

A Theory of Liquidity and Regulation of Financial Intermediation

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First version received January 2007; final version accepted October 2008 (Eds.)

This paper studies a Diamond–Dybvig model of providing insurance against unobservable liquidity shocks in the presence of unobservable trades. We show that competitive equilibria are inefficient. A social planner finds it beneficial to introduce a wedge between the interest rate implicit in optimal allocations and the economy’s marginal rate of transformation. This improves risk sharing by reducing the attractiveness of joint deviations where agents simultaneously misrepresent their type and engage in trades on private markets. We propose a simple implementation of the optimum that imposes a constraint on the portfolio share that financial intermediaries invest in short-term assets.

1. INTRODUCTION

A key role of financial intermediaries is to provide insurance against liquidity shocks. Accordingly, the regulation of financial intermediaries is an important concern for central banks and is a frequent topic of debate in the policy-making community. In this paper, we answer several important questions. Can markets provide and allocate liquidity insurance efficiently? If not, can we precisely identify the origin of the market failure? Can a regulator design a simple policy rule to improve on the allocations provided by competitive markets alone?

Liquidity is a catch-all term referring to several different concepts (see, e.g. von Thadden, 1999). This paper discusses the desire of agents to insure against liquidity shocks that might affect them in the future. We focus, in particular, on the aggregate amount of resources set aside to satisfy liquidity shocks. In the model, this corresponds to the fraction of savings invested in short-term assets, which we refer to as aggregate liquidity. We identify a market failure leading to the underprovision of liquidity. We then show that a simple regulation, working through a general equilibrium channel by lowering long-term interest rates, can restore efficiency. The regulatory intervention is justified not by concerns about individual intermediaries but rather by the inadequacy of the aggregate amount of investment in short-term assets in the financial system as a whole.

More specifically, we study a model where financial intermediaries act as providers of insurance against liquidity shocks, in the spirit of Diamond and Dybvig (1983), Jacklin (1987), and

Allen and Gale (2004).¹ The Diamond–Dybvig model is an established workhorse for positive and normative analysis of financial intermediation. Its simplicity allows for a precise understanding of the nature of potential market failures and the mechanics of prescribed policy interventions. In this model, some agents receive liquidity shocks that affect their consumption opportunities. Agents who receive high liquidity shocks value early consumption only and derive a higher indirect marginal utility of income.²

We impose two informational frictions. Our results are driven by their interactions. The first friction is that liquidity shocks are private information to the agents. Its consequences are well understood. In a model where there is no other friction, an argument similar to that of Prescott and Townsend (1984) or Allen and Gale (2004) can be used to establish that the first welfare theorem holds. The allocations provided by competitive financial intermediaries are constrained efficient. Consequently, this first friction alone does not justify regulating financial intermediation.

The second friction derives from the limits to the observability of consumption. We assume that consumers can borrow and lend to each other on a private market by engaging in hidden side trades. Since the contributions of Allen (1985) and Jacklin (1987), the possibility of agents to engage in hidden side trades has been recognized as an important constraint on risk sharing. This second friction can be interpreted as the case where contracts with financial intermediaries cannot be made exclusive. Arguably, both unobservability of certain financial market transactions and nonexclusivity become more relevant with the increasing sophistication of financial markets. Agents can and do engage in a variety of financial market transactions and routinely deal with several different intermediaries.

We formalize unobservable trades by considering private markets in which agents can trade after they are allocated consumption profiles by either an intermediary or a social planner. Incentive compatibility together with the possibility of private trades requires the equalization of the present value of resources given to all agents, discounted at the interest rate prevailing on the private market. Efficient liquidity insurance provision requires redistribution of resources in the present value sense towards agents with a higher marginal utility of income. In the model, this corresponds to agents affected by a liquidity shock: early consumers.

We first define and characterize the competitive equilibrium in the presence of hidden trades. The competitive equilibrium features limited risk sharing. The reason for this is that arbitrage among intermediaries makes the interest rate on the private market and the marginal rate of transformation equal. We then define and characterize the constrained efficient allocation in the presence of retrading. By affecting the total amount of resources available in each period, the social planner can introduce a wedge between the interest rate prevailing on the private market and the marginal rate of transformation. We show that lowering the interest rate relaxes incentive constraints and improves risk sharing. The intuition is as follows. The planner wants to allocate a higher present value of resources—discounted at the rate of return on the long-term asset—to agents affected by a liquidity shock. However, the planner is constrained by the possibility that late consumers will portray themselves as early consumers and save. Lowering the interest rate reduces the return on such deviations and relaxes incentive compatibility constraints. We then analytically characterize the optimal interest rate. We show that the constrained efficient allocation with retrading coincides with the constrained efficient allocation without retrading and with the unconstrained “first-best” solution. For the case of Diamond–Dybvig preferences, the social planner can completely negate the frictions imposed by retrading and private information

1. See also Bhattacharya and Gale (1987), Hellwig (1994), Diamond (1997), Holmstrom and Tirole (1998), Von Thadden (1999), Allen and Gale (2003), Caballero and Krishnamurthy (2004), and Lorenzoni (2006). For an excellent survey of the literature, see Freixas and Rochet (1997) and Gorton and Winton (2002).

2. Our results would carry over to the case of investment opportunity shocks affecting financially constrained firms.

and achieves the unconstrained optimal allocation. This is in stark contrast with the allocation achieved in a competitive equilibrium where the possibility of unobservable trades poses severe constraints on provision of insurance. While the general point that a government intervention can improve on the allocation in a market system with asymmetric information is well known,³ a contribution of this paper is to provide a clear understanding of the rationale and the direction of the required intervention in the context of a widely used and policy-relevant model.

We propose a simple implementation of the constrained efficient allocation that relies on a natural regulation imposed on financial intermediaries in a competitive market. The regulation is a liquidity floor that stipulates a minimal portfolio share to be held in the short-term asset by intermediaries. The liquidity floor increases the amount of the first period aggregate resources and drives the interest rate on the private markets down. We show how the liquidity floor can be chosen to implement the optimal solution. This simple regulation resembles the different forms of reserve requirements imposed on banks. However, our requirement is concerned with regulating the aggregate amount of liquidity. In practice, reserve requirements were mostly developed as an answer to different concerns pertaining to systemic risk or the fear of bank runs. According to our analysis, they also contribute to mitigating the inefficiency that we highlight. The market failure and the required regulation that we consider are novel but are close in spirit to some arguments that were made in the early stages of financial regulation during the National Banking era, as described in a classical study by Sprague (1910*a*) and in a modern exposition by Chari (1989).

Our paper is most closely related to Jacklin (1987) and to Allen and Gale (2004). Jacklin (1987) compares a competitive equilibrium *with* private markets to the social optimum *without* private markets and reaches the conclusion that the prohibition of private markets leads to a Pareto improvement. We solve for the optimum with private market and show how it can be implemented by a liquidity floor. Our paper focuses on a different mechanism of inefficiency of competitive markets than Allen and Gale (2004). The result of Allen and Gale (2004) that their equilibrium is inefficient relies on the exogenously imposed incompleteness of markets for trades between intermediaries when there are aggregate shocks. When the markets for aggregate shocks are complete or in the absence of aggregate shocks, Allen and Gale (2004) conclude that there is no role for regulation. We show how the planner can manipulate the interest rate on the private markets. We then demonstrate that a liquidity requirement can improve upon the competitive equilibrium even when there are complete markets for insurance against aggregate shocks or when there are no aggregate shocks. The characterization of the mechanism through which liquidity requirements affect interest rates and improve upon the market allocation is new to the banking literature.

While the focus of this paper is financial intermediation, we also contribute to the literature on optimal allocations in the presence of hidden trades.⁴ In particular, Golosov and Tsyvinski (2007) study an optimal dynamic Mirrlees taxation model with endogenous private markets. There are two main differences between our paper and their work. The first difference is in the nature of the shocks. In Golosov and Tsyvinski (2007) as in most of models of dynamic taxation (see, *e.g.* Golosov, Kocherlakota and Tsyvinski, 2003; Golosov, Tsyvinski and Werning, 2006; Kocherlakota, 2006; Farhi and Werning, 2007), private information (skill shocks) is dynamic and separable from consumption. In our setup, shocks affect the marginal rate of substitution for consumption and the marginal utility of income. The second difference pertains to the strength of the results that we obtain. Golosov and Tsyvinski (2007) are able to identify only the direction of a

3. See Hart (1975), Newbery and Stiglitz (1984), and, for the area of financial intermediation, Allen and Gale (2004).

4. See, *e.g.* Arnott and Stiglitz (1986, 1990), Greenwald and Stiglitz (1986), Hammond (1987), Geanakoplos and Polemarchakis (1986), Chiappori, Macho, Rey and Salanie (1994), Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (2001), Albanesi (2006), and Bisin and Rampini (2006).

local policy change that leads to a Pareto improvement. We characterize the *globally optimal* allocation in the presence of private markets and show that optimal liquidity regulation implements the constrained optimum.

2. SETUP AND A BENCHMARK MODEL

We consider a standard model of financial intermediation similar to Diamond and Dybvig (1983) and to Allen and Gale (2004). The economy lasts three periods, $t = 0, 1, 2$. There are two assets (technologies) in the model. The short asset is a storage technology that returns one unit of consumption good at $t + 1$ for each unit invested at t . Investment in the long asset has to be done at $t = 0$ to yield $\hat{R} > 1$ units of the consumption good at $t = 2$. Therefore, the time interval from $t = 0$ to $t = 2$ in this model is interpreted, as in the Diamond–Dybvig model, as the time it takes to costlessly liquidate the long-term asset.

The economy is populated by a unit continuum of *ex ante* identical agents or investors. Suppose that there are two types of agents denoted by $\theta \in \{0, 1\}$. At $t = 0$, all individuals are (*ex ante*) identical and receive an endowment e . At $t = 1$, each consumer gets a draw of his type. With probability $\pi \in (0, 1)$, he is an agent of type $\theta = 0$, and with probability $(1 - \pi)$, he is an agent of type $\theta = 1$. The fraction of agents of each type is therefore π and $1 - \pi$, respectively. We introduce the “baseline” utility function, $u : \mathbb{R}^+ \rightarrow \mathbb{R}$, twice continuously differentiable, increasing, strictly concave and satisfies Inada conditions, $u'(0) = +\infty$ and $u'(+\infty) = 0$. In terms of the baseline function u , preferences of an agent of type θ are given by utility function $U : \mathbb{R}^+ \times \mathbb{R}^+ \times \{0, 1\} \rightarrow \mathbb{R}$, which is assumed to take the form

$$U(c_1, c_2, \theta) = (1 - \theta)u(c_1) + \theta\rho u(c_1 + c_2),$$

where c_1 is agent’s consumption in period 1, c_2 is agent’s consumption in period 2, and ρ is a constant, which is the same for agents of both types. In addition, we assume, as in Diamond and Dybvig (1983), that the coefficient of relative risk aversion is everywhere greater than or equal to 1:

$$\frac{-cu''(c)}{u'(c)} \geq 1 \text{ for all } c > 0, \quad (1)$$

and that $\hat{R}^{-1} < \rho < 1$ (which implies $\rho\hat{R} > 1$). This assumption is needed to ensure that in the various optima, the social planner wants to redistribute to the early consumers and such types consume more than their endowment.

Agents of type $\theta = 0$ are affected by liquidity shocks. They value consumption in the first period only. Agents of type $\theta = 1$ are indifferent between consuming in the first and the second period. We use these preferences throughout the main body of the paper. In the extensions, we consider a more general class of preferences that demonstrate the somewhat specific properties of the Diamond–Dybvig setup. A key informational friction is that types of agents are private, *i.e.* observable only by the agent himself but not by others.

We denote by $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ an *allocation* of consumption across consumers. An allocation is *feasible* if it satisfies:

$$\pi \left[c_1(0) + \frac{c_2(0)}{\hat{R}} \right] + (1 - \pi) \left[c_1(1) + \frac{c_2(1)}{\hat{R}} \right] \leq e. \quad (2)$$

We now define and characterize a benchmark economy in which the only friction is unobservability of types. In this environment, agents are given consumption allocations depending on their types. Agents cannot engage in any unobservable transaction and their consumption is therefore observable. A constrained efficient program, *i.e.* the problem of the social planner, which we

call problem SP^2 or a “second best” problem, is given by:

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}} \pi u(c_1(0)) + (1 - \pi)\rho u(c_1(1) + c_2(1)) \tag{3}$$

s.t.

$$\pi \left(c_1(0) + \frac{c_2(0)}{\hat{R}} \right) + (1 - \pi) \left(c_1(1) + \frac{c_2(1)}{\hat{R}} \right) \leq e, \tag{4}$$

$$u(c_1(0)) \geq u(c_1(1)), \tag{5}$$

$$u(c_1(1) + c_2(1)) \geq u(c_1(0) + c_2(0)). \tag{6}$$

The planner maximizes expected utility of an agent subject to the feasibility constraint (4) and two incentive compatibility constraints. Constraint (5) ensures that an agent of type $\theta = 0$ does not want to pretend to be an agent of type $\theta = 1$. Constraint (6) ensures that an agent of type $\theta = 1$ does not want to pretend to be an agent of type $\theta = 0$.⁵

We can also define an unconstrained optimum that we call SP^1 in which there is no private information—the “first best” program. That program differs from the problem SP^2 in that the incentive compatibility constraints (5) and (6) are omitted.

Following Diamond and Dybvig (1983), it is easy to show that the incentive compatibility constraints are not binding at the optimum of (3). In other words, solutions to problems SP^1 and SP^2 coincide. Then, the solution of this problem is given by:

$$c_2(0) = c_1(1) = 0, \tag{7}$$

$$u'(c_1(0)) = \rho \hat{R} u'(c_2(1)), \tag{8}$$

$$\pi c_1(0) + (1 - \pi) \frac{c_2(1)}{\hat{R}} = e. \tag{9}$$

Moreover, $c_1(0) > e$ and $c_2(1) < \hat{R}e$. The planner redistributes resources to consumers of type $\theta = 0$ who are given a higher present value of consumption than the value of their endowment. Late consumers, those with $\theta = 1$, receive consumption that is less than the present value of their endowment.

We can also define a competitive equilibrium problem in which there is a continuum of intermediaries providing insurance to agents. The intermediaries are subject to the same constraint as the social planner and do not observe the types of agents. We omit a formal definition here. A version of the first welfare theorem would hold here as shown by Prescott and Townsend (1984) and Allen and Gale (2004). The competitive equilibrium allocations would coincide with the solution to the problem SP^2 . The key to this result is that consumption is observable—agents cannot engage in unobservable trades.

3. A MODEL WITH PRIVATE MARKETS

The allocations described in the previous section may not be achieved if agents can engage in private transactions. Allen (1985) and Jacklin (1987) were the first to point out that the possibility of such trades may restrict risk sharing across agents. We now describe how to model

5. In specifying incentive compatibility constraints as (5) and (6), we are neglecting the possibility that this revelation game may have a “bank run equilibrium” of the sort considered by Diamond and Dybvig (1983). Implicitly, we are assuming that the planner is able to select the outcome of the revelation game.

unobservable consumption. This formalization will be central to defining and characterizing both competitive equilibria and constrained efficient allocations with private markets. Consider an environment in which all consumers have access to a market in which they can trade assets among themselves unobservably.⁶ Formally, suppose that consumers are offered a menu of contracts $\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}$. A consumer treats the contract and the equilibrium interest rate R on the private market as given and chooses his optimal reporting strategy θ' that determines his endowment of consumption $(c_1(\theta'), c_2(\theta'))$. Unlike in the environment without private markets, the actual after-trade consumption (x_1, x_2) may differ from the consumption specified in the contract since it is impossible to preclude a consumer from borrowing and lending an amount s on the private market.⁷ Given a menu of consumption allocations $\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}$ and an interest rate R , an agent of type θ solves:

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}, R, \theta) = \max_{x_1, x_2, s, \theta'} U(x_1, x_2; \theta), \quad (10)$$

subject to:

$$x_1 + s = c_1(\theta'), \quad (11)$$

$$x_2 = c_2(\theta') + Rs. \quad (12)$$

Let us define a menu of consumption allocations as $\tilde{C} = \{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}$ to simplify the notation. In what follows, we define $x_1(\tilde{C}, R, \theta)$, $x_2(\tilde{C}, R, \theta)$, $s(\tilde{C}, R, \theta)$, $\theta'(\tilde{C}, R, \theta)$ as a solution to problem (10). We formally define an equilibrium in the private market as follows.

Definition 1. An equilibrium in the private market given the menu of endowments \tilde{C} consists of interest rate R and for each agent of type θ : allocations $x_1(\tilde{C}, R, \theta)$, $x_2(\tilde{C}, R, \theta)$, trades $s(\tilde{C}, R, \theta)$, and choices of reported types $\theta'(\tilde{C}, R, \theta)$ such that

- (i) $x_1(\tilde{C}, R, \theta)$, $x_2(\tilde{C}, R, \theta)$, $s(\tilde{C}, R, \theta)$, $\theta'(\tilde{C}, R, \theta)$ constitute a solution to problem (10);
- (ii) the feasibility constraints on the private market are satisfied for $t = 1, 2$:

$$\pi x_t(\tilde{C}, R, 0) + (1 - \pi) x_t(\tilde{C}, R, 1) \leq \pi c_t(\theta'(\tilde{C}, R, 0)) + (1 - \pi) c_t(\theta'(\tilde{C}, R, 1)). \quad (13)$$

3.1. Competitive equilibrium with private markets CE^3

We now formally describe competitive equilibria and show how risk sharing is hindered by the possibility of that agents engage in unobservable trades in private markets.

Consider a market with a continuum of intermediaries. We assume throughout the paper that all activities at the intermediary level are observable. In period 0, before the realization of idiosyncratic shocks, consumers deposit their initial endowment with an intermediary. The intermediary provides a menu of consumption allocations $\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}$. In the presence of private markets, intermediaries need to take into account, in addition to unobservable types, that consumers are able to engage in transactions in the private market. Contracts are offered competitively, and there is free entry for intermediaries. Therefore, each consumer signs a contract with

6. All our analysis is easily extended to the case in which agents can trade not only among themselves but also with other intermediaries. This case would bring this model closer to an interpretation as an environment of nonexclusive contracts. A key assumption that allows us to extend our results to that case is that portfolios of the intermediaries (investment in short and long assets) are observable while transactions with individual consumers are not. Our choice of modelling side trades as private markets allows us to economize on notation without affecting the substance of the results.

7. It can be shown that a consumer trades only a risk-free security (*e.g.* Golosov and Tsyvinski, 2007).

the intermediary who promises the highest *ex ante* expected utility. We denote the equilibrium utility of a consumer by \underline{U} .

We assume that intermediaries can trade bonds b among themselves. We denote by q the price of a bond b in period $t = 1$ that pays one unit of consumption good in period 2. All intermediaries take this price as given. They also pay dividends d_1, d_2 to the owners.⁸

It is important to note that intermediaries take the interest rate on the private market R as given. The maximization problem of the intermediary that faces intertemporal price q , interest rate on the private market R and reservation utility of consumers \underline{U} is

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}, (d_1, d_2), b} d_1 + \frac{d_2}{\hat{R}} + qb - \frac{b}{\hat{R}} \tag{14}$$

s.t.

$$\pi \left(c_1(0) + \frac{c_2(0)}{\hat{R}} \right) + (1 - \pi) \left(c_1(1) + \frac{c_2(1)}{\hat{R}} \right) + d_1 + \frac{d_2}{\hat{R}} + qb - \frac{b}{\hat{R}} \leq e, \tag{15}$$

$$\theta = \theta'(\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}, R, \theta), \quad \forall \theta, \tag{16}$$

$$\pi \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}, R, 0) + (1 - \pi) \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}, R, 1) \geq \underline{U}. \tag{17}$$

The first constraint in the intermediary’s problem is the budget constraint. The second constraint is incentive compatibility that states that, given the profile of consumptions $\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}$ and the possibility to borrow or lend at an interest rate R , consumers choose to truthfully reveal their types, *i.e.* the true type θ is a solution to the problem (10). We can restrict the intermediaries to truth-telling mechanisms because the Revelation Principle applies. The last constraint states that the intermediary cannot offer a contract, which delivers a lower expected utility than the equilibrium utility \underline{U} from the contracts offered by other intermediaries. In equilibrium, all intermediaries act identically and make zero profits. The definition of the competitive equilibrium is then as follows.

Definition 2. A competitive equilibrium with private markets, CE^3 , is a set of allocations $\{c_1^*(\theta), c_2^*(\theta)\}_{\theta \in [0,1]}$, a price q^* , dividends $\{d_1^*, d_2^*\}$, bond trades b^* , utility \underline{U}^* , and the interest rate on the private market R^* such that

- (i) each intermediary chooses $\{\{c_1^*(\theta), c_2^*(\theta)\}_{\theta \in [0,1]}, \{d_1^*, d_2^*\}, b^*\}$ to solve problem (14) taking q^*, R^* , and \underline{U}^* as given;
- (ii) consumers choose the contract of an intermediary that offers them the highest *ex ante* utility;
- (iii) the aggregate feasibility constraint (2) holds;
- (iv) the private market, given the menus $\{c_1^*(\theta), c_2^*(\theta)\}_{\theta \in [0,1]}$, is in an equilibrium of Definition 1 and R^* is an equilibrium interest rate on the private market;
- (v) intermediaries make zero profits;
- (vi) bond markets clear: $b^* = 0$.

It is easy to see that the interest rate on the markets for trades among intermediaries must be equal to the return on the production technology, so that $1/q^* = \hat{R}$. We now argue that an allocation satisfies the incentive compatibility constraint (16) if and only if the net present value

8. Since intermediaries make zero profits in equilibrium, we do not formally specify how these dividends are distributed.

of resources allocated to each type must be equalized when discounted at the market interest rate R :

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R}. \tag{18}$$

If the present values are not equated across types, an agent would pretend to claim a type that gives a higher present value of allocations and engage in trades on the private markets to achieve her desired consumption allocation.

Let us rewrite the problem of the intermediary in a more tractable form by considering its dual, simplifying the incentive compatibility constraint using (18) and the fact that $d_1 + \frac{d_2}{R} = 0$ and $b = 0$ since we are interested in symmetric allocations only:

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}} \pi \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R, 0) + (1 - \pi) \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R, 1), \tag{19}$$

s.t. (18) and

$$\pi \left(c_1(0) + \frac{c_2(0)}{\hat{R}} \right) + (1 - \pi) \left(c_1(1) + \frac{c_2(1)}{\hat{R}} \right) \leq e. \tag{20}$$

Let R^* denote equilibrium price on the private market corresponding to the competitive equilibrium in Definition 2. It is easy to show following Hellwig (1994) and Allen and Gale (2004) that $R^* = \hat{R}$ and

$$\begin{aligned} c_1(1) &= e, & c_2(0) &= 0, \\ c_1(1) &= 0, & c_2(1) &= \hat{R}e. \end{aligned}$$

An intuition for this result is that, if $R^* \neq \hat{R}$, arbitrage opportunities are created. An intermediary would find it profitable to invest either in the long-run or in the short-run assets and let agents trade in the private market.

This result means that risk sharing is severely limited in a competitive equilibrium with side trades. As in Jacklin (1987) and Allen and Gale (2004), the present values of consumption entitlements (evaluated at \hat{R}) are equated across consumers of different types:

$$c_1(0) + \frac{c_2(0)}{\hat{R}} = c_1(1) + \frac{c_2(1)}{\hat{R}}.$$

3.2. Constrained efficient allocation with private markets

We now define and characterize the constrained efficient problem with private markets. We call the program SP^3 or the “third best” program. Consider a social planner that cannot observe or shut down trades on private markets and cannot observe agents’ types. The difference with the problem SP^2 is that, in addition to the private information faced by SP^2 , planner SP^3 faces constraints that agents may trade on the private market. The social planner SP^3 chooses the allocation $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ that maximizes the *ex ante* utility of consumers. The revelation principle shows that, without loss of generality, the social planner can offer a contract $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ so that all consumers choose to report their types truthfully to the planner and not trade in the private market.

Formally, the constrained efficient allocation $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ is the solution to the problem SP^3 given by:

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}} \pi U(c_1(0), c_2(0); 0) + (1 - \pi) U(c_1(1), c_2(1); 1), \tag{21}$$

s.t.

$$\pi \left(c_1(0) + \frac{c_2(0)}{\hat{R}} \right) + (1 - \pi) \left(c_1(1) + \frac{c_2(1)}{\hat{R}} \right) \leq e, \tag{22}$$

$$U(c_1(\theta), c_2(\theta); \theta) \geq \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R; \theta) \quad \forall \theta, \tag{23}$$

where R is an equilibrium interest rate on the private market, given the profile of endowments $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ according to Definition 1.

We now show that choosing consumption allocations in the constrained efficient problem (21) is equivalent to the problem of a planner choosing an interest rate R on the private market and allocating the same income (present value of consumption allocations) I to agents of different types. The planner can introduce a wedge between the interest rate R on the private market and the rate of return on the long-run asset \hat{R} . The incentive compatibility constraint again presents itself as a requirement that the same present value of resources I is allocated across agents of different types.

Formally, we proceed as follows. Let

$$V(I, R; \theta) = \max_{x_1, x_2} U(x_1, x_2; \theta) \tag{24}$$

subject to

$$x_1 + \frac{x_2}{R} \leq I \tag{25}$$

be the *ex post* indirect utility of an agent of type θ if her income is I and the interest rate on the private market is R . Denote the solutions to this problem (uncompensated demands) by $x_1^u(I, R; \theta)$ and $x_2^u(I, R; \theta)$.

Consider the problem of a social planner who chooses the interest rate R and income I to maximize the expected indirect utility of agents subject to feasibility constraints:

$$\max_{I, R} \pi V(I, R; 0) + (1 - \pi) V(I, R; 1) \tag{26}$$

subject to

$$\pi \left(x_1^u(I, R; 0) + \frac{x_2^u(I, R; 0)}{\hat{R}} \right) + (1 - \pi) \left(x_1^u(I, R; 1) + \frac{x_2^u(I, R; 1)}{\hat{R}} \right) \leq e, \tag{27}$$

where $x_1^u(I, R; \theta), x_2^u(I, R; \theta)$ are defined above as solutions to (24).

We now prove the equivalence of the problems (21) and (26).

Lemma 1. *Let I^* and R^* be solutions to (26) and $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta \in \{0,1\}}$ be solution to (24) given I^* and R^* . Then, $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ defined by*

$$c_t(\theta) = x_t^u(I^*, R^*; \theta), \quad \forall \theta \in \{0, 1\} \quad \forall t \in \{1, 2\} \tag{28}$$

is solution to problem (21). Conversely, if $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ solves problem (21) then there exist I^ and R^* , which solve (26) if $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta \in \{0,1\}}$ are given by (28), and such that $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta \in \{0,1\}}$ solve (24) for $I = I^*$ and $R = R^*$.*

Proof. In the Appendix.

The above lemma reduces the dimensionality of the problem of the planner to choosing I and R as opposed to allocations of consumption across all types. The intuition for the lemma is

as follows. The planner effectively has only two instruments: an income I and an interest rate R . The planner is constrained to choose only allocations that give the same income to the agents because of the possibility of participating in the private markets. However, the planner also has an advantage over the competitive markets, whereas allocating different amounts of aggregate assets available across time the planner can affect the interest rate on the private markets. This additional instrument is not an artefact of this model and also appears in other problems with private markets such as dynamic optimal taxation problems (Golosov and Tsyvinski, 2007).

Using this lemma, we can now provide a characterization of the constrained efficient allocation.

Theorem 1. *Solutions to the constrained efficient problem with private markets, SP^3 , constrained efficient problem without private markets, SP^2 , and informationally unconstrained problem, SP^1 , coincide. Moreover, the interest rate R^* on the private market corresponding to the solution of SP^3 is such that $R^* \in (1, \rho \hat{R}]$. If $u(c) = \log(c)$ then $R^* = \rho \hat{R}$.*

Proof. In the appendix.

This theorem is one of the central results of the paper. A social planner, even in the presence of hidden trades, can achieve allocations superior to the ones achieved by competitive markets. Moreover, we fully characterize the constrained efficient allocation and show that for the case of Diamond–Dybvig preferences, it coincides with the unconstrained, full information optimum, SP^1 .⁹ The intuition for the result is that lowering the interest rate relaxes the incentive compatibility constraints. Consider a relevant deviation in the model. An agent of type $\theta = 1$ wants to claim to be an agent of type $\theta = 0$ and then save the allocation $c_1(0)$ at the private market interest rate R^* . An interest rate on the private markets $R^* < \hat{R}$ reduces the profitability of this deviation. In the case of Diamond–Dybvig preferences, lowering the interest rates allows perfect screening of the different types and achieves not only the constrained efficient allocation SP^3 but also the unconstrained optimum SP^1 . Note that the manipulation of the equilibrium interest rate by the planner is indirect and happens through the general equilibrium effect of changing the profile of endowments. The planner can increase the amount of investment in the short asset (amount of allocations paid in the first period) and correspondingly reduces the amount of investment in the long asset (amount of allocations paid in the second period). Lemma 1 showed that such a manipulation of endowments induces the desired change of the interest rate in the private market.

More generally, lowering the interest rate benefits agents who value consumption in the first period more. If these agents also have a higher marginal utility of income—as is the case for Diamond–Dybvig preferences—this leads to an improvement in the provision of liquidity insurance and in the *ex ante* welfare. Recall that the unobservability of agents' types and possibility of trades require that agents of different types receive the same present value of consumption evaluated at the private market interest rate R :

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R}.$$

However, the amount of resources evaluated at the real rate of return may differ across agents

$$c_1(\theta) + \frac{c_2(\theta)}{\hat{R}} \neq c_1(\theta') + \frac{c_2(\theta')}{\hat{R}}.$$

9. An interesting recent paper by Lorenzoni (2006) analyses a similar mechanism in the model of financial intermediation with money.

In our case, the change in the interest rate transfer resources to the agent $\theta = 0$ affected by a liquidity shock, who is marginally more valuable to the regulator.

3.3. *Implementing constrained efficient allocations—liquidity requirements*

In this subsection, we show that there exists an intervention—a *liquidity floor*—that implements the constrained efficient allocation SP^3 . The liquidity requirement is a constraint imposed on all intermediaries, *i.e.* a constraint on problem (14). It is stated in a ratio form that specifies the lower bound i on the share of the short asset in the intermediary’s portfolio. Specifically, if an intermediary attracts a mass m of agents then the value of her portfolio is me , and the share of short-term assets is given by:

$$m [\pi c_1(0) + (1 - \pi)c_1(1)] / (me) \geq i,$$

i.e.

$$\pi c_1(0) + (1 - \pi)c_1(1) \geq ie. \tag{29}$$

An attractive feature of a liquidity requirement is that it does not require the regulator to observe the individual contracts $\{c_0(\theta), c_1(\theta)\}$ signed by different agents with intermediaries. Only the aggregate portfolio allocation of each intermediary needs to be observed. It is important to design our regulation to note that intermediaries might differ by size, *i.e.* by the measure of agents who deposit their endowments. In equilibrium, agents are indifferent between all intermediaries. Although the equilibrium allocation is uniquely determined, the size distribution of intermediaries is indeterminate. There could be only one intermediary, a continuum of same-size intermediaries, and anything between these two polar cases. However, our proposed regulation can be designed to circumvent the difficulties arising from this indeterminacy and can be made operational simultaneously for every possible size distribution of intermediaries.

We now intuitively describe the effects that a binding liquidity requirement has on the interest rate on private markets. Let \hat{X} be the investment in the short asset that arises in a competitive equilibrium as in Definition 2. Suppose that a liquidity floor i is set higher than the amount of the first period claims provided by competitive markets:

$$ie \geq \hat{X}.$$

When a liquidity floor is imposed, the aggregate endowment in the first period is equal to i rather than \hat{X} . Private trading markets in which agents participate after receiving their allocation from the intermediaries are an exchange economy. The liquidity floor increases the aggregate endowment of the first period good in the private market (and, correspondingly, decreases the aggregate endowment of the second period good). Therefore, this requirement has a general equilibrium effect in indirectly lowering the interest rate R below \hat{R} .

In the absence of regulations, the interest rate R on the private market is equal to \hat{R} . Imposing a liquidity floor lowers the interest rate and implements the constrained efficient allocation by putting a limit on this arbitrage.

Proposition 1. *Let the liquidity floor i^* defined in (29) be given by*

$$i^* = \pi I^* / e, \tag{30}$$

where I^* is the solution to (26). Then, competitive equilibrium allocation specified in Definition 2 with the imposed liquidity floor (formally, an additional constraint (29)) coincides with the constrained efficient allocation SP^3 .

Proof. In the appendix.

This proposition is important as it specifies a simple regulation that implements the optimum. Note that this regulation does not prohibit private markets. Rather, it affects the investments and holdings of assets by financial intermediaries. In general, deriving implementations of constrained efficient allocations is a difficult task in environments where private trades are possible. An abstract treatment of a related problem is given in Bisin *et al.* (2001) who show that, in a general class of environments with anonymous markets, taxes can achieve Pareto improvements. The difference with our setup is that they do not define the constrained efficient problem SP^3 but rather show that a local linear tax can improve upon the market allocation. Golosov and Tsyvinski (2007) study a dynamic model of optimal taxation and define the optimal program similar to our SP^3 . They also show that a linear tax on savings may locally improve upon the competitive equilibrium allocation.

Our results can also be related to results concerning regulation in Allen and Gale (2004). Allen and Gale (2004) consider an environment where each intermediary faces a shock affecting the distribution of liquidity shocks that her subscribers will face. These shocks are idiosyncratic—the overall distribution of liquidity shocks in the population is constant—and there are no markets to share the corresponding risks—only a risk-free bond can be traded. Because markets are incomplete, there might exist welfare improving interventions. The authors show that when relative risk aversion is greater than one a liquidity floor improves upon the competitive market allocation. The intuition bears some similarity with our model. In equilibrium, insurance is provided to early consumers so that they absorb more resources than late consumers, in terms of net present value. If markets were complete, an intermediary facing an unusually large amount of early consumers would receive net resources from others. Raising the interest rate redistributes resources towards intermediaries with a low fraction of late consumers and partly substitutes for the payments these intermediaries would have received under complete markets, thereby improving welfare. There are several differences with our model. First, Allen and Gale (2004) impose exogenously that some markets are missing. If markets were complete then there would be no scope for intervention in their model. By contrast, in our model, the justification for intervention does not rely on any such exogenous constraint on the environment. Second, they do not solve for the optimal regulation. By contrast, we not only solve for the optimal liquidity requirement but also show that this policy implements the constrained efficient allocation.

4. SOME HISTORICAL BACKGROUND

In this section, we argue that some elements of our model can be connected to the debates that were taking place in the period of the National Banking System in the U.S. (1863–1913). We follow the discussion of the classic work by Sprague (1910*a*, 1910*b*) and the modern exposition and interpretation by Chari (1989). Sprague and Chari are mostly interested in banking crises and panics while we focus on liquidity provision more generally. However, some of their arguments identify frictions that resemble the ones that we are emphasizing.¹⁰

The National Banking System of reserves was a three-tier structure: regional banks (the first tier), designated banks in reserve cities (the second tier), and designated banks in New York City (the third tier). The different tiers of the banking system were subject to different reserve requirements. Banks in the first and second tiers could decide whether to hold their reserves in cash or to deposit them in institutions of the upper tier. As a result, lower tier banks had an incentive to deposit their reserves in New York City banks rather than holding liquid assets or

10. The discussion that follows may also be interpreted as supporting the analysis of Allen and Gale (2004).

cash. In effect, the reserves of the lower tier banks deposited in New York City were loans and did not contribute to the overall amount of reserves in the system.

At the time, the demand for withdrawals fluctuated with the quality of the crops and was hard to predict. In other words, liquidity shocks were prevalent. The National Banking System experienced several major banking crises. Many commentators argued that these crises were in part due to the insufficient amount of aggregate reserves in the form liquid assets set aside by the financial system. Sprague (1910*a*, pp. 96–97), commenting on the crisis of 1873, wrote that “The aggregate [reserves] held by all national banks of the United States does not finally much exceed 10 per cent of their direct liabilities.” This amount was much lower than the statutory requirement.

The blame was put on the practice of paying interest on the reserves deposited to the New York City banks. Sprague (1910*b*) writes “But this practice of paying interest on bankers’ deposits, as it now obtains, has other and more far-reaching consequences. It is an important cause of the failure to maintain a reserve of lending power in periods of business activity and the fundamental cause of the failure” and “The abandonment of the practice of paying interest upon deposits will remove a great inducement to divide . . . reserves between cash in hand and deposits in cities” (Sprague, 1910*a*, p. 97). In other words, interest rates on reserves in New York banks were too high, crowding out liquid assets.¹¹

In response to the crisis of 1873, the New York Clearing House Association was created. Its main purpose was to improve the allocation of liquidity by allowing banks to draw on each other’s reserves. However, financial innovation progressively undermined the role of the Clearing House Association. In particular, the rise of trust companies in the beginning of the 1900’s contributed significantly to the severe crisis of 1907 (Moen and Tallman, 1992). The trust companies accounted for a significant amount of assets—nearly as much as banks—and had very small reserve requirements: they did not fall under the banking regulations and were not part of the New York Clearing House Association. These trusts, however, were engaged in significant transactions with the banks that were members of the Association and member banks often used trusts to circumvent reserve requirements (Sprague, 1910*a*, p. 227).

These episodes illustrate several features of financial intermediation that are also at play in our model. First, aggregate liquidity—and not only concerns about the liquidity or solvency of any particular individual intermediary—matters. Second, high interest rates inefficiently divert resources from low-return liquid assets. Finally, side trades and financial innovation can severely undermine financial regulations. A lesson for our times is that an efficient regulation should have a wide scope and cover a variety of financial institutions – *e.g.* mutual funds and hedge funds.

5. GENERALIZATIONS AND EXTENSIONS

5.1. *General preferences*

In this subsection, we informally consider a more general specification of preferences.¹² We argue that the specification of preferences matters for the form of the optimal regulation. Also, the ability of agents to engage in trades may lead to constrained efficient allocations inferior to those without trades.

Suppose that there is a continuum of possible types and the preference shock is denoted by $\theta \in [\theta_L, \theta_H] \subset [0, 1]$, where $\theta_L < \theta_H$. We focus on three types of preferences. *Discount factor*

11. See also the address of George S. Coe, a prominent financier of that time, to the New York Clearing House Association, discussing why individual banks have an incentive to underinvest in the assets with short-term maturity (Sprague, 1910*a*, pp. 377–378).

12. A more extensive analysis containing the formal statements of the results and all the proofs is in the working paper version (Farhi, Golosov and Tsyvinski, 2008).

shocks: $u(c_1, c_2; \theta) = \hat{u}(c_1) + \theta \hat{u}(c_2)$. The first feature of these preferences is that an agent with a higher θ shock has a higher marginal utility of consumption in the second period. The second feature of these preferences is that an agent with higher θ has higher lifetime marginal utility of income. *Liquidity shocks*: $u(c_1, c_2; \theta) = \frac{1}{\theta} \hat{u}(c_1) + \hat{u}(c_2)$. In this case, a low θ shock increases marginal utility of consumption in the first period. The second feature of these preferences is that an agent with lower θ has a higher lifetime marginal utility of income than an agent with higher θ . These preferences are a straightforward generalization of the Diamond–Dybvig setup. *Valuation-neutral shocks*: $u(c_1, c_2; \theta) = (1 - \theta) \log(c_1) + \theta \log(c_2)$. In this case, agents differ in marginal utility of consumption across periods, but all agents have the same marginal value of income.

A liquidity requirement is a constraint imposed on all intermediaries that requires that investment in the short asset for any intermediary be higher than a certain share of short-term assets in the case of a *liquidity floor* or lower than a certain share in the case of a *liquidity cap*. Let R^* be the interest rate on the private market in the solution to the constrained efficient problem SP^3 .

In the case of liquidity shocks, the planner wants to allocate a higher present value of resources to agents with lower θ as they have higher marginal lifetime utility of income. An agent with $\theta' > \theta$ wants to engage in the following deviation: pretend to be an agent with a lower type θ and save on the private market. An interest rate $R^* < \hat{R}$ makes such deviations less profitable, and the liquidity requirement that implements SP^3 is a liquidity floor. In the case of discount factor shocks, the direction of the deviation is reverse: to pretend to be an agent of higher θ and to borrow on the private market. Increasing the interest rate on the private market to $R > \hat{R}$ discourages such deviations, and the liquidity requirement that implements SP^3 is a liquidity cap. In the case of both the liquidity and the discount shocks, the constrained efficient solution with private markets SP^3 and the the solution to the optimal problem SP^2 without private markets do not coincide. If preferences are valuation-neutral shocks then $R^* = \hat{R}$, and no regulations are needed. Moreover, the solutions to the problem SP^2 coincide with the solutions to SP^3 .

The case of Diamond–Dybvig preferences is conceptually close to the case of liquidity shocks preferences in this section. The direction of the relevant deviation and the ability to affect agents with a lower interest rate directly extend to the more general preferences. The important difference between these two cases is that, in general, solutions to the constrained efficient problem with trades, SP^3 , and constrained efficient problem without trades, SP^2 (or problem without private information, SP^1) do not coincide. For Diamond–Dybvig preferences, the solutions to SP^1 , SP^2 , and SP^3 coincide. The reason for that is that agents of type $\theta = 0$ and $\theta = 1$ have very different marginal rates of substitution. In the case of $\theta = 0$, the marginal rate of substitution between periods $t = 1$ and $t = 2$ is zero, and in the case of $\theta = 1$, the marginal rate of substitution is one. Such differences in preferences allow the social planner to perfectly screen the agents of different types. In general, this is not the case, and the solutions to constrained efficient programs SP^2 and SP^3 would not coincide.

5.2. Aggregate shocks

It is easy to extend the model to the case in which the economy experiences aggregate shocks to e and \hat{R} that are known in period $t = 0$. Suppose that there are N aggregate states $n = \{1, 2, \dots, N\}$ and the state is observable. We denote the probabilities of these states by η_n . We notice that it is technologically impossible for the society to transfer resources across states. Therefore, the problem with aggregate shocks can be reduced to solving N independent problems described in case without aggregate shocks and is, essentially, a comparative statics exercise with respect to the aggregate shock.

5.3. Idiosyncratic shocks and heterogeneity of intermediaries

We first note that if there are complete interbank markets then this model reduces to the case described in previous sections in which all intermediaries are identical. The intuition for this result is simple: in period 0, intermediaries can trade bonds with the payoff contingent on the shocks realized in period 1. We now illustrate the result for the case with no aggregate uncertainty in which intermediaries face return shocks.

Formally, we proceed as follows. At time $t = 1$, an intermediary can face a rate of return shock $n \in \{1, 2, \dots, N\}$ with probability η_n , in which case the return on the long-term asset is $\hat{R}(n)$. We assume that there is no aggregate uncertainty and that

$$\sum_{n=1}^N \eta_n \hat{R}(n) = \hat{R},$$

$$\sum_{n=1}^N \eta_n = 1.$$

At time 0, there are interbank markets in which intermediaries trade N Arrow–Debreu securities. The price the security that pays 1 if state n occurs and 0 otherwise is q_n . Prices q_n are determined by a market clearing condition. It is immediate to see that intermediaries choose to fully insure themselves at $t = 0$ against idiosyncratic shocks. The problem of each intermediary then reduces to the case of no idiosyncratic shocks described above.

Our result that an optimal liquidity requirement that is uniform across all intermediaries is not necessarily true in more general models in which intermediaries have a dimension of *ex ante* or *ex post* heterogeneity. Consider, *e.g.* a model of Allen and Gale (2004) where there are shocks to intermediaries in addition to the shocks to the agents as well as incomplete markets for aggregate shocks. They show that a liquidity requirement improves upon the competitive market allocation. However, the optimal liquidity requirement in their model likely has to be conditioned on the risks of individual intermediaries. The same would be true in our model if intermediaries have shocks that cannot be fully insured and there is a nontrivial heterogeneity of intermediaries. A more general lesson is that, if there are several heterogeneous intermediaries operating in competitive markets, optimal liquidity requirement has to be conditioned on observable characteristics of such intermediaries (*e.g.* their risk).

5.4. Direct access to technology

Another variation of our setup is the case in which some agents have access to technology that yields \hat{R} directly without the need for financial intermediaries, while other agents need an intermediary to access the technology. If we modified our assumption that all activities at the level of intermediaries are observable and instead supposed that the regulator could observe the aggregate investment in the technology yielding \hat{R} , then our results would also hold. The constrained optimum in that model would be implemented by a tax on returns to investment of those who can access the technology and by a liquidity adequacy requirement on the financial intermediaries providing liquidity insurance for agents who cannot access the technology.

6. CONCLUSIONS

The theoretical mechanism of this paper addresses a critique of the financial intermediation literature: retrading puts significant limitations on the provision of insurance against liquidity shocks. We showed that a social planner can significantly improve the provision of insurance

against liquidity shocks even if agents can trade privately. Indeed, in the case of the widely used Diamond–Dybvig model, the social planner can achieve the first best. A simple intervention—liquidity requirement—can implement the constrained efficient allocation. The simplicity of the Diamond–Dybvig model allows for a transparent characterization of the market failure that we analyse and of the direction of the intervention needed to correct it.

Currently, other regulations—*e.g.* those aimed at controlling the risks taken by financial intermediaries—are already in place and contribute to alleviating the inefficiencies that we are describing. Understanding the precise interaction between prudential and liquidity, regulations are an important topic of research.

APPENDIX

A.1. Proof of Lemma 1

The proof of equivalence consists of two steps. First, we take a solution $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ to problem (21) and show that it may be implemented for some I and R satisfying (27), in the sense that $(c_1(\theta), c_2(\theta))$ would solve (24) for any θ . Second, we take I^* and R^* that solve problem (26) and show that in that case, $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ given by $c_t(\theta) = x_t^u(I, R; \theta)$ are feasible in problem (21).

Take any solution $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ to problem (21) and let R be the equilibrium interest rate for the private market given these endowments. Note that the incentive compatibility constraint (23) can be rewritten as

$$U(c_1(\theta), c_2(\theta); \theta) \geq \tilde{V}\left(c_1(\theta') + \frac{c_2(\theta')}{R}, R; \theta\right) \text{ for all } \theta, \theta',$$

meaning that agent of type θ does not get more than utility from endowment by pretending to be of type θ' and then trading in private market. By (24), this implies

$$\tilde{V}\left(c_1(\theta) + \frac{c_2(\theta)}{R}, R; \theta\right) \geq \tilde{V}\left(c_1(\theta') + \frac{c_2(\theta')}{R}, R; \theta\right) \text{ for all } \theta, \theta'.$$

Since \tilde{V} is strictly increasing in its first argument, this is equivalent to

$$c_1(\theta) + \frac{c_2(\theta)}{R} = \max_{\theta'}\left(c_1(\theta') + \frac{c_2(\theta')}{R}\right) \text{ for all } \theta, \theta'.$$

Consequently,

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R};$$

denote this value by I . Let us prove that for (I, R) , $(c_1(\theta), c_2(\theta))$, solve (24) for any θ ; this would automatically imply (27) if we take $x_t^u(I, R; \theta) = c_t(\theta)$ for all t, θ , because $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ satisfies (22). But $(c_1(\theta), c_2(\theta))$ constitute a solution (10) and, moreover, these are still a solution under the additional constraint $\theta' = \theta$, which is implicit in (24). This proves that any solution $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ to problem (21) may be implemented through the appropriate choice of I and R ; note that the values of maximands in (21) and (26) are equal for these parameter values; hence, the maximum in problem (26) is at least as large as one in problem (21) (recall that $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ is a solution to problem (21)).

Conversely, suppose that (I^*, R^*) solve problem (26); let $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ be given by $c_t(\theta) = x_t^u(I, R; \theta)$, where $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta \in \{0,1\}}$ is solution to (24) for (I^*, R^*) . We need to check that $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ is feasible for problem (26), *i.e.* satisfies (22) and (23). Clearly, (22) immediately follows from (27). Note that since (25) is binding, we must have

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R} = I^*,$$

meaning that

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R^*, \theta) = V(I^*, R^*; \theta) \text{ for all } \theta,$$

because with income I^* for both types and in the presence of private markets, a consumer does as well by reporting truthfully $\theta' = \theta$ as he would if he reported $\theta' \neq \theta$ in (10). Now (23) follows from the fact that $(x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta))$ solves (24). We have proved that if (I^*, R^*) solves problem (26) then the induced allocations are feasible in the social planner’s problem (21). Again, note that for these parameter values, the maximands are equal, which implies that the maximum in problem (21) is at least as large as one in problem (26).

We have proved that the maximums in problems (21) and (26) coincide. Consequently, any $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ that solves problem (21) induces (I^*, R^*) , which solves (26) and which is such that (28) holds, and vice versa. This proves the equivalence of problems. We have proved Lemma 1. \parallel

A.2. Proof of Theorem 1

We proceed as follows. We show that there exists (I^*, R^*) for which $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta \in [0,1]}$, found as solutions to (24), coincide with the solution to problem SP^2 (and thus problem SP^1 since the solutions to those coincide). By Lemma 1, if we define $\{c_1(\theta), c_2(\theta)\}_{\theta \in [0,1]}$ by (28), conditions (22) and (23) will be satisfied, provided that (27) is satisfied. This means that the solution to problems SP^1 and SP^2 is feasible for problem SP^3 . But this implies that it is also a solution to SP^3 because SP^3 is obtained from SP^1 by imposing additional constraints. Given that solution to SP^1 is unique, we conclude that so is solution to SP^3 , in particular, income I and interest rate R are determined uniquely.

Define (I^*, R^*) as solutions the following system of equations:

$$u'(I) = \rho \hat{R} u'(RI), \tag{31}$$

$$I = \frac{e}{\pi + (1-\pi)\frac{R}{\hat{R}}}. \tag{32}$$

In particular, R^* is a solution to

$$\frac{u'\left(\frac{e}{\pi + (1-\pi)\frac{R}{\hat{R}}}\right)}{u'\left(\frac{R e}{\pi + (1-\pi)\frac{R}{\hat{R}}}\right)} - \rho \hat{R} = 0. \tag{33}$$

Denote by $f(R)$ the left-hand side of (33). Note that $f(R)$ is increasing and $f(1) = 1 - \rho \hat{R} < 0$, so that the solution to (33) involves $R^* > 1$. Since the coefficient of relative risk aversion is everywhere greater than 1 and $R^* > 1$:

$$\frac{u'\left(\frac{e}{\pi + (1-\pi)\frac{R^*}{\hat{R}}}\right)}{u'\left(\frac{R^* e}{\pi + (1-\pi)\frac{R^*}{\hat{R}}}\right)} \geq R;$$

this shows that $R^* \leq \rho \hat{R}$. Note that if $u(c) = \log(c)$, we have $R^* = \rho \hat{R}$. We have shown that $R^* \in (1, \rho \hat{R}]$. Moreover, this argument establishes existence and uniqueness of (I^*, R^*) : indeed, as we just showed R^* is uniquely determined by (33) and then I^* is uniquely determined by (32).

Now let us solve (24) for (I^*, R^*) given by (31) and (32). First, take $\theta = 0$; then (24) becomes

$$\max_{x_1, x_2} u(x_1)$$

s.t. (25); the solution is, obviously, $x_1^u(I^*, R^*, 0) = I^*$, $x_2^u(I^*, R^*, 0) = 0$. Now take $\theta = 1$; the problem becomes

$$\max_{x_1, x_2} \theta \rho u(x_1 + x_2)$$

s.t. (25); since $R^* > 1$, the solution is $x_1^u(I^*, R^*, 1) = R^* I^*$, $x_2^u(I^*, R^*, 1) = 0$. Let us check that (8) and (9) are satisfied (for $c_t(\theta) = x_t^u(I^*, R^*; \theta) \forall t, \theta$). Indeed,

$$u'(c_1(0)) = u'(I^*) = \rho \hat{R} u'(R^* I^*) = \rho \hat{R} u'(c_2(1));$$

we used (31) here, and, similarly,

$$\pi c_1(0) + (1-\pi)\frac{c_2(1)}{\hat{R}} = \pi I^* + (1-\pi)\frac{R^* I^*}{\hat{R}} = I^* \left(\pi + (1-\pi)\frac{R^*}{\hat{R}} \right) = e,$$

because (I^*, R^*) solves (32). This proves that solution $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta \in [0,1]}$ of problem (24) satisfies (7), (8), and (9) and therefore is a solution to SP^1 and SP^2 . For this allocation, the feasibility constraint (4) is satisfied and therefore (27) follows (one could also check (27) by plugging $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta \in [0,1]}$ found above). Therefore, (I^*, R^*) lead to a feasible solution of (24), and the allocations obtained for (I^*, R^*) coincide with allocations that solve SP^1 and SP^2 . Since SP^3 is a constrained version of these, then (I^*, R^*) is a solution to SP^3 . This proves Theorem 1. \parallel

A.3. Proof of Proposition 1

The program of an intermediary when the interest rate is R^* and the liquidity requirement is i^* is

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}} \pi \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R^*, 0) + (1 - \pi) \rho \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R^*, 1)$$

s.t.

$$\pi c_1(0) + (1 - \pi)c_1(1) \geq i^* e, \tag{34}$$

$$c_1(0) + \frac{c_2(0)}{R^*} = c_1(1) + \frac{c_2(1)}{R^*}, \tag{35}$$

$$\pi \left(c_1(0) + \frac{c_2(0)}{\hat{R}} \right) + (1 - \pi) \left[c_1(1) + \frac{c_2(1)}{\hat{R}} \right] \leq e. \tag{36}$$

Here, (35) stems from (18) since we assumed that the interest rate faced by intermediaries and consumers is R^* , while (36) is simply (20).

Since, as we proved, $1 < R^* \leq \rho \hat{R} < \hat{R}$, then for any menu of contracts $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$, each agent of type $\theta = 0$ will, after trading on the private market, end up with first period good only, and each agent of type $\theta = 1$ will end up with second period good only (under the condition that the market clears). To put it differently,

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R^*, 0) = u\left(c_1(0) + \frac{c_2(0)}{R^*}\right)$$

(in the parentheses, we have the wealth of agents of type $\theta = 0$) and

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}, R^*, 1) = \rho u\left(R^* \left(c_1(1) + \frac{c_2(1)}{R^*}\right)\right).$$

Since markets clear, the intermediaries may as well offer contracts with $c_2(0) = c_1(1) = 0$, merely providing each consumer with what he would get as a result of the trade. We can therefore write the program as

$$\max_{c_1(0), c_2(1)} \pi u(c_1(0)) + (1 - \pi) \rho u(c_2(1))$$

s.t.

$$\pi c_1(0) \geq i^* e, \tag{37}$$

$$\pi c_1(0) + (1 - \pi) \frac{c_2(1)}{\hat{R}} \leq e. \tag{38}$$

This problem is convex. Hence, if we prove that $c_1(0) = I^*$ and $c_2(1) = R^* I^*$ satisfies both the constraints and the first-order conditions, this would imply that the solution to SP^3 is also the unique competitive equilibrium with the liquidity floor. It is easy to check that constraints are satisfied as equalities; indeed,

$$\pi c_1(0) = \pi I^* = i^* e$$

by (30), and

$$\pi I^* + (1 - \pi) \frac{R^* I^*}{\hat{R}} = I^* \left(\pi + (1 - \pi) \frac{R^*}{\hat{R}} \right) = e,$$

because (I^*, R^*) satisfies (32). Because of complementary slackness conditions, we only need to verify that the Lagrange multipliers λ on the constraint (37) and μ on the constraint (38) are nonnegative. The first-order conditions are as follows:

$$\pi u'(I^*) = \pi (-\lambda + \mu),$$

$$(1 - \pi) u'(R^* I^*) = (1 - \pi) \mu \frac{1}{\hat{R}}.$$

Rearranging and using the fact that $u'(I^*) = \rho \hat{R} u'(R^* I^*)$ (which holds since (I^*, R^*) satisfies (31)), we get

$$\mu = \hat{R} u'(R^* I^*) > 0,$$

$$\lambda = (1 - \rho) \hat{R} u'(R^* I^*) > 0.$$

Moreover, no agent wants to borrow or lend at the interest rate $R^* > 1$, so the market indeed clears for such allocation. This completes the proof of Proposition 1.

Acknowledgements. Golosov and Tsyvinski acknowledge support by the National Science Foundation. Golosov thanks University of Chicago, Ente “Luigi Einaudi”, and Minneapolis Fed for hospitality. Tsyvinski thanks Ente “Luigi Einaudi” for hospitality. We are grateful to the editor Bruno Biais and three anonymous referees for detailed comments. We are especially grateful to one of the referees who provided exceptionally detailed comments. We also thank Daron Acemoglu, Stefania Albanesi, Franklin Allen, Marios Angeletos, Ricardo Caballero, V. V. Chari, Georgy Egorov, Ed Green, Christian Hellwig, Skander Van den Heuvel, Bengt Holmström, Oleg Itskhoki, Guido Lorenzoni, Chris Phelan, Bernard Salanié, Jean Tirole, Ivan Werning, and Ruilin Zhou for comments. We thank the audiences at the Bank of Canada, Harvard, MIT, Minneapolis Federal Reserve, Society of Economic Dynamics, UT Austin, and Wharton for useful comments.

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