom(C)} \{u - \zeta(C(u, l) - l)\}$ and define the value function

$$\bar{K}(v, \hat{\beta}) \equiv \max_{\{u_t, l_t\}_t} \mathbb{E} \sum_{t=0}^{\infty} \hat{\beta}^t \left[u_t - \zeta(C(u_t, l_t) - l_t) - m\right],$$

subject to $(u_t, l_t) \in dom(C)$, for $t \geq 0$, and

$$\mathbb{E} \sum_{t=0}^{\infty} \tilde{\beta}^t u_t = v,$$

where $u_t : \{\theta_H, \theta_L\}^t \times \{0, 1\}^t \to [0, \bar{U})$, for all t, and where the expectation is over histories of job opportunities θ^t and job statuses ι^t, starting from $\iota_0 = 0$. Function $\bar{K}(v, \hat{\beta})$ is differentiable with $\bar{K}(v, \hat{\beta}) + const \geq K(v, 0)$. Also, since the argument of the sum is non-positive, $\bar{K}(v, \tilde{\beta}) \geq \bar{K}(v, \hat{\beta})$. Problem $\bar{K}(v, \tilde{\beta})$ is the value of a standard allocation problem whose solution $\{u_t^*, l_t^*\}_t$ depends only on whether the agent is employed or not. In particular, an employed agent receives (u_H^*, l_H^*) while an unemployed agent receives (u_L^*). These values satisfy the first order conditions

$$\begin{aligned} C_2(u_H^*, l_H^*) - 1 &= 0, \\ 1 - \zeta C_1\left(u_j^*, l_j^*\right) &= \gamma_v, \ j \in \{H, L\}, \end{aligned}$$

with $l_L^* = 0$, for some Lagrange multiplier γ_v. As $v \to \frac{\bar{U}}{1-\tilde{\beta}}$, we have that u_H^*, $u_L^* \to \bar{U}$ and, thus, $\gamma_v \to -\infty$. By the envelope theorem, $\bar{K}_1(v, \tilde{\beta}) = \gamma_v \to -\infty$. Also, $\bar{K}(v, \tilde{\beta}) \to -\infty$, which implies $K(v, 0) \to -\infty$. Finally, the latter two results imply $K_1(v, 0) \to -\infty$. ■

Lemma C.7 *Suppose* $K_1(w, 1) \leq K_1(w, 0)$ *for all* $w \geq 0$, *then the incentive compatibility constraint holds with equality.*

Proof. Let's consider a constrained problem where we add an additional constraint

$$u_H + \tilde{\beta}\left[(1 - q) w_H^1 + q w_H^0\right] \leq u_L + \tilde{\beta} w_L.$$

We want to show that this constraint is always slack. Suppose it is not slack, which can be only for $\sigma = 1$. By Lemma C.6, the problem is convex, thus, we can set up the Lagrangian

$$\begin{aligned}\mathcal{L} = \max_{\substack{u_H,u_L,l,\\ w_H^1,w_H^0,w_L}} \quad & p\left[u_H - \zeta\left(C\left(u_H,l\right)-l\right)+\hat{\beta}\left(\left(1-q\right)K\left(w_H^1,1\right)+qK\left(w_H^0,0\right)\right)\right]+\\ & \left(1-p\right)\left[u_L-\zeta C\left(u_L,0\right)+\hat{\beta}K\left(w_L,0\right)\right]+\\ & \mu_v(\{u_H+\tilde{\beta}\left[\left(1-q\right)w_H^1+qw_H^0\right]\}-\{u_L+\tilde{\beta}w_L\})-\\ & \gamma_v(p\{u_H+\tilde{\beta}\left[\left(1-q\right)w_H^1+qw_H^0\right]\}+\left(1-p\right)\{u_L+\tilde{\beta}w_L\}-v),\end{aligned}$$

subject to $(u_H, u_L, l) \in \mathcal{C}$, $w_H^0, w_L \in dom(K(\cdot, 0))$, $w_H^1 \in dom(K(\cdot, 1))$, for some multipliers $\mu_v < 0$ and γ_v. The first-order conditions w.r.t. u_H, u_L, w_H^1, w_H^0, and w_L are, respectively,

$$\begin{aligned} p\left[1-\zeta C_1\left(u_{H,v},l_v\right)\right]+\mu_v &\leq \gamma_v p,\\ \left(1-p\right)\left[1-\zeta C_1\left(u_{L,v},0\right)\right]-\mu_v &\leq \gamma_v\left(1-p\right),\\ p\hat{\beta}K_1\left(w_{H,v}^1,1\right)+\tilde{\beta}\mu_v &\leq \gamma_v\tilde{\beta}p,\\ p\hat{\beta}K_1\left(w_{H,v}^0,0\right)+\tilde{\beta}\mu_v &\leq \gamma_v\tilde{\beta}p,\\ \left(1-p\right)\hat{\beta}K_1\left(w_{L,v},0\right)-\tilde{\beta}\mu_v &\leq \gamma_v\tilde{\beta}\left(1-p\right),\end{aligned}$$

with equality if the solution is interior. They can be re-arranged as

$$\begin{aligned} 1-\zeta C_1\left(u_{H,v},l_v\right)+\frac{\mu_v}{p} &\leq \gamma_v,\\ 1-\zeta C_1\left(u_{L,v},0\right)-\frac{\mu_v}{1-p} &\leq \gamma_v,\\ \frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{H,v}^1,1\right)+\frac{\mu_v}{p} &\leq \gamma_v,\\ \frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{H,v}^0,0\right)+\frac{\mu_v}{p} &\leq \gamma_v,\\ \frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{L,v},0\right)-\frac{\mu_v}{1-p} &\leq \gamma_v.\end{aligned}$$

We assume that $w_{H,v}^1$, $u_{H,v}$ are interior, the other cases are immediate. If $\mu_v < 0$, then,

$$\begin{aligned} K_1\left(w_{H,v}^1,1\right) &> K_1\left(w_{L,v},0\right)\geq K_1\left(w_{L,v},1\right),\\ C_1\left(u_{H,v},l_v\right) &< C_1\left(u_{L,v},0\right).\end{aligned}$$

Thus, $w_{H,v}^1 < w_{L,v}$ and $u_{H,v} < u_{L,v}$. The incentive constraint then implies $w_{H,v}^0 > w_{L,v}$. Thus, $w_{H,v}^0$ is interior and its first order condition holds with equality. Also, by concavity,

$$K_1\left(w_{L,v},0\right)\geq K_1\left(w_{H,v}^0,0\right)$$

or

$$\frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{L,v},0\right)-\frac{\mu_v}{1-p} > \frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{H,v}^0,0\right)+\frac{\mu_v}{p},$$

which violates the first order conditions. Therefore, we must have $\mu_v = 0$. ■

Using Lemma C.7, we can rewrite $\kappa(v,\sigma,0)$ as

$$\kappa(v,\sigma,0) = p\sigma F^H(v) + (1-p\sigma) F^L(v),$$

where

$$F^H(v) \equiv \max_{u_H,l,w_H^1,w_H^0} u_H - \zeta(C(u_H,l)-l) + \hat{\beta}\left((1-q)K\left(w_H^1,1\right) + qK\left(w_H^0,0\right)\right),$$

subject to $v = u_H + \tilde{\beta}[(1-q)w_H^1 + qw_H^0]$, $(u_H,l) \in dom(C)$, $w_H^0 \in dom(K(\cdot,0))$ and $w_H^1 \in dom(K(\cdot,1))$, and

$$F^L(v) \equiv \max_{u_L,w_L} u_L - \zeta C(u_L,0) + \hat{\beta}K(w_L,0),$$

subject to $v = u_L + \tilde{\beta}w_L$, $(u_L,0) \in dom(C)$ and $w_L \in dom(K(\cdot,0))$.

Lemma C.8 *Suppose $K_1(w,1) \leq K_1(w,0)$ for all $w \geq 0$, then $\kappa(v,\cdot,0)$ is linear for all $v \geq 0$.*

Proof. The statement follows immediately from the equation above. ■

Lemma C.9 *Suppose $K_1(w,1) \leq K_1(w,0)$, for all $w \geq 0$, then $F_1^H(v) \leq F_1^L(v)$ for all $v \geq 0$.*

Proof. Since the maximization problem defining $F^j(v)$, $j \in \{H,L\}$, is convex, there exist Lagrange multipliers γ_v^j, $j \in \{H,L\}$, such that the solutions $(u_{H,v}, l_v, w_{H,v}^0, w_{H,v}^1)$ and $(u_{L,v}, w_{L,v})$ satisfy the first-order conditions

$$\begin{aligned}
1-\zeta C_1(u_{H,v},l_v) &\leq \gamma_v^H, \\
1-\zeta C_1(u_{L,v},0) &\leq \gamma_v^H, \\
\frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{L,v},0) &\leq \gamma_v^L, \\
\frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{H,v}^0,0\right) &\leq \gamma_v^H, \\
\frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{H,v}^1,1\right) &\leq \gamma_v^H.
\end{aligned}$$

In addition, standard Benveniste-Scheinkman arguments prove that $F^j(v)$, $j \in \{H,L\}$, is differentiable and $F_1^j(v) = \gamma_v^j$, $j \in \{H,L\}$.

Suppose $\gamma_v^H > \gamma_v^L$ for some v. We assume that $w_{H,v}^1$, $u_{H,v}$ are interior, the other cases are immediate. Then, *(i)* $K_1(w_{H,v}^1,1) > K_1(w_{L,v},0) \geq K_1(w_{L,v},1)$, which implies $w_{H,v}^1 < w_{L,v}$ by concavity of $K(\cdot,1)$; *(ii)* $C_1(u_{H,v},l_v) \leq C_1(u_{L,v},0) < C_1(u_{L,v},l_v)$ by $C_{12} > 0$ and $l_v > 0$, which implies $u_{H,v} < u_{L,v}$ by convexity of $C(\cdot,l)$. The constraints then imply $w_{H,v}^0 > w_{L,v}$. Thus, $w_{H,v}^0$ is interior and its first order condition holds with equality. As a result

$$\frac{\hat{\beta}}{\tilde{\beta}}K_1\left(w_{H,v}^0,0\right) = \gamma_v^H > \gamma_v^L \geq \frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{L,v},0)$$

and, by concavity of $K(\cdot,0)$, $w^0_{H,v} < w_{L,v}$, which is a contradiction. Therefore, $\gamma^H_v \leq \gamma^L_v$, for all $v \geq 0$. ■

Lemma C.10 *Suppose $K_1(w,1) \leq K_1(w,0)$ for all $w \geq 0$, then (a) $\kappa_1(v,1) \leq \kappa_1(v,\sigma,0)$ and (b) $\kappa_{12}(v,\sigma,0) \leq 0$ for all $v \geq 0$ and σ.*

Proof. *(a)* Since $F^H_1(v) \leq F^L_1(v)$ for $v \geq 0$, by Lemma C.9, we have $\kappa_1(v,1) = F^H_1(v) \leq p\sigma F^H_1(v) + (1-p\sigma) F^L_1(v) = \kappa_1(v,\sigma,0)$.

(b) By Lemma C.8, $\kappa(v,\cdot,0)$ is linear and, thus, differentiable with $\kappa_2(v,\sigma,0) = p\left[F^H(v) - F^L(v)\right]$. Therefore, by Lemma C.9, $\kappa_{12}(v,\sigma,0) = p\left[F^H_1(v) - F^L_1(v)\right] \leq 0$. ■

By part *(b)* of Lemma C.10, $\kappa_1(v,0,0) \geq \kappa_1(v,1,0)$, thus, we can follow the same steps in Proposition 2 in the main text to prove the statement of Proposition C.1.

Step (3)

We are left to verify that $K_1(w,1) \leq K_1(w,0)$ for all $w \geq 0$. We will use the contraction mapping theorem. More specifically, we let T^0 and T^1 be two operators that map next period's value functions $K(\cdot,0)$ and $K(\cdot,1)$ into current period's functions $\tilde{K}(\cdot,0)$ and $\tilde{K}(\cdot,1)$. These operators are defined by the problems (C.15) and (C.16). The next lemma shows that T^0 and T^1 map submodular functions – i.e. functions K that satisfy $K_1(w,1) \leq K_1(w,0)$ for all $w \geq 0$ – into functions that satisfy the same property. Since the set of submodular functions is closed, the contraction mapping theorem then implies that the value functions are submodular.

Lemma C.11 *Suppose $K_1(w,1) \leq K_1(w,0)$ for all $w \geq 0$ and let $\tilde{K}(\cdot,\iota) = T^i(K(\cdot,\iota))$, $\iota \in \{0,1\}$, then $\tilde{K}_1(w,1) \leq \tilde{K}_1(w,0)$ for all $w \geq 0$.*

Proof. Arguments in step (2) imply that $\tilde{K}(\cdot,0)$ is the concave envelope of

$$\max\{\kappa(v,1,0) - \chi p, \kappa(v,0,0)\}.$$

In particular, by part (b) of Lemma C.10, there are thresholds v' and v'' such that *(i)* $\tilde{K}(w,0) = \kappa(w,1,0) - \chi p$, for $w < v'$; *(ii)* $\tilde{K}(w,0) = \kappa(w,0,0)$, for $w > v''$; and *(iii)* $\tilde{K}(w,0)$ is obtained by randomization between $\kappa(v',1,0) - \chi p$ and $\kappa(v'',0,0)$, for $v' \leq w \leq v''$. In the latter case, we also have $\tilde{K}_1(w,0) = \kappa_1(v',1,0) = \kappa_1(v'',0,0)$.

Suppose w is in the first region. Then, by part (a) of Lemma C.10,

$$\tilde{K}_1(w,1) = \kappa_1(w,1) \leq \kappa_1(w,1,0) = \tilde{K}_1(w,0).$$

The same argument applies for the second region. Finally, suppose w is in the intermediate region. Then, by concavity of $\kappa(\cdot,1)$ and part (a) of Lemma C.10,

$$\tilde{K}_1(w,1) = \kappa_1(w,1) \leq \kappa_1\left(v',1\right) \leq \kappa_1\left(v',1,0\right) = \tilde{K}_1(w,0).$$

■

5 Proofs for Section 4.2

In this section, we consider the special case with $q = 0$. When the sustainability constraint is slack, i.e. $\chi = 0$, this economy corresponds to a stationary version of Hopenhayn and Nicolini (1997). It is immediate to adapt the value functions (C.15) and (C.16). First, for the unemployed,

$$\begin{aligned}\kappa(v,\sigma,0) = \max_{u_H,u_L,l,w_H,w_L} \quad & p\sigma\left[u_H - \zeta\left(C(u_H,l)-l\right) + \hat{\beta}K(w_H,1)\right] + \\ & (1-p\sigma)\left[u_L - \zeta C(u_L,0) + \hat{\beta}K(w_L,0)\right],\end{aligned} \tag{C.17}$$

subject to $(u_H, u_L, l) \in \mathcal{C}$, $w_H \in dom(K(\cdot,1))$, $w_L \in dom(K(\cdot,0))$ and

$$\begin{aligned}(u_H + \tilde{\beta}w_H) - (u_L + \tilde{\beta}w_L) &\geq 0,\\ (1-\sigma)\left[(u_H + \tilde{\beta}w_H) - (u_L + \tilde{\beta}w_L)\right] &= 0,\\ p\sigma(u_H + \tilde{\beta}w_H) + (1-p\sigma)(u_L + \tilde{\beta}w_L) &= v.\end{aligned}$$

Also, the optimal choice of σ satisfies

$$k(v,0) = \max_{\sigma} \kappa(v,\sigma,0) - \chi p\sigma.$$

Finally, the value function $K(\cdot,0)$ is given by $K(\cdot,0) = co(k(\cdot,0))$.

For the employed agent, we have

$$\kappa(v,1) = \max_{u_H,l,w_H} u_H - \zeta\left(C(u_H,l)-l\right) + \hat{\beta}K(w_H,1), \tag{C.18}$$

subject to $(u_H, l) \in dom(C)$, $w_H \in dom(K(\cdot,1))$ and

$$u_H + \tilde{\beta}w_H = v,$$

thus,

$$K(v,1) = \kappa(v,1) - \chi.$$

Under the assumption that $q = 0$ we can strengthen many of the results of Section 4. In particular, in Lemma C.9 the inequality is strict: $F_1^H(v) < F_1^L(v)$, for all $v > 0$. In turn, the latter immediately implies that the inequalities in Lemma C.10 are also strict: $\kappa_1(v,1) < \kappa_1(v,\sigma,0)$ and $\kappa_{12}(v,\sigma,0) < 0$, for all $v > 0$ and σ. Finally, Lemma C.11 implies that $\tilde{K}_1(w,1) < \tilde{K}_1(w,0)$, for all $w > 0$.

We begin with the benchmark in which the government can commit.

5.1 Case with commitment (Hopenhayn and Nicolini (1997))

We start with case in which the sustainability constraint is slack, that is, the case with $\chi = 0$. As a result, $\hat{\beta} = \tilde{\beta}$. In addition, the optimal reporting strategy is $\sigma = 1$. Therefore, if we let $\tilde{K} \equiv (K - v)/\zeta$, we can rewrite the value functions above as

$$\tilde{K}(v,0) = \max_{u_H,u_L,l,w_H,w_L} p\left[-\left(C(u_H,l)-l\right) + \tilde{\beta}\tilde{K}(w_H,1)\right] + (1-p)\left[-C(u_L,0) + \tilde{\beta}\tilde{K}(w_L,0)\right], \tag{C.19}$$

subject to $(u_H, u_L, l) \in \mathcal{C}$, $w_H \in dom(\tilde{K}(\cdot, 1))$, $w_L \in dom(\tilde{K}(\cdot, 0))$ and

$$\begin{aligned} (u_H + \tilde{\beta} w_H) - (u_L + \tilde{\beta} w_L) &\geq 0, \\ p(u_H + \tilde{\beta} w_H) + (1-p)(u_L + \tilde{\beta} w_L) &= v. \end{aligned}$$

Similarly,

$$\tilde{K}(v, 1) = \max_{u_H, l, w_H} -(C(u_H, l) - l) + \tilde{\beta} \tilde{K}(w_H, 1), \tag{C.20}$$

subject to $(u_H, l) \in dom(C)$, $w_H \in dom(\tilde{K}(\cdot, 1))$ and

$$u_H + \tilde{\beta} w_H = v.$$

Let $(u_{H,v}, u_{L,v}, l_v, w_{H,v}, w_{L,v})$ denote the optimal allocations in (C.19).

Lemma C.12 *Suppose $\chi = 0$. The incentive constraint binds for all v. In addition, $w_{L,v} < v = w_{H,v}$ and $u_{H,v} = (1-\tilde{\beta})v < u_{L,v}$ for all $v > 0$.*

Proof. Notice that Lemma C.6 does not depend on χ, thus, it still holds when $\chi = 0$. Standard arguments prove that $\tilde{K}(\cdot, \iota)$, $\iota \in \{0, 1\}$, is strictly concave. We show that the incentive constraint binds. Since the problem is convex, there exist two Lagrange multipliers, γ_v and $\mu_v \geq 0$, such that the solution to (C.19) satisfies the first order conditions (we assume that $w_{H,v}$ and $u_{H,v}$ are interior, the other cases are immediate)

$$\begin{aligned} -\zeta C_1(u_{H,v}, l_v) + \frac{\mu_v}{p} &= \gamma_v, \\ -\zeta C_1(u_{L,v}, 0) - \frac{\mu_v}{1-p} &\leq \gamma_v, \\ K_1(w_{H,v}, 1) + \frac{\mu_v}{p} &= \gamma_v, \\ K_1(w_{L,v}, 0) - \frac{\mu_v}{1-p} &\leq \gamma_v. \end{aligned}$$

Suppose $\mu_v = 0$ for some $v \geq 0$. Then, using $K_1(w, 1) \leq K_1(w, 0)$, $l_v > 0$, $C_{12} > 0$ and Lemma C.6, the conditions above imply $w_{H,v} \leq w_{L,v}$ and $u_{H,v} < u_{L,v}$, which violate the incentive constraint. Therefore, $\mu_v > 0$ for all $v \geq 0$.

Since the incentive constraint holds with equality, we can replace the two constraints in problem (C.19) with

$$\begin{aligned} u_H + \tilde{\beta} w_H &= v, \\ u_L + \tilde{\beta} w_L &= v. \end{aligned}$$

As a result, we can use problem (C.20) to rewrite problem (C.19) as follows:

$$\tilde{K}(v, 0) = \max_{u_L, w_L} p\tilde{K}(v, 1) + (1-p)\left[-C(u_L, 0) + \tilde{\beta}\tilde{K}(w_L, 0)\right],$$

subject $u_L + \tilde{\beta} w_L = v$, $(u_L, 0) \in dom(C)$ and $w_L \in dom(\tilde{K}(\cdot, 0))$.

For $v > 0$, standard Benveniste-Scheinkman arguments and the first order condition for u_L imply

$$\tilde{K}_1(v, 0) \geq p\tilde{K}_1(v, 1) + (1-p)\tilde{K}_1(w_{L,v}, 0),$$

with equality if $w_{L,v}$ is interior. Also, $w_{H,v}$ satisfies

$$\tilde{K}_1(v,1) = \tilde{K}_1(w_{H,v},1).$$

If $w_{L,v} > 0$, these conditions, together with $\tilde{K}_1(w,1) < \tilde{K}_1(w,0)$ and the strict concavity of $\tilde{K}(\cdot,1)$, yield $w_{L,v} < v = w_{H,v}$. The incentive constraints then imply $u_{H,v} = (1-\tilde{\beta})v < u_{L,v}$. If $w_{L,v} = 0$ then the result is immediate since $v > 0$. ■

5.2 Case without commitment

Consider now the case in which the sustainability constraint binds. Let $(u_{H,v}, u_{L,v}, l_v, w_{H,v}, w_{L,v})$ be the solution to problem (C.17). Also, by Proposition C.1 we can focus on $\sigma \in \{0,1\}$ and simplify the notation by letting

$$\kappa^{UI}(v) \equiv \kappa(v,1,0), \ \kappa^{DI}(v) \equiv \kappa(v,0,0).$$

We characterize optimal allocations.

Lemma C.13 *Suppose $\chi > 0$. In steady state,*
(i) if an agent is in the region with full information revelation, he remains in that region as long as he is unemployed;
(ii) the continuation utility of an unemployed agent falls monotonically;
(iii) the continuation utility of an employed agent converges to a value that is independent of the agent's past history.

Proof. *Part (i).* By Lemma C.7, we can replace the two constraints in problem (C.17) with

$$\begin{aligned} u_H + \tilde{\beta} w_H &= v, \\ u_L + \tilde{\beta} w_L &= v. \end{aligned}$$

Let $\underline{v}$ and $\bar{v}$, with $\underline{v} \le \bar{v}$, be the thresholds of Proposition C.1. We assume that $\underline{v} > 0$, otherwise $w_{L,\underline{v}} = \underline{v} = 0$ and the statement is trivial. By construction $\kappa_1^{UI}(\underline{v}) = K_1(\underline{v},0) = K_1(\bar{v},0) = \kappa_1^{DI}(\bar{v})$.

We first prove that a newly-born agent must receive a continuation utility that is either in the region with no information revelation or in the region where randomization occurs.

Uing (C.18) we can rewrite problem (C.17) as

$$\kappa(v,\sigma,0) = p\sigma(K(v,1)+\chi) + (1-p\sigma)\max_{u_L,w_L}\left[u_L - \zeta C(u_L,0) + \hat{\beta}K(w_L,0)\right], \qquad \text{(C.21)}$$

subject to $u_L + \tilde{\beta} w_L = v$, $(u_L, 0) \in dom(C)$ and $w_L \in dom(K(\cdot,0))$. If $w_{L,v} > 0$, standard Benveniste-Scheinkman arguments imply

$$\begin{aligned} \kappa_1^{UI}(v) &= pK_1(v,1) + (1-p)\frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{L,v},0), \qquad \text{(C.22)} \\ \kappa_1^{DI}(v) &= \frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{L,v},0). \end{aligned}$$

We prove that $K_1(\underline{v}, 0) = K_1(\bar{v}, 0) \geq 0$, which implies our initial statement. Suppose that $K_1(\underline{v}, 0) = K_1(\bar{v}, 0) < 0$. Let $v_0 > 0$ be the value such that $K_1(v_0, 0) = \kappa_1^{UI}(v_0) = 0$ (If such value does not exist then newly-born agents receive $v_0 = w_{L,v_0} = 0$ and the arguments below follow immediately). By concavity, $v_0 < \underline{v}$ and, by Proposition C.1, $\sigma_{v_0}^* = 1$. Suppose $w_{L,v_0} > 0$, the case with $w_{L,v_0} = 0$ is immediate. From (C.22),

$$
\begin{aligned}
K_1(v_0, 0) &= \kappa_1^{UI}(v_0) \\
&= pK_1(v_0, 1) + (1-p)\frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{L,v_0}, 0).
\end{aligned}
$$

Since $v_0 > 0$, $K_1(v_0, 1) < K_1(v_0, 0) = 0$, thus, $K_1(w_{L,v_0}, 0) > 0$ and $w_{L,v_0} < v_0$. Therefore, the unemployed agent will remain in the region with $\sigma_{v_0}^* = 1$. However, since $\chi > 0$, the sustainability constraint would eventually be violated. Therefore, agents must be born either in the region with no information revelation or in the region where randomization occurs.

The same arguments prove that an agent who is born in the region with full information revelation will remain in that region as long as he is unemployed.

Part (ii). Suppose $\underline{v} > 0$, otherwise, $\underline{v} = w_{L,\underline{v}} = 0$ and the statement is trivial. Take v_1 and v_2, with $0 < v_1 < v_2 \leq \underline{v}$, we first prove that $w_{L,v_1} \leq w_{L,v_2}$. Suppose instead that $w_{L,v_1} > w_{L,v_2}$. Then the constraint in (C.21) implies $0 \leq u_{L,v_1} < u_{L,v_2}$ and the first order condition for u_{L,v_1} is

$$1 - \zeta C_1(u_{L,v_1}, 0) \leq \frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{L,v_1}, 0).$$

Also, since u_{L,v_2} is interior, its first order condition is

$$1 - \zeta C_1(u_{L,v_2}, 0) \geq \frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{L,v_2}, 0).$$

By concavity, $K_1(w_{L,v_1}, 0) \leq K_1(w_{L,v_2}, 0)$ and $C_1(u_{L,v_1}, 0) < C_1(u_{L,v_2}, 0)$, which lead to a contradiction. Therefore, $w_{L,v_1} \leq w_{L,v_2}$. The same arguments prove that the inequality is strict if w_{L,v_1} is interior. Now, let $\{v_t\}_t$ be the sequence of continuation utilities of an agent, *conditional* on him remaining unemployed. Also, let $v_0 = \underline{v}$, that is, the agent starts at the lower threshold of Proposition C.1. We prove by induction that this sequence decreases monotonically. From part *(i)*, $v_1 = w_{L,\underline{v}} < v_0$. Suppose that the statement is true at some time $t \geq 1$, i.e. $v_t \leq v_{t-1}$. Then, monotonicity of $w_{L,v}$ implies $v_{t+1} \equiv w_{L,v_t} \leq w_{L,v_{t-1}} = v_t$, with strict inequality if v_{t+1} is interior.

Part (iii). Consider the problem of an employed agent (C.18). Standard Benveniste-Scheinkman arguments imply

$$K_1(v, 1) = \frac{\hat{\beta}}{\tilde{\beta}}K_1(w_{H,v}, 1).$$

By concavity, $w_{H,v} > v$, if $K_1(v, 1) > 0$, and $w_{H,v} < v$, if $K_1(v, 1) < 0$. Therefore, when an agent is employed, his continuation utility reverts to the value v_{SS} such that $K_1(v_{SS}, 1) = 0$. ■

Lemma C.14 *For any steady state, there is a payoff-equivalent steady state such that newly-born agents are randomly assigned into two different groups:*

(1) the group with no information revelation, where agents remain until they die;

(2) the group with full information revelation, where agents remain until they either find a job or die;

Finally, the lifetime utility of agents in group (1) is higher than the lifetime utility of agents in group (2).

Proof. Let $\underline{v}$ and $\bar{v}$, with $\underline{v} \leq \bar{v}$, be the thresholds of Proposition C.1 (if these thresholds do not exist then the statement of the lemma is trivial). Newly-born agents receive utility $v_0 > 0$ such that $K_1(v_0, 0) = 0$ (if such value does not exist then $v_0 = w_{L,v_0} = 0$ and the arguments in the proof of Lemma C.13 show that the sustainability constraint would eventually be violated). Similarly for $\underline{v} = \bar{v} = 0$. Thus, we consider the case with $\bar{v} > 0$.

In the proof of part *(i)* of Lemma C.13 we showed that $K_1(\underline{v}, 0) = K_1(\bar{v}, 0) \geq 0$. Suppose first that $K_1(\underline{v}, 0) = K_1(\bar{v}, 0) > 0$. Then by concavity newly-born agents receive lifetime utility $v_0 > \bar{v}$ satisfying $K_1(v_0, 0) = \kappa_1^{DI}(v_0) = 0$. If $u_{L,v_0} = 0$, then $w_{L,v_0} > 0$ and standard Benveniste-Scheinkman arguments imply

$$0 = K_1(v_0, 0) = \kappa_1^{DI}(v_0) = \frac{\hat{\beta}}{\tilde{\beta}} K_1(w_{L,v_0}, 0). \tag{C.23}$$

Instead, if $u_{L,v_0} > 0$, then the first order condition for u_L is

$$1 - \zeta C_1(u_{L,v_0}, 0) \geq \frac{\hat{\beta}}{\tilde{\beta}} K_1(w_{L,v_0}, 0),$$

with strict inequality only if $w_{L,v_0} = 0$. Benveniste-Scheinkman arguments then imply

$$0 = K_1(v_0, 0) = \kappa_1^{DI}(v_0) = 1 - \zeta C_1(u_{L,v_0}, 0) \geq \frac{\hat{\beta}}{\tilde{\beta}} K_1(w_{L,v_0}, 0).$$

The latter, however, implies that the inequality cannot be strict, otherwise, by concavity, $w_{L,v_0} > v_0$, contradicting $w_{L,v_0} = 0$. Therefore, $K_1(w_{L,v_0}, 0) = 0$, thus, agents will remain in the region of no information revelation forever and the statement of the lemma is trivially satisfied.

We next consider the case with $K_1(\underline{v}, 0) = K_1(\bar{v}, 0) = 0$. We first prove a few properties that steady states must satisfy.

(1) The mass of agents with v such that $K_1(v, 0) < 0$ is zero.

First, notice that agents cannot receive any such v at birth. Then, take an agent with continuation utility $v \leq \underline{v}$. By part *(iii)* of Lemma C.13, the continuation utility w of this agent will fall monotonically as long as he remains unemployed. By concavity of $K(\cdot, 0)$, we will thus have $K_1(w, 0) \geq 0$ as long as the agent remains unemployed. Now consider an agent with $v > \underline{v}$ and $K_1(v, 0) = 0$. If this agent is in the region where randomization occurs, then his continuation utility will either be $\underline{v}$ (so that we are back to the previous case) or $\bar{v}$. In the latter case, equation (C.23) and the arguments below it yield $K_1(w_{L,\bar{v}}, 0) = 0$. Similarly, for any $v > \bar{v}$ such that $K_1(v, 0) = 0$. Therefore, if an agent is born with v such that $K_1(v, 0) \geq 0$, he can never enter the region with $K_1(v,0) < 0$. Finally, we show that any interval (v_1, v_2) with $K_1(v,0) < 0$, $v \in (v_1, v_2)$, must have a zero mass of agents. Suppose that there is any such interval with a positive mass of agents. By part *(i)*, agents with $v \in (v_1, v_2)$ will reveal no information, thus, by equation (C.23) and the arguments below it,

$$0 > K_1(v,0) = \kappa_1^{DI}(v) = \frac{\hat{\beta}}{\tilde{\beta}} K_1(w_{L,v}, 0).$$

In turn, the latter implies $w_{L,v} < v$, thus, that the mass of agents on (v_1, v_2) will shrink over time. Therefore, in steady state, there cannot exist intervals (v_1, v_2) with $K_1(v,0) < 0$, $v \in (v_1, v_2)$, with a positive mass of agents. □

(2) Agents with v such that $\kappa_1^{DI}(v) = 0$ receive the same $u_{L,v}$.

Consider problem κ^{DI}. First observe that, by equation (C.23) and the arguments below it, we must have $w_{L,v} > 0$ and $K_1(w_{L,v}, 0) = 0$. Thus, the first order condition for $u_{L,v}$ is

$$1 - \zeta C_1(u_{L,v}, 0) \leq \frac{\hat{\beta}}{\tilde{\beta}} K_1(w_{L,v}, 0) = 0$$

for all v such that $\kappa_1^{DI}(v) = 0$. Strict concavity of $-C(\cdot, 0)$ implies that either $u_{L,v} = 0$ or $u_{L,v} = \hat{u}$, where $\hat{u}$ is the unique solution to $1 - \zeta C_1(\hat{u}, 0) = 0$. We show that we must have $u_{L,v} = \hat{u}$. If not, then $u_{L,v} = 0$ and the constraint would require $w_{L,v} = v/\tilde{\beta} > v \geq \bar{v} > 0$. Therefore, the agent would remain in the region with continuation values $v > 0$ such that $\kappa_1^{DI}(v) = 0$ and receive period utility of 0, which is a contradiction. □

(3) The value function κ^{UI} has a unique maximizer.

By definition, κ^{UI} coincides with (C.21) when $\sigma = 1$. The latter is a weighted sum of two functions: $K(\cdot, 1) + \chi$ and κ^{DI}, which are, respectively, strictly concave and concave by standard arguments. Therefore, κ^{UI} is strictly concave and, hence, it has a unique maximizer. □

(4) There is a point $\hat{v}$ such that $\kappa_1^{DI}(\hat{v}) = 0$ and $w_{L,\hat{v}} = \hat{v}$.

Let $\hat{v} \equiv \hat{u}/(1 - \tilde{\beta})$ be the lifetime utility of an agent whose continuation utility coincides with $\hat{v}$ forever. We want to show that $\hat{v}$ is such that $\kappa_1^{DI}(\hat{v}) = 0$. Take any $\tilde{v} \geq \bar{v}$ such that $\kappa_1^{DI}(\tilde{v}) = K_1(\tilde{v}, 0) = 0$. By property 2, $u_{L,\tilde{v}} = \hat{u}$ and, by the arguments in the proof of that property, $w_{L,v} > 0$. Suppose that $w_{L,\tilde{v}} < \tilde{v}$, the other case is analogous. Consider the sequence $\{(\hat{u}, w^i)\}_i$ where $w^i = \hat{u} + \tilde{\beta} w^{i-1}$ and $w^0 = \tilde{v}$. Notice that $w^i > w^{i-1}$ for all i and $w^i \to \hat{v}$. Since $w^i > \tilde{v} \geq \bar{v}$, by Proposition C.1, $\sigma^*_{w^i} = 0$ for all i. Consider the problem $\kappa_1^{DI}(w^1)$. By the arguments in the proof of that property, $w_{L,w^1} > 0$ and the first order condition for u_{L,w^1} is

$$1 - \zeta C_1(u_{L,w^1}, 0) \leq \frac{\hat{\beta}}{\tilde{\beta}} K_1(w_{L,w^1}, 0).$$

By construction, $w_{L,w^1} = w^0 = \tilde{v}$ and $u_{L,w^1} = \hat{u}$ satisfy this condition, which proves that $(\hat{u}, w^0)$ is the optimal allocation delivering w^1. Also,

$$\kappa_1^{DI}(w^1) = K_1(w^1, 0) = \frac{\hat{\beta}}{\tilde{\beta}} K_1(w^0, 0) = 0.$$

Proceeding recursively, we prove that

$$\kappa_1^{DI}\left(w^{i+1}\right) = K_1\left(w^{i+1}, 0\right) = \frac{\hat{\beta}}{\bar{\beta}} K_1\left(w^i, 0\right) = 0$$

for all i. By continuity, $\hat{v}$ must also satisfy the same condition. $\square$

We now construct a payoff-equivalent steady state satisfying the property stated in the lemma. We focus on the case with $\underline{v} > 0$, the case with $\underline{v} = 0$ is analogous. Notice that, by part *(ii)* of Lemma C.13, if an unemployed agent is in the region with full information revelation, he will remain in this region as long as he remains unemployed. Let v_0 be the lifetime utility of a newly-born agent. If $v_0 > \bar{v}$, then $\kappa_1^{DI}(v_0) = 0$ and, by property (2), the agent receives $u_{L,v_0} = \hat{u}$. If, instead, $v_0 \in [\underline{v}, \bar{v}]$, then, by Proposition C.1, v_0 is delivered through randomization between $\underline{v}$ (with probability p_w) and $\bar{v}$ (with probability $1 - p_w$). Let $\underline{p} \equiv \int \mathcal{I}_{w \geq \underline{v}} p_w \pi(dw, 0)$ be the mass of agents who receive $\underline{v}$. Notice that, by property (3), $\underline{v}$ is the unique point such that $\kappa_1^{UI}(\underline{v})$. In addition, conditional on remaining alive, agents receiving $\underline{v}$ will either find a job or, by part *(ii)* of Lemma C.13, receive a continuation utility $w_{L,\underline{v}} < \underline{v}$. In both cases, they will leave $\underline{v}$. Similarly, let $\bar{p} \equiv \int \mathcal{I}_{w \geq \underline{v}} \pi(dw, 0) - \underline{p}$ be the mass of agents who receive $v \geq \bar{v}$. In the proof of part *(i)* of Lemma C.13 we showed that, in steady state, there cannot be agents with v such that $K_1(v, 0) < 0$. Thus, by property (2), all agents with $v \geq \bar{v}$ must receive $u_{L,v} = \hat{u}$. Finally, in steady state, the mass of agents entering the region with $v \geq \underline{v}$ – which coincides with the mass of newly-born agents – must be exactly equal to the mass of agents leaving it – which equals the sum of those who are at $\underline{v}$ and those who die. Therefore,

$$\delta = \underline{p} + (1 - \delta)\bar{p}. \tag{C.24}$$

Consider now a candidate steady state with the following properties. First, the allocations delivering each continuation utility v coincide with their counterparts in the original steady state. Second, newly-born agents are randomly split into two groups: with probability $\hat{p}$ an agent is assigned to the first group where he receives $\underline{v}$, and with probability $1-\hat{p}$ he is assigned to the second group where he receives $\hat{v}$. Also, we let $\hat{p} \equiv 1 - \bar{p}(1-\delta)/\delta$, which is a well-defined probability by (C.24).

By property (4), the continuation utility of agents in the second group remains $\hat{v}$ forever, while those in the first group will either find a job or, by part *(ii)* of Lemma C.13, receive a continuation utility $w_{L,\underline{v}} < \underline{v}$. As a result, in the candidate steady state, agents in the second group are the only ones who receive utility $v \geq \bar{v}$. Let $p_{\hat{v}}$ denote the mass of agents in the second group. In steady state, it must satisfy

$$\delta(1 - \hat{p}) = (1 - \delta)\, p_{\hat{v}},$$

which implies $p_{\hat{v}} = \bar{p}$. Therefore, the mass of agents at $\hat{v}$ in the candidate steady state coincides with the mass of agents receiving $v > \bar{v}$ in the original steady state. Similarly, the mass of agents at $\underline{v}$, denoted with $p_{\underline{v}}$, must equal $\delta\hat{p}$, i.e. the mass of newly-born agents who are assigned to the first group. We have

$$\begin{aligned} \delta\hat{p} &= \delta - \bar{p}(1 - \delta) \\ &= \underline{p}, \end{aligned}$$

therefore, the mass of agents at $\underline{v}$ in the candidate steady state coincides with the mass of agents receiving $\underline{v}$ in the original steady state.

To sum up, the candidate steady state differs from the original steady state only because agents with $v \geq \bar{v}$ are now bunched at $\hat{v}$. As a result, both aggregate resources and information revealed to the planner are the same as in the original steady state. Therefore, the candidate steady state is also a steady state.

Finally, since $\hat{v} \geq \underline{v}$, agents in the second group receive lifetime utility that is higher than those in the first group. ■

We conclude with the proof of Lemma 4 in the main text. There, we used v_0^{UI} to denote the initial lifetime utility of an agent who is assigned to the group which reveals information. Arguments above show that v_0^{UI} coincides with $\underline{v}$, that is, the threshold in Proposition C.1, which, by property (3) in the proof of Lemma C.14, is the unique maximizer of κ^{UI}.

Proof of Lemma 4. The proof follows from two steps. First, in the proof of part *(i)* of Lemma C.13, we showed that $w_{L,v_0^{UI}} < v_0^{UI}$ if $v_0^{UI} > 0$. Second, from part *(ii)* of Lemma C.13, we showed that $w_{L,v}$ is decreasing, thus, $v < v_0^{UI}$ implies $w_{L,v} \leq w_{L,v_0^{UI}} < v_0^{UI}$. ■

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