Supplemental Material for
"Sticky Discount Rates"[*]

Masao Fukui   Niels Joachim Gormsen   Kilian Huber

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Supplement A   Comparing Different Investment Rules

The aim of this section is to explain why investment rules based on the stochastic discount factor and on a discount rate lead to similar investment decisions, as long as the discount rate is chosen in a certain way. Moreover, the section clarifies why textbooks recommend that firms should set their discount rate equal to the cost of capital. The discussion here is based on Gormsen and Huber (2023).

Setup   In models with uncertainty, firms can generally maximize market value by using the stochastic discount factor to discount future cash flows. Textbooks aimed at managers nonetheless tend to present simpler rules based on a discount rate. We illustrate that the two methods lead to similar investment outcomes using the example of a simple project with uncertain returns. This project generates expected revenue $\mathbb{E}_t[\text{Revenue}_{t+j}]$ in period $t+j$ and costs Cost_t in period t.

Using the Stochastic Discount Factor   The first decision rule states that the firm accepts the project if the net present value, discounted using the stochastic discount factor M_{t+j}, is positive:

$$\mathbb{E}_t\left[M_{t+j}\text{Revenue}_{t+j}\right] - \text{Cost}_t > 0. \tag{S1}$$

Using the definition of covariance, we can rewrite equation S2 as:

$$\mathbb{E}_t\left[\text{Return}_{t,t+j}\right] > R^f_{t,t+j} - \text{Cov}_t\left[M_{t+j},\text{Return}_{t,t+j}\right] R^f_{t,t+j}, \tag{S2}$$

where $R^f_{t,t+j} = \mathbb{E}_t\left[M_{t+j}\right]^{-1}$ is the risk-free interest rate between t and $t+j$ and $\text{Return}_{t,t+j} = \frac{\text{Revenue}_{t+j}}{\text{Cost}_t}$ is the return to the project.

Using a Discount Rate   The second rule states that the firm accepts the project if the net present value of the project, discounted using a discount rate δ_t, is positive:

$$\sum_{s=0}^{\infty}(1+\delta_t)^{-s}\,\mathbb{E}_t[\text{Revenue}_{t+s} - \text{Cost}_{t+s}] = (1+\delta_t)^{-j}\,\mathbb{E}_t[\text{Revenue}_{t+j}] - \text{Cost}_t > 0. \tag{S3}$$

This rule can also be rewritten as saying that the firm should invest if the return to the project exceeds a "hurdle" rate, such that:

$$\mathbb{E}_t\left[\text{Return}_{t,t+j}\right] > (1+\delta_t)^j. \tag{S4}$$

The two rules in equations S2 and S4 are equivalent, as long as the firm sets the discount rate such that:

$$(1+\delta_t)^j = R^f_{t,t+j} - \text{Cov}_t\left[M_{t+j},\text{Return}_{t,t+j}\right] R^f_{t,t+j}. \tag{S5}$$

Hence, for a given project, the rules based on the stochastic discount factor and the discount rate lead to the same investment outcome if the chosen discount rate satisfies equation S5.

Choosing the Discount Rate and the Cost of Capital   To determine the discount rate given by equation S5, the firm can use financial prices. Assume that the firm issues just one financial asset (e.g., only equity). By definition, the expected return to the financial asset of firm i over one period is equal to 1 plus the firm's "financial cost of capital," given by r^{fin}_{it}. The basic asset pricing equation implies that the

expected return to the financial asset over the lifetime of the project is:

$$(1 + r_{it}^{\text{fin}})^j = \mathbb{E}_t \left[R_{t,t+j}^i \right] = R_{t,t+j}^f - \text{Cov}_t \left[M_{t+j}, R_{t,t+j}^i \right] R_{t,t+j}^f. \tag{S6}$$

If the covariance between the stochastic discount factor and the project return is identical to the covariance between the stochastic discount factor and the financial asset return (i.e., $\text{Cov}_t \left[M_{t+j}, R_{t,t+j}^i \right] = \text{Cov}_t \left[M_{t+j}, \text{Return}_{t,t+j} \right]$), then the rules in equations S2 and S4 are equivalent for a firm that sets the discount rate equal to its financial cost of capital. Intuitively, if the project under consideration exhibits the same risk profile as the firm's existing investments, then the financial cost of capital tells the firm how financial markets price the risk of the project.

Generalizations The above results generalize to firms with multiple liabilities (e.g., debt and equity). In such cases, r_{it}^{fin} is the weighted average cost of capital, where the expected return is separately estimated for each asset type and weights are calculated using the value of outstanding assets of that type relative to firm total assets, accounting for differential tax treatments of different assets.

The results can also be extended to investments with more complex cash flows. For instance, consider an investment consisting of multiple sub-projects, indexed by s, where each project requires a cost in period t and pays uncertain revenue in one period $t + j$. In that case, the firm could still apply a decision rule as in equations S2 and S4, by summing over the individual sub-projects s.

If $\text{Cov}_t \left[M_{t+j}, R_{t,t+j}^i \right] \neq \text{Cov}_t \left[M_{t+j}, \text{Return}_{t,t+j} \right]$, firms cannot infer the riskiness of an individual project using expected returns on the firm's existing financial assets. Instead, firms should then adjust the discount factor by a project-specific risk premium.

Supplement B   Non-Linear Characterization of the Firm Problem

Due to the constant returns to scale assumptions, each individual firm's problem is independent of its size, k_t. Let $\iota_t = I_t / k_t$ be the investment rate of the firm. It is

easy to verify that the firm's value functions is linear in the capital stock:

$$V_t^I(k_t, \delta) = v_t^I(\delta) P_t k_t, \tag{S7}$$

where $v_t^d(\delta)$ denotes the real marginal value of unit capital. It solves the following Bellman equation:

$$v_t^I(\delta) = \max_{\iota_t} \omega_t - \iota_t - \varphi(\iota_t) + \mathbb{E}_t \frac{1 + \pi_{t+1}}{1 + \delta} \{(1 - \xi) + \iota_t\} v_{t+1}^I(\delta), \tag{S8}$$

where $1 + \pi_{t+1} \equiv P_{t+1}/P_t$ is the gross inflation rate, ω_t are real profits from unit capital,

$$\omega_t \equiv \max_l \frac{1}{P_t}(p_t F_t(1, l) - W_t l), \tag{S9}$$

and $\varphi(\iota) \equiv \Phi(\iota, 1)$. The first-order optimality condition for investment is

$$1 + \varphi'(\iota_t) = \mathbb{E}_t \frac{1 + \pi_{t+1}}{1 + \delta} v_{t+1}^I(\delta) \tag{S10}$$

Likewise, the firm's financial market value is also linear in capital:

$$V_t^a(k) = v_t^a P_t k, \quad V_t^n(k, \delta) = v_t^n(\delta) P_t k, \tag{S11}$$

where v_t^a and v_t^n solve the following recursion

$$v_t^a = \max_{\delta^*} \omega_t - \iota_t - \varphi(\iota_t) + \mathbb{E}_t \frac{1 + \pi_{t+1}}{1 + i_t} \{(1 - \xi) + \iota_t\} \left[\theta v_{t+1}^n(\delta^*) + (1 - \theta)v_{t+1}^a\right]$$
$$\tag{S12}$$

$$\text{s.t.} \quad \iota_t = \bar{\iota}_t(\delta^*) \tag{S13}$$

and

$$v_t^n(\delta^*) = \omega_t - \iota_t - \varphi(\iota_t) + \mathbb{E}_t \frac{1 + \pi_{t+1}}{1 + i_t} \{(1 - \xi) + \iota_t\} \left[\theta v_{t+1}^n(\delta^*) + (1 - \theta)v_{t+1}^a\right]$$
$$\tag{S14}$$

$$\text{s.t.} \quad \iota_t = \bar{\iota}_t(\delta^*) \tag{S15}$$

The first-order optimality condition for the choice of the discount rate is

$$
\mathbb{E}_t \left[\frac{1 + \pi_{t+1}}{1 + i_t} [\theta v_{t+1}^n(\delta^*) + (1 - \theta)v_{t+1}^a] - (1 + \varphi'(\iota_t)) \right] \frac{d\bar{\iota}_t}{d\delta_t^*}
$$
$$
+ \mathbb{E}_t \frac{1 + \pi_{t+1}}{1 + i_t} \{(1 - \xi) + \iota_t\} \theta \frac{dv_{t+1}^n(\delta^*)}{d\delta^*} = 0. \tag{S16}
$$

Supplement C   Heterogeneous-Agent Model with Sticky Discount Rates

We consider a richer heterogeneous-agent New Keynesian model with investment along the lines of Kaplan et al. (2018) and Auclert et al. (2020). Instead of assuming that a fraction μ of households is hand-to-mouth, we allow these households to access financial markets subject to uninsurable idiosyncratic income risk. We call these households workers. We index workers with superscript h and permanent income households with superscript p.

Workers experience idiosyncratic productivity shocks e, which follow a Markov process. Workers save in risk-free assets that give a deterministic return of $1 + r_t$. Workers face a borrowing constraint of the form $b_t \geq \underline{b}$ for liquid asset holdings. As in the baseline, unions make labor supply decisions that are the same for all households, L_t. We assume that lump-sum transfers are imposed proportionally to household idiosyncratic productivity. The worker's problem in recursive form is

$$
U_t(b, e) = \max_{c, b' \geq \underline{b}} u(c_t) + \beta^h \mathbb{E}_t[U_{t+1}(b', e')]
$$
$$
\text{s.t.} \quad c + b' = (1 + r_t)b + e \left[(W_t/P_t)L_t(1 - \tau^l) - T_t \right].
$$

Let $c_t^h(b, e, a)$ denote the policy function. Aggregate consumption of workers is $C_t^h \equiv \int c_t^h(b, e) dH_t(b, e)$, where H_t denotes the joint distribution of assets and idiosyncratic worker productivity in period t.

Permanent-income households solve the identical problem as in the main text and face no idiosyncratic risk. Aggregate consumption is

$$
C_t = \mu C_t^h + (1 - \mu)C_t^p. \tag{S17}
$$

Our baseline two-agent structure is nested as a special case where $\beta^h \to 0$.

There is a risk-neutral financial intermediary that issues liquid deposits to households and invests them in a diversified portfolio of firm shares and government bonds. The no-arbitrage conditions imply

$$\mathbb{E}_t \left[\frac{V_{t+1} + D_{t+1}}{V_t} \right] = \mathbb{E}_t[1 + r^p_{t+1}] = \mathbb{E}_t[1 + r_{t+1}], \qquad \text{(S18)}$$

where V_t is the value of firm shares and D_t is the dividend paid by firms.

Following Auclert et al. (2023), we assume wages are sticky and prices are flexible. The wage Phillips curve is becomes

$$\hat{\pi}^w_t = \psi_w \left[\sigma \hat{C}_t + \sigma \int_0^1 \vartheta^i (\chi^i_t - \chi^i)/\chi^i di + \nu \hat{L}_t - \hat{W}_t + \hat{P}_t \right] + \frac{1}{1+r} \mathbb{E}_t \hat{\pi}^w_{t+1}, \quad \text{(S19)}$$

where $\vartheta^i = \frac{\tilde{\vartheta}^i u'(C^i)}{\int_0^1 \tilde{\vartheta}^i u'(C^i) di}$ is the weight placed on the utility of household $i \in [0,1]$, and $\psi_w \equiv (1 - \gamma_w)(1 - \beta \gamma_w)/\gamma_w$. The rest of the model remains unchanged.

We calibrate the economy as follows. We set the share of workers to 80%, $\mu = 0.8$, reflecting that the majority of household assets are concentrated at the top of the wealth distribution. The discount factor of permanent-income households matches the steady-state annualized cost of capital of 7%. The income process is an AR(1) process with autocorrelation 0.966 and a standard deviation of 0.13, following McKay et al. (2016). We discretize the income process using the methodology in Rouwenhorst (1995) with 7 grid points. The borrowing limit of workers is zero, $\underline{b} = 0$. We choose the discount factor of workers, β^h, to target an average annual marginal propensity to consume (MPC) out of a one-time transfer of 0.5, in line with the estimates of Fagereng et al. (2021). Finally, we set $\vartheta^i = 1$ so that the New Keynesian wage Phillips curve only depends on aggregate values. The wage stickiness parameter is $\gamma_w = 0.85$ and prices are flexible, $\gamma_p = 0$. The remaining calibration is the same as in the baseline model.

Figures S1, S2, and S3 present impulse response functions to the government spending, patience, and inflation target shocks. The qualitative conclusions are similar to the baseline model. Quantitatively, we find a larger amplification effect relative to our baseline economy even though the impact MPC is similar in both
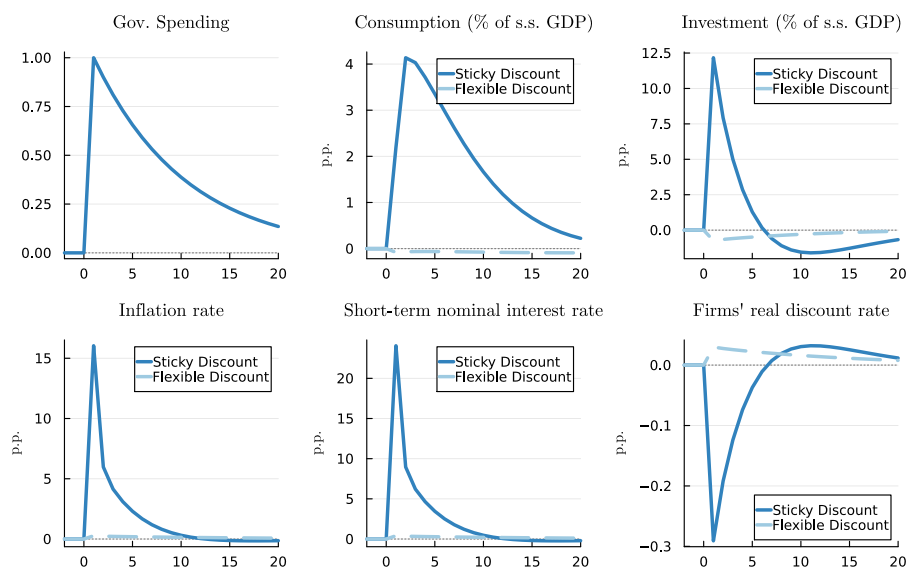
Figure S1: Heterogeneous Agents: Impulse Responses to a Government Spending Shock

The figure plots the impulse response to a government spending shock for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

models, consistent with Auclert et al. (2023).[S1]

[S1]In our baseline model, the MPC of hand-to-mouth households is 1 and the MPC of permanent income households is $1 - \beta = 0.017$. Since 30% of households are hand-to-mouth, the average MPC is $0.3 \times 1 + 0.7 \times 0.017 \approx 0.31$. In our heterogenous-agent model, the average quarterly impact MPC is 0.25.
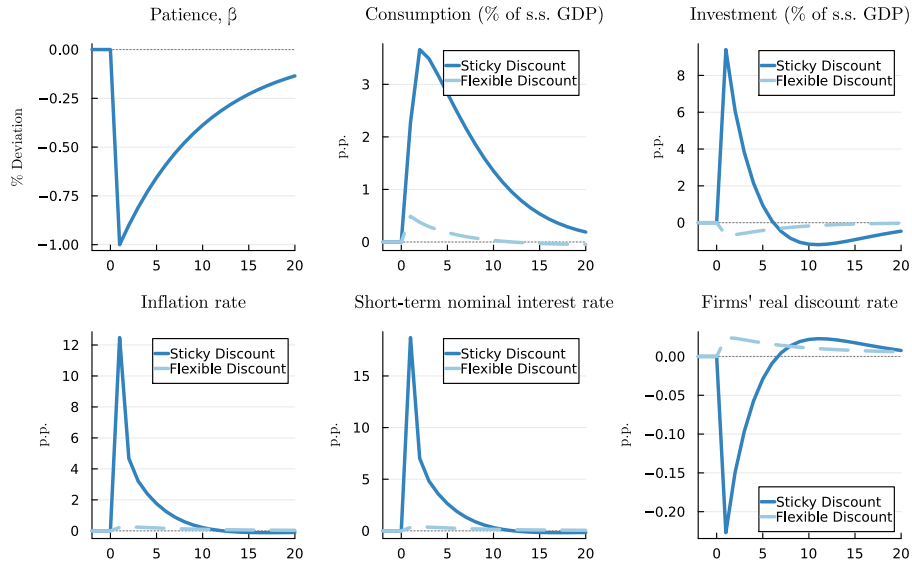
Figure S2: Heterogeneous Agents: Impulse Responses to a Household Patience Shock

The figure plots the impulse response to a household patience shock for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.
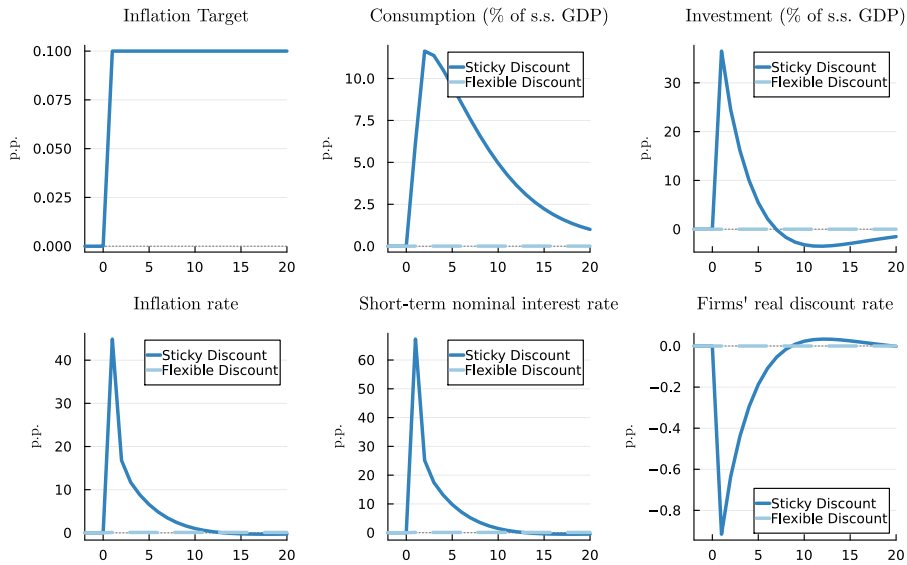

Figure S3: Heterogeneous Agents: Impulse Responses to an Inflation Target Shock

The figure plots the impulse response to an inflation target shock for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

Supplement D   Real Business Cycle Model with Sticky Discount Rates

We present a simple representative-agent real business cycle (RBC) model augmented with sticky discount rates by setting $\mu = 0$ and $\gamma_p = \gamma_w = 0$.

Figures S4 and S5 present impulse responses to a government spending shock and a household patience shock under $\mu = 0$ and $\gamma_w = \gamma_p = 0$. In both cases, sticky discount rates reverse the sign of the investment response. Unlike the two-agent New Keynesian model, the RBC model does not generate comovement between consumption and investment following the patience shock.

Figure S6 shows impulse responses to an inflation target shock. With flexible discount rates, the inflation target is entirely neutral for real outcomes. With sticky discount rates, the inflation target shock stimulates investment because of the direct link between expected inflation and real discount rates. This finding highlights that sticky discount rates are an independent source of monetary non-neutrality.
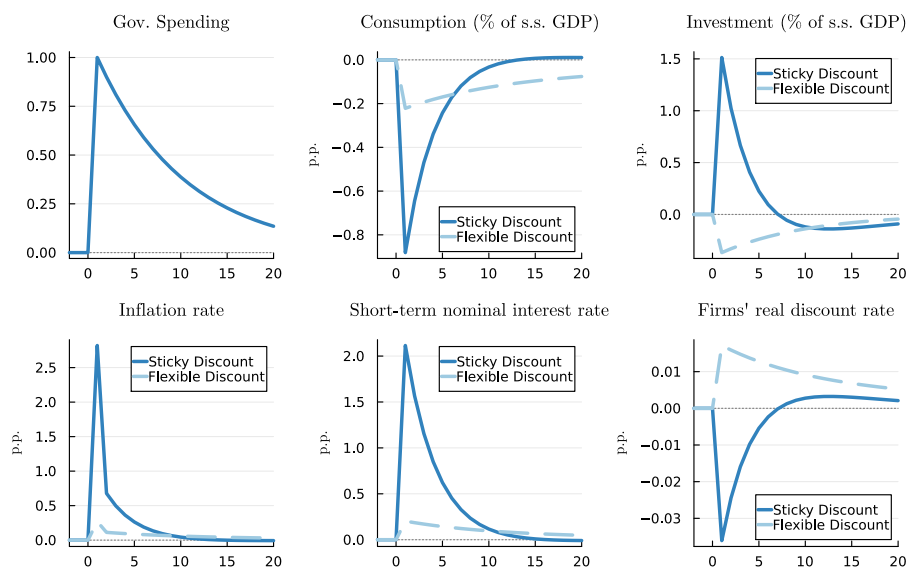

Figure S4: RBC: Impulse Responses to a Government Spending Shock

The figure plots the impulse response to a government spending shock under flexible prices ($\gamma_p = 0$) and no hand-to-mouth households ($\mu = 0$)for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.
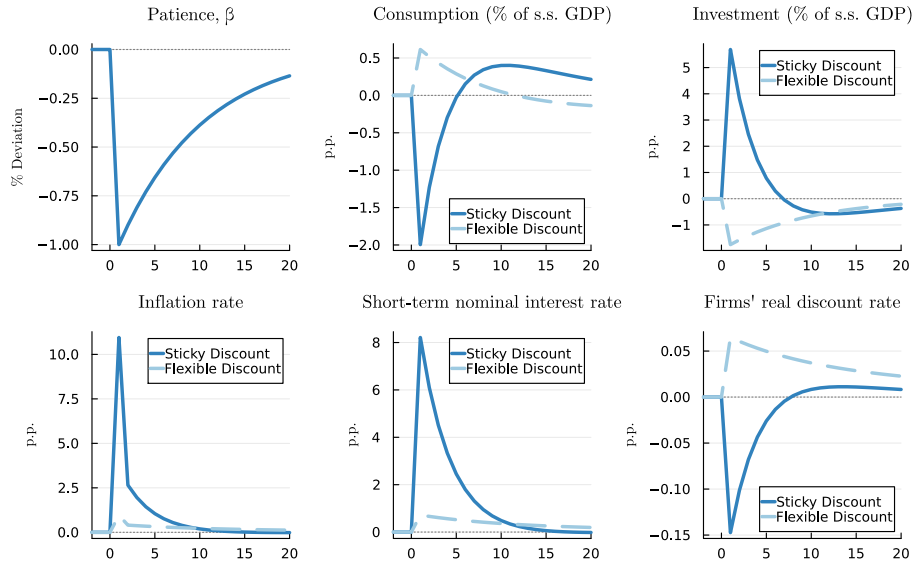
Figure S5: RBC: Impulse Responses to a Household Patience Shock

The figure plots the impulse response to a patience shock under flexible prices ($\gamma_p = 0$) and no hand-to-mouth households ($\mu = 0$)for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.
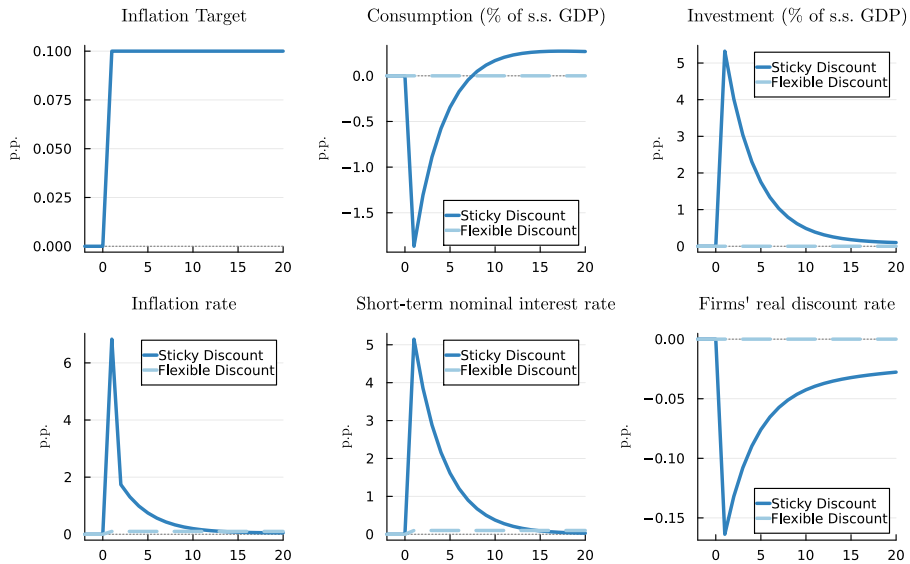

Figure S6: RBC: Impulse Responses to an Inflation Target Shock

The figure plots the impulse response to an increase in long-run inflation target by 0.1 p.p. under flexible prices ($\gamma_p = 0$) and no hand-to-mouth households ($\mu = 0$)for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

Supplement E   Optimal Monetary Policy: Derivation of the Linear-Quadratic Approximation of the Welfare Loss Function

We consider a linear-quadratic approximation of the optimal monetary policy problem around the efficient deterministic steady state. Let $\hat{x}_t = \log x_t/x$ denote the deviation of a variable x from its steady-state value. We will often invoke the relationship:

$$\frac{x_t - x}{x} \approx \hat{x}_t + \frac{1}{2}\hat{x}_t^2. \tag{S20}$$

We define welfare in the economy as

$$W = \sum_{t=0}^{\infty} \prod_{s=0}^{t} \beta_s \left[\sum_{i \in \{h,p\}} \mu^i \Gamma^i u(C_t^i) - v(L_t) \right], \tag{S21}$$

where Γ^i denotes the Pareto weight on household type $i \in \{h, p\}$, $\mu^h = \mu, \mu^p = 1 - \mu$, and $i = h, p$ denotes hand-to-mouth households and permanent income households, respectively. Following McKay and Wolf (2022), we assume that given the relative consumption of two types of agents in the steady state, the Pareto weights, $\{\Gamma^i\}$, and employment subsidy, τ^l, are such that the steady-state allocation is efficient. We denote χ_t^i as the consumption of each household type relative to aggregate consumption:

$$C_t^i = \chi_t^i C_t. \tag{S22}$$

The optimality of the steady state implies that the Pareto weights are such that the marginal utility of consumption is equalized across two agents:

$$\Gamma^i u'(\chi^i C) \equiv \bar{u}'(C) \quad \text{for } i = p, h. \tag{S23}$$

Moreover, the definition of the consumption shares satisfies

$$\sum_i \mu^i \chi_t^i = 1. \tag{S24}$$

The second-order approximation of utility from consumption is

$$\sum_i \mu^i \Gamma^i u(\chi_t^i C_t) \approx \bar{u}'(C)C \left(\hat{C}_t + \frac{1}{2}(1-\sigma)\hat{C}_t^2 \right) \tag{S25}$$

$$+ \bar{u}'(C)C \sum_i \mu^i (\chi_t^i - \chi^i) - \sigma \bar{u}'(C)C \sum_i \mu^i \frac{1}{2}\frac{1}{\chi^i}(\chi_t^i - \chi^i)^2 \tag{S26}$$

$$+ \bar{u}'(C)C(1-\sigma)\hat{C}_t \sum_i \mu^i (\chi_t^i - \chi^i) \tag{S27}$$

$$= \bar{u}'(C)C \left(\hat{C}_t + \frac{1}{2}(1-\sigma)\hat{C}_t^2 - \sigma \sum_i \mu^i \frac{1}{2}\frac{1}{\chi^i}(\chi_t^i - \chi^i)^2 \right), \tag{S28}$$

where the second line follows from (S24).

Therefore, the second-order approximation of (S21) around the steady state is

$$W \approx \sum_{t=0}^{\infty} \beta^t \left[\bar{u}'(C)C \left(\hat{C}_t + \frac{1}{2}(1-\sigma)\hat{C}_t^2 - \frac{1}{2}\sigma \sum_i \mu^i \frac{1}{\chi^i}(\chi_t^i - \chi^i)^2 \right) \right.$$

$$\left. - v'(L)L \left(\hat{L}_t + \frac{1+\nu}{2}\hat{L}_t^2 \right) + \bar{u}'(C)C \sum_{s=0}^{t} \hat{\beta}_s \hat{C}_t - v'(L)L \sum_{s=0}^{t} \hat{\beta}_s \hat{L}_t \right] + t.i.p.,$$

$$\tag{S29}$$

where $t.i.p.$ denotes a set of terms independent of policies. The resource constraint is given by

$$\int \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di \left[C_t + I_t + \int \varphi \left(\frac{\iota_t(i)}{\iota_t} \iota_t \right) di K_t + G_t \right] = A_t F \left(K_t, L_t^f \right), \tag{S30}$$

where i indexes a price-setting firm. The aggregate labor demand, L_t^f, consists of a variety of labor services supplied through labor unions:

$$L_t = \int \left(\frac{W_t(\ell)}{W_t} \right)^{-\varepsilon_w} d\ell L_t^f, \tag{S31}$$

where ℓ indexes a labor union. Let $\hat{p}_t(i) = \log P_t(i) - \log P_t$ and $\hat{w}_t(\ell) = \log W_t(\ell) - \log W_t$.

The misallocation resulting from price dispersion can be expressed as:

$$\int \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di \approx 1 - \varepsilon \int \hat{p}_t(i) di + \frac{\varepsilon^2}{2} \int \hat{p}_t(i)^2 di. \tag{S32}$$

Since $\int \left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon} di = 1$ by the definition of the price index, we also have

$$1 \approx 1 + (1-\varepsilon) \int \hat{p}_t(i) di + \frac{(1-\varepsilon)^2}{2} \int \hat{p}_t(i)^2 di. \tag{S33}$$

Combining the previous two expressions, price dispersion is

$$\int \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di \approx \frac{\varepsilon}{2} \int \hat{p}_t(i) di \tag{S34}$$

$$= \frac{\varepsilon}{2} \mathrm{var}(\hat{p}_t(i)). \tag{S35}$$

Likewise, wage dispersion is

$$\int \left(\frac{W_t(\ell)}{W_t} \right)^{-\varepsilon_w} d\ell \approx \frac{\varepsilon_w}{2} \int \hat{W}_t(\ell) d\ell \tag{S36}$$

$$= \frac{\varepsilon_w}{2} \mathrm{var}(\hat{w}_t(\ell)). \tag{S37}$$

In a similar vein, we can define $\hat{\iota}^d(i) = \log(\iota_t(i)/\iota_t)$ and express the misallocation from investment dispersion as

$$\int \varphi \left(\frac{\iota_t(i)}{\iota_t} \iota_t \right) di \approx \frac{1}{2} \varphi''(\iota) \iota^2 \int \hat{\iota}^d(i)^2 di + \frac{1}{2} \varphi''(\iota) \iota^2 \hat{\iota}_t^2$$

$$= \frac{1}{2} \varphi''(\iota) \iota^2 \mathrm{var}(\hat{\iota}^d(i)) + \frac{1}{2} \varphi''(\iota) \iota^2 \hat{\iota}_t^2. \tag{S38}$$

Therefore, the second-order approximation of (S30) is

$$C \left(\hat{C}_t + \frac{1}{2} \hat{C}_t^2 \right) + I \left(\hat{I}_t + \frac{1}{2} \hat{I}_t^2 \right) + \frac{1}{2} \phi K \iota^2 \mathrm{var}(\hat{\iota}^d(i)) + \frac{1}{2} \phi K \iota^2 \hat{\iota}_t^2 + G_t + \frac{1}{2} G_t^2$$

$$= Y \left[-\frac{\varepsilon}{2} \mathrm{var}(\hat{p}_t(i)) + \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right]. \tag{S39}$$

The Cobb-Douglas production function implies that $Y_t = F_t(K, L) = A_t K^\alpha L^{1-\alpha}$ and the second-order approximation gives

$$\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2 = \hat{A}_t + \frac{1}{2}(\hat{A}_t)^2 + \alpha(\hat{K}_t + \frac{1}{2}\hat{K}_t^2) + (1-\alpha)(\hat{L}_t^f + \frac{1}{2}(\hat{L}_t^f)^2) - \frac{1}{2}\alpha(1-\alpha)(\hat{L}_t^f - \hat{K}_t)^2$$
$$+ (1-\alpha)\hat{A}_t\hat{L}_t^f + \alpha\hat{A}_t\hat{K}_t. \tag{S40}$$

The second-order approximation of (S31) is

$$\hat{L}_t + \frac{1}{2}\hat{L}_t^2 = \hat{L}_t^f + \frac{1}{2}(\hat{L}_t^f)^2 + \frac{\varepsilon_w}{2}\text{var}(\hat{w}_t(\ell)). \tag{S41}$$

Combining (S40) and (S41),

$$\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2 = \hat{A}_t + \frac{1}{2}(\hat{A}_t)^2 + \alpha(\hat{K}_t + \frac{1}{2}\hat{K}_t^2) + (1-\alpha)\left(\hat{L}_t + \frac{1}{2}\hat{L}_t^2 - \frac{\varepsilon_w}{2}\text{var}(\hat{w}_t(\ell))\right)$$
$$- \alpha(1-\alpha)(\hat{L}_t - \hat{K}_t)^2 + (1-\alpha)\hat{A}_t\hat{L}_t + \alpha\hat{A}_t\hat{K}_t. \tag{S42}$$

The second-order approximation of the capital accumulation equation, $K_{t+1} = (1-\xi)K_t + I_t$, is

$$K\left(\hat{K}_{t+1} + \frac{1}{2}\hat{K}_{t+1}^2\right) = (1-\xi)K\left(\hat{K}_t + \frac{1}{2}\hat{K}_t^2\right) + I\left(\hat{I}_t + \frac{1}{2}\hat{I}_t^2\right). \tag{S43}$$

Using (S39), one can rewrite $\bar{u}'(C)C\hat{C}_t$ as

$$\bar{u}'(C)C\hat{C}_t = \bar{u}'(C)\left\{Y\left[-\frac{\varepsilon}{2}\text{var}(\hat{p}_t(i)) - \frac{\varepsilon_w}{2}\text{var}(\hat{w}_t(\ell)) + \alpha(\hat{K}_t + \frac{1}{2}\hat{K}_t^2) + (1-\alpha)(\hat{L}_t + \frac{1}{2}\hat{L}_t^2)\right.\right.$$
$$\left. - \frac{1}{2}\alpha(1-\alpha)(\hat{L}_t - \hat{K}_t)^2\right] - \frac{1}{2}C\hat{C}_t^2 - I\left(\hat{I}_t + \frac{1}{2}\hat{I}_t^2\right) - \frac{1}{2}\varphi''(\iota)\iota^2 K\text{var}(\hat{\iota}^d(i))$$
$$\left. - \frac{1}{2}\varphi''(\iota)\iota^2 K\hat{\iota}_t^2 + Y\left[(1-\alpha)\hat{A}_t\hat{L}_t + \alpha\hat{A}_t\hat{K}_t\right]\right\} + t.i.p. \tag{S44}$$

Using (S43), we can express

$$\bar{u}'(C)I\hat{I}_t = \bar{u}'(C)\left[K\left(\hat{K}_{t+1} + \frac{1}{2}\hat{K}_{t+1}^2\right) - (1-\xi)K\left(\hat{K}_t + \frac{1}{2}\hat{K}_t^2\right) - I\frac{1}{2}\hat{I}_t^2\right]. \tag{S45}$$

Using (S45) and (S44), we can rewrite (S29) as

$$
\begin{aligned}
W \approx \bar{u}'(C) \sum_t \beta^t \Big[&-Y\frac{\varepsilon}{2}\mathrm{var}(\hat{p}_t(i)) - (1-\alpha)Y\frac{\varepsilon_w}{2}\mathrm{var}(\hat{w}_t(\ell)) - \frac{1}{2}\alpha(1-\alpha)Y(\hat{L}_t - \hat{K}_t)^2 \\
&- \frac{1}{2}\phi\xi^2 K\mathrm{var}(\hat{\imath}^d(i)) - \frac{1}{2}\phi\xi^2 K(\hat{I}_t - \hat{K}_t)^2 - \frac{1}{2}\sigma C\hat{C}_t^2 - (1-\alpha)Y\frac{\nu}{2}\hat{L}_t^2 \\
&+ C\sum_{s=0}^t \hat{\beta}_s\hat{C}_t - (1-\alpha)Y\sum_{s=0}^t \hat{\beta}_s\hat{L}_t + (1-\alpha)Y\hat{A}_t\hat{L}_t + \alpha Y\hat{A}_t\hat{K}_t \\
&- \sigma C\sum_i \mu^i \frac{1}{2}\frac{1}{\chi^i}(\chi_t^i - \chi^i)^2 \Big] + t.i.p.,
\end{aligned}
$$

where we have used the fact that

$$
v'(L) = (1-\alpha)Y/L\bar{u}'(C) \tag{S46}
$$

$$
1 = \beta(\alpha Y/K + (1-\xi)) \tag{S47}
$$

hold in steady state.

Price dispersion can be expressed as a function of inflation

$$
\sum_t \beta^t \mathrm{var}(\hat{p}_t(i)) = \frac{\gamma_p}{(1-\gamma_p)(1-\gamma_p\beta)} \sum_t \beta^t \hat{\pi}_t^2. \tag{S48}
$$

Likewise, wage dispersion is

$$
\sum_t \beta^t \mathrm{var}(\hat{w}_t(\ell)) = \frac{\gamma_w}{(1-\gamma_w)(1-\gamma_w\beta)} \sum_t \beta^t (\hat{\pi}_t^w)^2. \tag{S49}
$$

Now, we seek to express the investment misallocation term, $\mathrm{var}(\hat{\imath}_t^d(i))$. Recall that the investment rate is

$$
\hat{\imath}_t(i) = \frac{1}{\phi\xi}\left[-\frac{1+r}{r}\hat{\delta}(i) + \hat{V}_{t+1} \right]. \tag{S50}
$$

Investment misallocation can be written as the dispersion in discount rates

$$
\mathrm{var}(\hat{\imath}_t^d(i)) = \frac{1}{(\phi\xi)^2}\left(\frac{1+r}{r}\right)^2 \mathrm{var}(\hat{\delta}_t(i)). \tag{S51}
$$

S15

The evolution of the aggregate discount rate is dictated by

$$(\hat{\delta}_t - \hat{\delta}_{t-1}) = \theta\mathbb{E}_i[\hat{\delta}_t(i) - \hat{\delta}_{t-1}] + (1-\theta)\mathbb{E}_i[\hat{\delta}_t^* - \hat{\imath}_{t-1}] \tag{S52}$$

$$= (1-\theta)\mathbb{E}_i[\hat{\delta}_t^* - \hat{\delta}_{t-1}], \tag{S53}$$

where $\hat{\delta}_t^*$ denotes the discount rate of firms with an adjustment opportunity in period t. We can rewrite $\text{var}(\hat{\delta}^d(i))$ as

$$\text{var}(\hat{\delta}_t(i)) = \text{var}(\hat{\delta}_t(i) - \hat{\imath}_{t-1}) \tag{S54}$$

$$= \mathbb{E}_i[(\hat{\delta}_t(i) - \hat{\delta}_{t-1})^2] - (\hat{\delta}_t - \hat{\delta}_{t-1})^2 \tag{S55}$$

$$= \theta\mathbb{E}_i[(\hat{\delta}_{t-1}(i) - \hat{\delta}_{t-1})^2] + (1-\theta)\mathbb{E}_i[(\hat{\delta}_t^* - \hat{\delta}_{t-1})^2] - (\hat{\imath}_t - \hat{\imath}_{t-1})^2 \tag{S56}$$

$$= \theta\text{var}(\hat{\delta}_{t-1}(i)) + (1-\theta)(\hat{\delta}_t^* - \hat{\imath}_{t-1})^2 - (\hat{\delta}_t - \hat{\delta}_{t-1})^2 \tag{S57}$$

$$= \theta\text{var}(\hat{\delta}_{t-1}(i)) + \frac{\theta}{1-\theta}(\hat{\delta}_t - \hat{\delta}_{t-1})^2 \tag{S58}$$

$$= \sum_{s=0}^{t} \theta^{t-s}\frac{\theta}{1-\theta}(\hat{\delta}_t - \hat{\delta}_{t-1})^2. \tag{S59}$$

In turn, the cumulative discounted sum is

$$\sum_{t=0}^{\infty} \beta^t\text{var}(\hat{\delta}_t(i)) = \frac{\theta}{(1-\theta)(1-\beta\theta)}\sum_{t=0}^{\infty}\beta^t(\hat{\delta}_t - \hat{\delta}_{t-1})^2. \tag{S60}$$

We plug (S60) into (S51) to obtain

$$\sum_{t=0}^{\infty} \beta^t\text{var}(\hat{\imath}_t^d(i)) = \frac{1}{(\phi\xi)^2}\left(\frac{1+r}{r}\right)^2\frac{\theta}{(1-\theta)(1-\beta\theta)}\sum_{t=0}^{\infty}\beta^t(\hat{\delta}_t - \hat{\delta}_{t-1})^2. \tag{S61}$$

Substituting (S48) and (S61) into (S46), the quadratic loss function is

$$\sum_{t=0}^{\infty} \beta^t\frac{1}{2}\mathbb{L}_t, \tag{S62}$$

where

$$\mathbb{L}_t \equiv \left[\omega_{KL}(\hat{L}_t - \hat{K}_t)^2 + \omega_{IK}(\hat{I}_t - \hat{K}_t)^2 + \omega_C \hat{C}_t^2 + \omega_L \hat{L}_t^2 + \omega_\pi \hat{\pi}_t^2 + \omega_\pi^w (\hat{\pi}_t^w)^2 + \omega_\delta (\hat{\delta}_t - \hat{\delta}_{t-1})^2 \right.$$

$$\left. + \omega_C \sum_i \mu^i \frac{1}{\chi^i} (\chi_t^i - \chi^i)^2 - 2 \sum_{s=0}^t \hat{\beta}_s (C\hat{C}_t - (1-\alpha)Y\hat{L}_t) - 2Y\hat{A}_t(\alpha\hat{K}_t + (1-\alpha)\hat{L}_t) \right]$$

(S63)

$$\omega_{KL} = \alpha(1-\alpha)Y, \quad \omega_{IK} = \phi\xi^2 K, \quad \omega_C = \sigma C, \quad \omega_L = \nu(1-\alpha)Y, \qquad \text{(S64)}$$

$$\omega_\pi = \varepsilon Y \frac{\gamma_p}{(1-\gamma_p)(1-\gamma_p\beta)}, \quad \omega_\pi^w = \varepsilon^w(1-\alpha)Y \frac{\gamma_w}{(1-\gamma_w)(1-\gamma_w\beta)}, \qquad \text{(S65)}$$

$$\omega_\delta = \frac{1}{\phi} K \left(\frac{1+r}{r} \right)^2 \frac{\theta}{(1-\theta)(1-\beta\theta)}. \qquad \text{(S66)}$$

The optimal monetary policy problem is to minimize (S62) subject to the fol-

lowing log-linearized equilibrium conditions:

$$C\hat{C}_t + I\hat{I}_t + Y\hat{G}_t = Y\hat{A}_t + \alpha Y\hat{K}_t + (1-\alpha)Y\hat{L}_t$$

$$\hat{K}_{t+1} = (1-\xi)\hat{K}_t + \xi\hat{I}_t$$

$$\hat{I}_t - \hat{K}_t = \frac{1}{\xi\phi}\left(\hat{q}_t - \frac{1+r}{r}(\hat{\delta}_t - i_t^l)\right)$$

$$\hat{\delta}_t = \theta\hat{\delta}_{t-1} + (1-\theta)\hat{\delta}_t^*$$

$$\hat{\delta}_t^* = \frac{1+r-\theta}{1+r}\hat{i}_t^l + \frac{\theta}{1+r}\hat{\delta}_{t+1}^*$$

$$\hat{q}_t = -i_t + \hat{\pi}_{t+1} + \frac{r+\xi}{1+r}\left[\sigma\hat{C}_{t+1} + \nu\hat{L}_{t+1} - (\hat{K}_{t+1} - \hat{L}_{t+1})\right] + \frac{1}{1+r}\hat{q}_{t+1}$$

$$\hat{i}_t^l = \frac{r}{1+r}\hat{i}_t + \frac{1}{1+r}\hat{i}_{t+1}^l$$

$$\hat{C}_t = \mathbb{E}_t\left[\hat{C}_{t+1} - \xi_r(1/\sigma)\left[\hat{\beta}_{t+1} + \hat{r}_{t+1}\right] + \xi_I\left[\hat{I}_t - \hat{I}_{t+1}\right]\right]$$

$$\hat{\pi}_t = \psi_p\left[\hat{W}_t - \hat{P}_t - \alpha(\hat{K}_t - \hat{L}_t)\right] + \frac{1}{1+r}\mathbb{E}_t\hat{\pi}_{t+1}$$

$$\hat{\pi}_t^w = \psi_w\left[\sigma\hat{C}_t + \sigma\sum_i \vartheta^i(\chi_t^i - \chi^i)/\chi^i + \nu\hat{L}_t - \hat{W}_t + \hat{P}_t\right] + \frac{1}{1+r}\mathbb{E}_t\hat{\pi}_{t+1}^w$$

$$\chi_t^h - \chi^h = (1-\alpha-\chi^h)\hat{C}_t + (1-\alpha)\frac{1}{C}G_t - \frac{1}{C}T_t + (1-\alpha)\frac{I}{C}\hat{I}_t$$

$$1 = \mu\chi_t^h + (1-\mu)\chi_t^p,$$

where we rewrote the New Keynesian wage Phillips curve using the consumption share. Since the objective function only involves second-order terms, the linear quadratic problem provides a valid approximation to the original non-linear optimal monetary policy problem (Benigno and Woodford 2012). Setting $\psi_p = \gamma_p = 0$ yields the expressions in the main text.

Now we rewrite the problem using the first-best allocation. Define the first-best allocation as the solution to the following problem:

$$\{C_t^n, I_t^n, K_{t+1}^n, L_t^n, I_t^n, \chi_t^{p,n}, \chi_t^{h,n}\}_{t=0}^{\infty} \in \arg\min_{\{C_t, I_t, K_{t+1}, L_t, I_t, \chi_t^p, \chi_t^h\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{1}{2}\mathbb{L}_t, \quad \text{(S67)}$$

$$\text{s.t.} \qquad C\hat{C}_t + I\hat{I}_t + Y\hat{G}_t = Y\hat{A}_t + \alpha Y\hat{K}_t + (1-\alpha)Y\hat{L}_t \qquad \text{(S68)}$$

$$\hat{K}_{t+1} = (1-\xi)\hat{K}_t + \xi\hat{I}_t, \qquad \text{(S69)}$$

where $\mathbb{L}_t$ is defined in (S63). Clearly, the solutions would feature $\chi_t^{h,n} = \chi_t^{p,n} = 0$, since there is no reason to create dispersion in the consumption distribution in the first-best allocation. Using $\{C_t^n, I_t^n, K_{t+1}^n, L_t^n, I_t^n\}$, we can rewrite (S62) as (S63).

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