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# What are the effects of monetary policy on output? Results from an agnostic identification procedure<sup>☆</sup>

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## Abstract

This paper proposes to estimate the effects of monetary policy shocks by a new agnostic method, imposing sign restrictions on the impulse responses of prices, nonborrowed reserves and the federal funds rate in response to a monetary policy shock. No restrictions are imposed on the response of real GDP to answer the key question in the title. I find that

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“contractionary” monetary policy shocks have no clear effect on real GDP, even though prices move only gradually in response to a monetary policy shock. Neutrality of monetary policy shocks is not inconsistent with the data.

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## 1. Introduction

What are the effects of monetary policy on output? This key question has been the focus of a substantial body of the literature. And the answer seems easy. The “Volcker recessions” at the beginning of the 1980s have shown just how deep a recession a sudden tightening of monetary policy can produce. Alternatively, look at Fig. 1, which juxtaposes movements in the federal funds rate from 1965 to 1996 with growth rates in real GDP, flipped upside-down for easier comparison. In particular, for the first half of that sample, it is striking, how rises in the federal funds rate are followed by falls in output (visible as *rises* in the dotted line, due to the upside-down flipping). The case is closed.

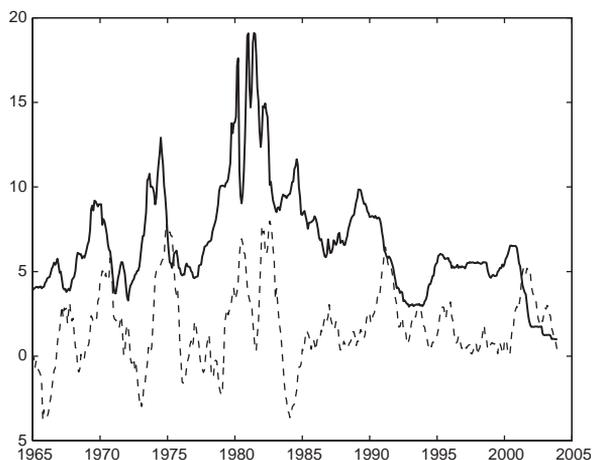


Fig. 1. This figure contrasts movements in the federal funds rate, shown as a thick, solid line with the scale on the left, with real annual GDP growth rates, transformed by multiplying with  $-1$  and adding 5, shown as a thinner, dotted line. The transformation of GDP growth has been done to aid the visual comparison, i.e., peaks in the figure are actually particularly low values for the growth rate. “Eyeball econometrics” suggests a strong cause-and-effect from federal funds rate movements to real GDP: whenever interest rates rise, growth rates fall (i.e. the dotted line rises) shortly afterwards. This is particularly visible for 1968–1983. It seems easy to conclude from this picture, that the question about the effects of monetary policy on output is answered clearly: contractionary monetary policy leads to contractions in real GDP.

Or is it? Eyeball econometrics such as Fig. 1 or case studies like the Volcker recessions can be deceptive: many things are going on simultaneously in the economy, and one may want to be careful to consider just a single cause-and-effect story. If the answer really is so obvious, it should emerge equally clearly from an analysis of multiple time series, which allows for additional channels of interaction and other explanations, at least in principle. Thus, many researchers have followed the lead of Sims (1972, 1980, 1986) and proceeded to analyze the key question in the title with the aid of vector autoregressions. Rapid progress has been made in the last 10 years. Bernanke and Blinder (1992) shifted the focus on the federal funds rate. The ‘price puzzle’, raised by Sims (1992), and other anomalies, led to the inclusions of e.g. nonborrowed reserves, total reserves as well as a commodity price index in VAR studies, see e.g. Eichenbaum (1992), Strongin (1995), Christiano and Eichenbaum (1992a, b), Leeper and Gordon (1992), Gordon and Leeper (1994), Christiano et al. (1996, 1997, 1999) and Kim (1999). Recently, Bernanke and Mihov (1998a, b) have reconciled a number of these approaches in a unifying framework, and Leeper et al. (1996) have summarized the current state of the literature, while adding new directions on their own. Additional excellent surveys are in Canova (1995), Christiano et al. (1999) and Bagliano and Favero (1998). There seems to be a growing agreement that this literature has reached a healthy state, and has provided a list of facts, which now theorists ought to explain, see e.g. Christiano et al. (1996, 1997, 1999) or Leeper and Sims (1994).

The key step in applying VAR methodology to the question at hand is in identifying the monetary policy shock. While this is usually done by appealing to certain informational orderings about the arrival of shocks, there also is a more informal side to the identification search: researchers like the results to look “reasonable”. According to conventional wisdom, monetary contractions should raise the federal funds rate, lower prices and reduce real output. If a particular identification scheme does not accomplish this, then the observed responses are called a puzzle, while successful identification needs to deliver results matching the conventional wisdom. The “facts” that are obtained this way are thus necessarily influenced by a priori theorizing. There is a danger that the literature just gets out what has been stuck in, albeit more polished and with numbers attached. Without being explicit about this a priori theorizing, it is hard to distinguish between assumptions and conclusions.

This circularity is well recognized in the literature, has already been clearly pointed out by Cochrane (1994), and has been dealt with in a variety of ways. Leeper et al. (1996) explicitly appeal to the reasonableness of impulse responses as an “informal” identification criterion. Galí (1992) directly asks whether the “IS-LM model fit[s] the postwar U.S. data” rather than indirectly presuming that this is the only model worth fitting. Cochrane (1994) and Rotemberg (1994) argue that economic theory is crucially important for identifying monetary policy shocks: a VAR analysis of these shocks only has a chance to be convincing, if the results look plausible to begin with. Christiano et al. (1999) propose to throw out all impulse responses inconsistent with some given set of theories, some of which are at odds with the conventional wisdom. Joint estimation of a theoretical model and a VAR is done in e.g. Altig et al. (2002).

Priors for a VAR from an explicitly formulated theory are constructed in [Del Negro and Schorfheide \(2003\)](#). In sum, the answer to the key question—here, the impact of monetary policy shocks on GDP—is often already substantially narrowed down by a priori theorizing, be it implicit or explicit.

What is therefore desirable as a complement to the existing literature is some way to make the a priori theorizing explicit (and use as little of it as possible), while at the same time leaving the question of interest open. This paper proposes to push this idea all the way, and to identify the effects of monetary policy shocks by directly imposing sign restrictions on the impulse responses. More specifically, I will assume that a “contractionary” monetary policy shock does not lead to increases in prices, increase in nonborrowed reserves, or decreases in the federal funds rate for a certain period following a shock. While theories with different implications can fairly easily be constructed, these assumptions may enjoy broad support and in any case are usually tacitly assumed in most of the VAR literature. In the approach here, they are brought out into the open and can therefore be subject to debate. Crucially, I impose no restrictions on the response of real GDP. Thus, the central question in the title is left agnostically open by design of the identification procedure: the data will decide. I call the procedure “agnostic” for this reason. One can think about the procedure as identifying all shocks which are consistent with these fairly weak a priori restrictions, and that the literature (insofar it delivers impulse responses also obeying the sign restrictions) uses further a priori identifying restrictions to only select a subset of these shocks.

This will not be a free lunch, nor should one expect it to be. When imposing the sign restrictions, one needs to take a stand on for how long these restrictions ought to hold after a shock. Furthermore, one needs to take a stand on whether a strong response in the opposite direction is more desirable than a weak one. I will try out a variety of choices and look at the answers.

Section 2 introduces the method with most of the technicalities postponed to Appendices A and B. Section 3 shows some results, based on the data set provided by [Bernanke and Mihov \(1998a, b\)](#), extended until the end of 2003. Section 4 concludes.

My approach is asymmetric in that I am agnostic about the response of output but not of some other variables. This is intentional: the response of output is the focus of this investigation. Nonetheless, it is interesting to also report findings about the other variables, keeping in mind that they are tainted by a priori sign restrictions. I find the following:

1. “Contractionary” monetary policy shocks have an ambiguous effect on real GDP. With  $\frac{2}{3}$  probability, a typical shock will move real GDP by up to  $\pm 0.2$  percent, consistent with the conventional view, but also consistent with e.g. monetary neutrality. Indeed, the usual label “contractionary” may thus be misleading, if output is moved *up*. Monetary policy shocks account for probably less than 25% of the variance for the 1-year or more ahead forecast revision of real output, and may easily account for less than 2% at any given horizon.

2. The GDP price deflator falls only slowly following a contractionary monetary policy shock. The commodity price index falls more quickly.
3. I also find, that monetary policy shocks account for only a small fraction of the forecast error variance in the federal funds rate, except at horizons shorter than half a year, as well as for prices.

While these observations confirm some of the results found in the empirical VAR literature so far, there are also some potentially important differences in particular with respect to my key question: “contractionary” monetary policy shocks do not necessarily seem to have contractionary effects on real GDP. Our conclusion from these results: one should feel less comfortable with the conventional view and the current consensus of the VAR literature than has been the case so far.

The new method introduced here complements the work by Blanchard and Quah (1989), Lippi and Reichlin (1994a, b) and in particular by Dwyer (1997), Faust (1998), Gambetti (1999), Canova and Pina (1999) and Canova and de Nicolo (2002): these authors also impose restrictions on the impulse responses to particular shocks. Like Faust, Dwyer and Canova–de Nicolo, my aim is to make explicit restrictions which are often used implicitly. But there are also important differences. I do not impose a particular shape of the impulse response as in Lippi and Reichlin (1994a) or Dwyer (1997) or impose a zero impulse response at infinity as in Blanchard and Quah (1989). Instead, I am content with restrictions on the sign at a few periods following the shock, making for substantial differences between their approach and ours. The intention here is to be minimalistic and to impose not (much) more than the sign restrictions themselves, as they can be reasonably agreed upon across many economists. Faust (1998) also only imposes sign restrictions to restrict monetary policy shocks. His focus is a different one. Faust examines the fragility of the consensus conclusion, that monetary shocks account for only a small fraction of GDP fluctuations, see Cochrane (1994), while this paper aims at estimating that response. Furthermore, Faust only imposed sign restrictions on impact. In my discussion (Uhlig, 1998), I have shown how his approach can be extended, when one wishes to impose the sign restrictions for several periods following the shock. The method by Canova and de Nicolo (2002) and its application in Canova and Pina (1999) identifies monetary disturbances by imposing sign restrictions on the cross-correlations of variables in response to shocks, adding restrictions until the maximum number of shocks is uniquely identified. The identification here proceeds differently by using impulse responses rather than cross-correlations, by using other criteria used to select among orthogonal decompositions satisfying the restrictions, and by not imposing increasingly stringent restrictions to eliminate candidate orthogonalizations.

I do not aim at a complete decomposition of the one-step ahead prediction error into all its components due to underlying structural shocks, but rather concentrate on identifying only one such shock, namely the shock to monetary policy. Similarly, Bernanke and Mihov (1998a, b) or Christiano et al. (1999) only identify a single shock or a subset of shocks. They impose considerably more structure than I do here. Again, the aim is to be minimalistic, and to use as little a priori reasoning about *other*

shocks as possible in order to identify the effects of monetary policy shocks. The identification of additional shocks can help in principle, as orthogonality between the shocks provides an additional restriction for identifying the monetary policy shock, and there may be those who argue that it is even necessary. The method can fairly easily be extended in this direction; if necessary, see [Mountford and Uhlig \(2002\)](#) for an example.

## 2. The method

There is not much disagreement about how to estimate VARs. A VAR is given by

$$Y_t = B_{(1)}Y_{t-1} + B_{(2)}Y_{t-2} + \cdots + B_{(l)}Y_{t-l} + u_t, t = 1, \dots, T, \quad (1)$$

where  $Y_t$  is an  $m \times 1$  vector of data at date  $t = 1 - l, \dots, T$ ,  $B_{(i)}$  are coefficient matrices of size  $m \times m$  and  $u_t$  is the one-step ahead prediction error with variance–covariance matrix  $\Sigma$ . An intercept and perhaps a time trend is sometimes added to (1).

The disagreement starts when discussing how to decompose the prediction error  $u_t$  into economically meaningful or fundamental innovations. This is necessary because one is typically interested in examining the impulse responses to such fundamental innovations, given the estimated VAR. In particular, much of the literature is interested in examining the impulse responses to a monetary policy innovation.

Suppose that there are a total of  $m$  fundamental innovations, which are mutually independent and normalized to be of variance 1: they can therefore be written as a vector  $v$  of size  $m \times 1$  with  $E[vv'] = I_m$ . Independence of the fundamental innovations is an appealing assumption adopted in much of the VAR literature: if, instead, the fundamental innovations were correlated, then this would suggest some remaining, unexplained causal relationship between them. We therefore also adopt the independence assumption here. What is needed is to find a matrix  $A$  such that  $u_t = Av_t$ . The  $j$ th column of  $A$  (or its negative) then represents the immediate impact on all variables of the  $j$ th fundamental innovation, one standard error in size. The only restriction on  $A$  thus far emerges from the covariance structure:

$$\Sigma = E[u_t u_t'] = A E[v_t v_t'] A' = A A'. \quad (2)$$

Simple accounting shows that there are  $m(m - 1)/2$  degrees of freedom in specifying  $A$ , and hence further restrictions are needed to achieve identification. Usually, these restrictions come from one of three procedures: from choosing  $A$  to be a Cholesky factor of  $\Sigma$  and implying a recursive ordering of the variables as in [Sims \(1986\)](#), from some structural relationships between the fundamental innovations  $v_{t,i}$ ,  $i = 1, \dots, m$  and the one-step ahead prediction errors  $u_{t,i}$ ,  $i = 1, \dots, m$  as in [Bernanke \(1986\)](#), [Blanchard and Watson \(1986\)](#) or [Sims \(1986\)](#), or from separating transitory from permanent components as in [Blanchard and Quah \(1989\)](#).

Here, I propose to proceed differently. First, note that I am solely interested in the response to a monetary policy shock: there is therefore a priori no reason to also identify the other  $m - 1$  fundamental innovations. [Bernanke and Mihov \(1998a, b\)](#)

and Christiano et al. (1999) similarly recognize this, and use a block-recursive ordering, to concentrate the identification exercise on only a limited set of variables which interact with the policy shock.

I propose to go all the way by only concentrating on finding the innovation corresponding to the monetary policy shock. This amounts to identifying a single column  $a \in \mathbb{R}^m$  of the matrix  $A$  in Eq. (2). It is useful to state a formal definition:

**Definition 1.** The vector  $a \in \mathbb{R}^m$  is called an *impulse vector*, iff there is some matrix  $A$ , so that  $AA' = \Sigma$  and so that  $a$  is a column of  $A$ .

Proposition A.1 in Appendix A shows, that any impulse vector  $a$  can be characterized as follows. Let  $\tilde{A}\tilde{A}' = \Sigma$  be the Cholesky decomposition of  $\Sigma$ . Then,  $a$  is an impulse vector if and only if there is an  $m$ -dimensional vector  $\alpha$  of unit length so that

$$a = \tilde{A}\alpha. \tag{3}$$

Given an impulse vector  $a$ , it is easy to calculate the appropriate impulse response as follows. Let  $r_i(k) \in \mathbb{R}^m$  be the vector response at horizon  $k$  to the  $i$ th shock in a Cholesky decomposition of  $\Sigma$ . The impulse response  $r_a(k)$  for  $a$  is then simply given by

$$r_a(k) = \sum_{i=1}^m \alpha_i r_i(k). \tag{4}$$

Further, find a vector  $\tilde{b} \neq 0$  with

$$(\Sigma - aa')\tilde{b} = 0$$

normalized so that  $b'a = 1$ . Then, the real number

$$v_t^{(a)} = b'u_t \tag{5}$$

is the scale of the shock at date  $t$  in the direction of the impulse vector  $a$ , and  $v_t^{(a)}a$  is a part of  $u_t$ , which is attributable to that impulse vector. Essentially,  $b$  is the appropriate row of  $A^{-1}$ .

Finally, consider the  $k$ -step ahead forecast revision  $E_t[Y_{t+k}] - E_{t-1}[Y_{t+k}]$  due to the arrival of new data at date  $t$ . The fraction  $\phi_{a,j,k}$  of the variance of this forecast revision for variable  $j$ , explained by shocks in the direction of the impulse vector  $a$  is given by

$$\phi_{a,j,k} = \frac{(r_{aj}(k))^2}{\sum_{i=1}^m (r_{ij}(k))^2},$$

where the additional index  $j$  picks the entry corresponding to variable  $j$ . With these tools, one can perform variance decompositions or counterfactual experiments.

To identify the impulse vector corresponding to monetary policy shocks, I impose that a contractionary policy shock does not lead to an increase in prices or in nonborrowed reserves and does not lead to a decrease in the federal

funds rate. These assumptions seem to be the least controversial implications of a contractionary monetary policy shock. Furthermore and crucially, these seem to be distinguishing characteristics of monetary policy shocks compared to other shocks prominently proposed in the literature. For example, money demand shocks are meant to be ruled out as a competing explanation by the requirement that nonborrowed reserves do not rise.

Obviously, this method of identification has its limits. For example, money demand shocks cannot be ruled out, if one takes the point of view that the federal reserve will not at least partially accommodate increases in money demand through an increase in nonborrowed reserves. Furthermore, combinations of other shocks could potentially look like monetary policy shocks. One way to avoid this problem would be to identify the other shocks explicitly, at the price of many additional assumptions. Furthermore, this problem is not new to this approach. For example, if the true data generating mechanism has more shocks than variables, and if one uses a conventional Cholesky decomposition to identify a monetary policy shock by the federal funds rate innovation ordered last, the monetary policy shock thus identified will actually be a linear combination of several underlying shocks, except in knife-edge cases. In sum, identification in any econometric exercise rests on assumptions: I do not claim that the identifying assumptions here are ironclad, but rather that they are particularly reasonable. Let me state the assumption explicitly. Choose some horizon  $K \geq 0$ .

**Assumption A.1.** A *monetary policy impulse vector* is an impulse vector  $a$ , so that the impulse responses<sup>1</sup> to  $a$  of prices and nonborrowed reserves are not positive and the impulse responses for the federal funds rate are not negative, all at horizons  $k = 0, \dots, K$ .

Given some VAR coefficient matrices  $B = [B'_1, \dots, B'_l]$ , some error variance–covariance matrix  $\Sigma$ , and some horizon  $K$ , let  $\mathcal{A}(B, \Sigma, K)$  be the set of all monetary policy impulse vectors. Because it is obtained from inequality constraints, the set  $\mathcal{A}(B, \Sigma, K)$  will typically either contain many elements or be empty. Therefore, one typically cannot obtain exact identification at this point, in contrast to more commonly used exact identification procedures. For that reason, we will eventually supplement the identification assumption above either by imposing a prior on  $\mathcal{A}(B, \Sigma, K)$  or by minimizing some criterion function  $f(\cdot)$  on the unit sphere, which penalizes violations of the relevant sign restrictions, see B.2.

As a first step, however, it is already informative to simply use the OLS estimate of the VAR,  $B = \hat{B}$  and  $\Sigma = \hat{\Sigma}$ , fix  $K$  or try out a few choices for  $K$ , and look at the entire range of impulse responses, as  $a \in \mathcal{A}(\hat{B}, \hat{\Sigma}, K)$  is varied, provided  $\mathcal{A}(\hat{B}, \hat{\Sigma}, K)$  is not empty. The set  $\mathcal{A}$  therefore results in an interval for the impulse responses, which we wish to calculate. One can think of this exercise as an extreme bounds analysis in the spirit of Leamer (1983). As usual in the literature, the bounds apply to each

<sup>1</sup>I will estimate my VAR using levels of the logs of variables, rather than e.g. first differences: therefore, the restrictions are indeed imposed on the impulse responses and not e.g. on the cumulative impulse responses.

response entry  $r_{a,j}(k)$  rather than to the entire function, i.e. there is probably not a single  $a$  such that the response will be at the bound for all variables  $j$  or all horizons  $k$ .

Numerically, this can and will be accomplished in a straightforward manner and brute force by generating many impulse vectors, calculating their implied impulse response functions, and checking whether or not the sign restrictions are satisfied. It is wise to calculate the Cholesky-responses  $r_i$  once, and then calculate the response for some given impulse vector by calculating a weighted sum of the  $r_i$  as in Eq. (4). I will generate these impulse vectors randomly, because this is easier to implement than other available alternatives: draw  $\tilde{a}$  from a standard normal in  $\mathbb{R}^m$ , flip signs of entries which violate sign restrictions, multiply with  $\tilde{A}^{-1}$  to calculate the corresponding  $\tilde{\alpha}$  and divide by its length to obtain a candidate draw for  $a$ . Check whether  $a \in \mathcal{A}(\hat{B}, \hat{\Sigma}, K)$  by checking the sign restrictions on the impulse responses for all relevant horizons  $k = 0, \dots, K$ . Generate, say, 10 000 candidate draws for  $a$ , and plot the maximum and the minimum of the impulse responses for those  $a$ , which satisfy these restrictions,  $a \in \mathcal{A}(\hat{B}, \hat{\Sigma}, K)$ . This is a consistent, although slightly biased estimate of the bounds. Results can be seen in Fig. 2: we will defer the description and discussion of these and all other results to Section 3.

In principle, the set  $\mathcal{A}(B, \Sigma, K)$  can be characterized analytically. A sign restriction for some variable  $j$  and at some horizon  $k$  amounts to a linear inequality on  $\alpha$  via Eq. (4), thereby constraining  $\alpha$  to some half space of  $\mathbb{R}^m$ . The set  $\mathcal{A}(B, \Sigma, K)$  is the intersection of all these half spaces. It is therefore convex, which implies that the range for variable  $j$  at horizon  $k$  of impulse responses satisfying the sign restrictions is intervals. The set  $\mathcal{A}(B, \Sigma, K)$  can be characterized by its extreme points, which in turn can be calculated using linear programming techniques. In practice, the number of inequality constraints imposed can be considerable: hence, imposing the inequality restrictions at horizon  $k = 0$  only (or imposing none), and relying on random trial-and-error for the rest is simpler to implement, and is done here.

I wish to move beyond estimation to inference in order to deal with the issue of nonexact identification of the impulse vector  $a$  and to deal with the sampling uncertainty in the OLS estimate of  $B$  and  $\Sigma$ . I propose two related, but different approaches, based on a Bayesian method. In the “pure-sign-restriction approach”, all impulse vectors satisfying the impulse response sign restrictions are considered equally likely: a formal statement is below and technical details are in Appendix B. In the “penalty-function approach”, I use an additional criterion to select the best of all impulse vectors, see Section B.2.

Let  $\tilde{A}(\Sigma)$  be the lower triangular Cholesky factor of  $\Sigma$ . Let  $\mathcal{P}_m$  be the space of positive definite  $m \times m$  matrices and let  $\mathcal{S}_m$  be the unit sphere in  $\mathbb{R}^m$ ,  $\mathcal{S}_m = \{\alpha \in \mathbb{R}^m : \|\alpha\| = 1\}$ . For both approaches, a Normal–Wishart prior is used rather than one of a variety of other recent suggestions in the literature, see Appendix B and the discussion at the end there. Using a different prior should not pose additional difficulties, and I suspect that the conclusions drawn here are reasonably robust to the choice of the prior. It would be interesting to check that more carefully: that, however, is beyond the scope of this paper.

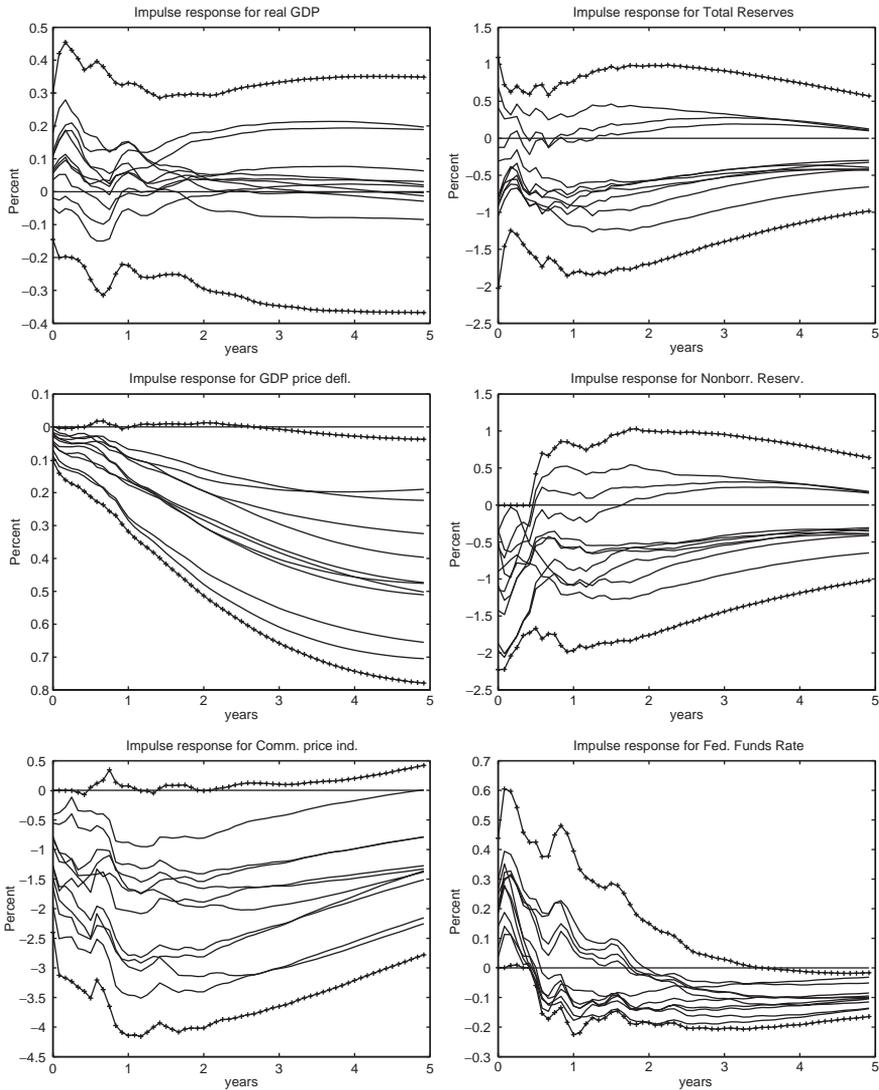


Fig. 2. This figure shows the possible range of impulse response functions when imposing the sign restrictions for  $K = 5$  at the OLSE point estimate for the VAR.

**Assumption A.2** (for the pure-sign-restriction approach). The parameters  $(B, \Sigma, \alpha)$  are drawn jointly from a prior on  $\mathbb{R}^{l \times m \times m} \times \mathcal{P}_m \times \mathcal{S}_m$ . The prior is proportional to a Normal–Wishart in  $(B, \Sigma)$ , whenever  $a = \tilde{A}(\Sigma)\alpha$  satisfies  $a \in \mathcal{A}(B, \Sigma, K)$  and zero elsewhere, i.e. is proportional to a Normal–Wishart density multiplied with an indicator variable on  $\tilde{A}(\Sigma)\alpha \in \mathcal{A}(B, \Sigma, K)$ .

By parameterizing the impulse vector, i.e. by formulating the prior as a product with an indicator variable in  $(B, \Sigma, \alpha)$ -space rather than  $(B, \Sigma, a)$ -space, an

undesirable scaling problem is avoided, see Section B.1. The flat prior on the unit sphere for  $\alpha$  is appealing for a number of reasons. In particular, the results will be independent of the chosen decomposition of  $\Sigma$ . For example, reordering the variables and choosing a different Cholesky decomposition in order to parameterize impulse vectors will not yield different results. Again, details are in Section B.1.

The penalty-function approach, described in Section B.2, exactly identifies a monetary policy shock by minimizing some penalty function. Both approaches have their merits. Deciding, which is more appropriate is a matter of taste and judgement, and depends on the question at hand. The penalty-function approach delivers impulse response functions with small standard errors as it seeks to go as far as possible in imposing certain sign restrictions. The penalty-function approach leaves the reduced-form VAR untouched, while the pure-sign-restriction is, in effect, *simultaneously* an estimation of the reduced-form VAR alongside the impulse vector: VAR parameter draws, which do not permit any impulse vector to satisfy the imposed sign restrictions, receive zero prior weight, and VAR parameter draws, which easily permit satisfaction of the sign restrictions, receive more weight.

The pure-sign-restriction approach is cleaner for my task at hand, since it literally only imposes the weak prior beliefs of e.g. prices not going up, following a surprise rise in interest rates. Therefore, I focus on it in the main body of the paper.

Numerically, I implement the pure-sign-restriction approach in the following way. Make assumption A.2. The posterior is given by the usual Normal–Wishart posterior for  $(B, \Sigma)$ , given the assumed Normal–Wishart prior for  $(B, \Sigma)$ , times the indicator function on  $\tilde{A}(\Sigma)\alpha \in \mathcal{A}(B, \Sigma, K)$ . To draw from this posterior, take a joint draw from both the posterior for the unrestricted Normal–Wishart posterior for the VAR parameters  $(B, \Sigma)$  as well as a uniform distribution over the unit sphere  $\alpha_{in} \mathcal{S}_m$ . Construct the impulse vector  $a$ , see Eq. (3), and calculate the impulse responses  $r_{k,j}$  at horizon  $k = 0, \dots, K$  for the variables  $j$ , representing the GDP deflator, the commodity price index, nonborrowed reserves and the federal funds rate. If all these impulse responses satisfy the sign restrictions, keep the draw. Otherwise discard it. Repeat sufficiently often. Calculate statistics, based on the draws kept.

Certainly, different priors are likely to generate different results. One can read Faust's (1998) contribution as searching for a prior that places all mass on the impulse vectors which explain the largest share of output variation (as well as studying the robustness with respect to the reduced-form VAR prior): he shows that up to 86% of the variance of output may be explainable with monetary policy shocks that way. Faust (1998) imposes far fewer sign restrictions than I do here, see his list on p. 230: indeed, the contribution of monetary policy shocks to the explanation of output variance decreases considerably, when imposing the same sign restrictions as here, as Fig. 6 of my discussion (Uhlig, 1998) shows. The sensitivity of the results to the choice of the prior may therefore not be too large. In sum, Faust's analysis provides a useful complement and robustness check to the method here.

### 3. Results

In this section, I present some results using my method. I have followed the empirical approach in [Bernanke and Mihov \(1998a, b\)](#), who have used real GDP, the GDP deflator, a commodity price index, total reserves, nonborrowed reserves and the federal funds rate for the U.S. at monthly frequencies from January 1965 to December 1996. To obtain monthly observations for all these series, some interpolation was required, see [Bernanke and Mihov \(1998a\)](#) and in particular their NBER 1995 working paper version for details. For the calculations here, I have recalculated and updated their data set, which now ends in December 2003. For the commodity price index, I have used the Dow Jones Spot Average (Symbol `_DJSD`), commercially available from Global Financial Data, Inc., and calculated monthly averages of the daily data. I have obtained all other time series from the St. Louis Fed website, using the series `GDPC1`, `GDPDEF`, `BOGNONBR`, `TOTRESNS` and `FEDFUNDS`. To obtain monthly series, I have used the interpolation method described in [Bernanke et al. \(1997\)](#) in the version described in [Moench and Uhlig \(2004\)](#). GDP has been interpolated with Industrial Production (`INDPRO`) and the GDP Deflator with CPI (`CPIAUCSL`) and PPI (`PPIFGS`). I have fitted a VAR with 12 lags in levels of the logs of the series except for using the federal funds rate directly. I did not include a constant or a time trend. This may result in a slight misspecification, but makes for more robust results because of the interdependencies in the specification of the prior between these terms and the roots in the autoregressive coefficients, see [Uhlig \(1994\)](#).

Before moving to results permitting inference, examine [Fig. 2](#), showing the range of impulse response functions, which satisfies the sign restrictions for  $k = 0, 1, \dots, K$  months after the shock, where  $K = 5$ . The VAR coefficients and the variance–covariance matrix  $\Sigma$  have been fixed at the MLE point estimate ([Fig. 3](#)). To generate this figure, 10 000 candidate draws for  $a$  have been generated. In addition to the bounds, 10 randomly selected impulse responses satisfying the sign restrictions have been drawn to show how typical responses in these bands might look. [Fig. 4](#) varies the restriction horizon  $K$ . One can already see that the bounds for the response of real GDP straddle the no-response line at zero, with the whole distribution moving up with a longer restriction horizon  $K$ . This turns out to be a rather typical feature of most of the Bayesian sampling results as well: we discuss these features in more detail in [Section 3.1](#).

[Fig. 3](#) shows the histograms of the initial responses of all variables, when drawing the orthogonalized impulse vectors uniformly from a sphere, as described for the pure-sign-restriction approach. One can clearly see how the sign restrictions appear to cut off part of the distribution for the initial response of prices and nonborrowed reserves. They also show how the uniform draws of the orthogonalized impulse vectors and restricting the signs of the impulse responses lead to a shaped distribution for the initial response.

For comparison to these results and the results below, [Fig. 5](#) shows results obtained from a conventional Cholesky decomposition of  $\Sigma$ , i.e. imposing lower triangularity on  $A$ . The Cholesky decomposition is popular in the literature because

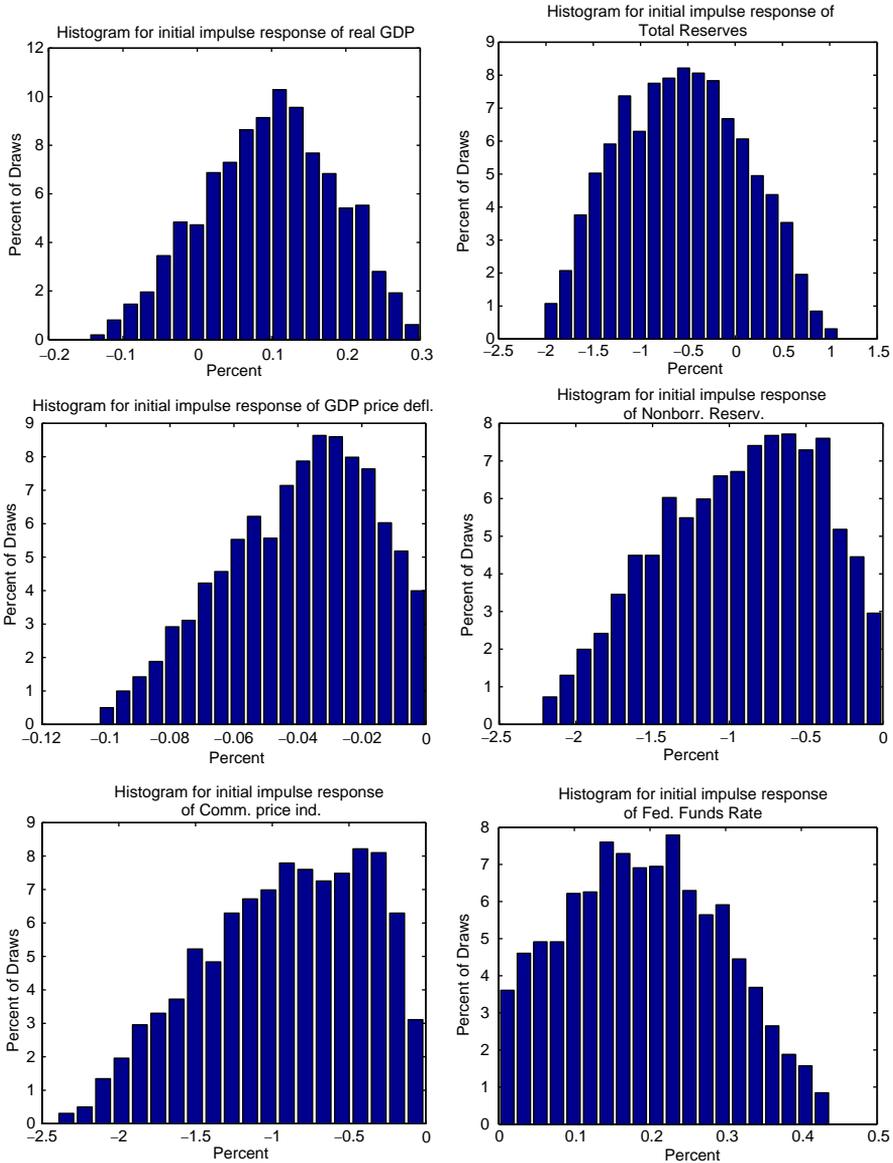


Fig. 3. This figure shows the distribution of the impact impulse response (i.e. at horizon 0), when imposing the sign restrictions for  $K = 5$  at the OLSE point estimate for the VAR.

it is easy to compute. This method requires a choice regarding the ordering of the variables as well as the choice of the variable, whose innovations are to be interpreted as monetary policy shocks. Here, I identify the monetary policy shock with the innovations in the federal funds rate ordered forth, before nonborrowed and total reserves.

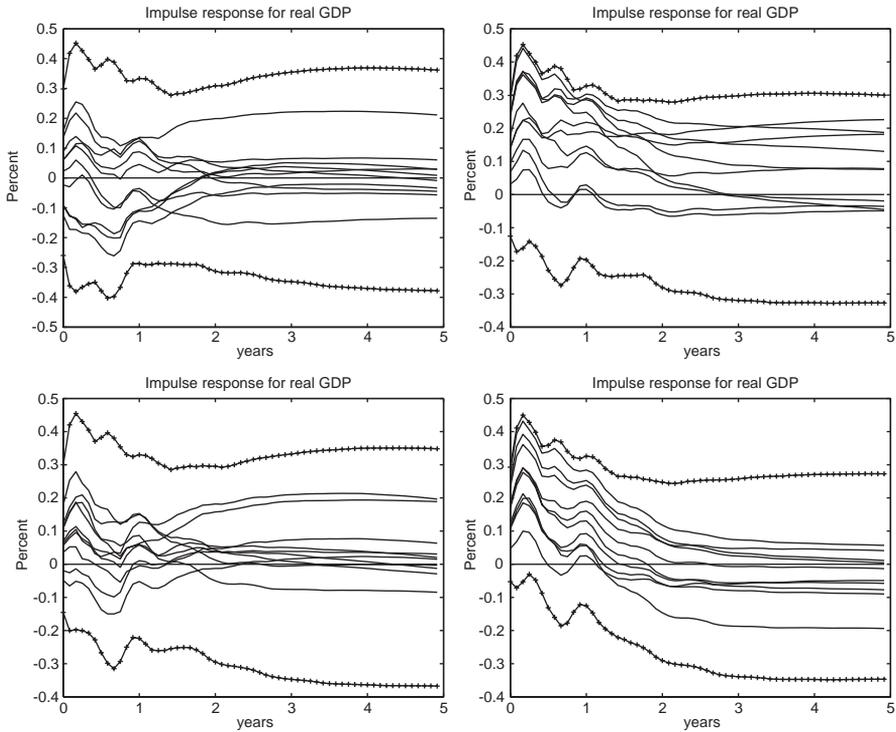


Fig. 4. Ranges for the impulse response of real GDP to a contractionary monetary policy shock one standard deviation in size. At the OLSE of the VAR, the collection of impulse responses consistent with the sign restriction cover the range shown. For the left column,  $K = 2$  and  $5$  have been used, whereas  $K = 11$  and  $23$  have been used in the right column.

Fig. 5 shows impulse responses for a horizon of up to 5 years after the shock. The top rows contain the results for real GDP and total reserves, the middle row contains the results for the GDP price deflator and for nonborrowed reserves and the bottom row contains the results for the commodity price index and the federal funds rate. Here as well as in all other plots, I show the median as well as the 16% and the 84% quantiles for the sample of impulse responses: if the distribution was normal, these quantiles would correspond to a one standard deviation band. A number of authors prefer two standard deviation bands, which would correspond to the 2.3% and the 97.7% quantiles. But given that I want to report the same statistics in all the figures and given that I based inference in the pure-sign-restriction approach on only 100 draws for computational reasons, I felt that I could not report these quantiles precisely enough. Furthermore, one standard deviation bands are popular in this literature as well. The results are fairly “reasonable” in that they confirm conventional undergraduate textbook intuition. The “reasonableness” of Fig. 5 is not an accident, but is to a good degree the result of the identification search alluded to in the Introduction, involving both a search over all the possibilities of ordering

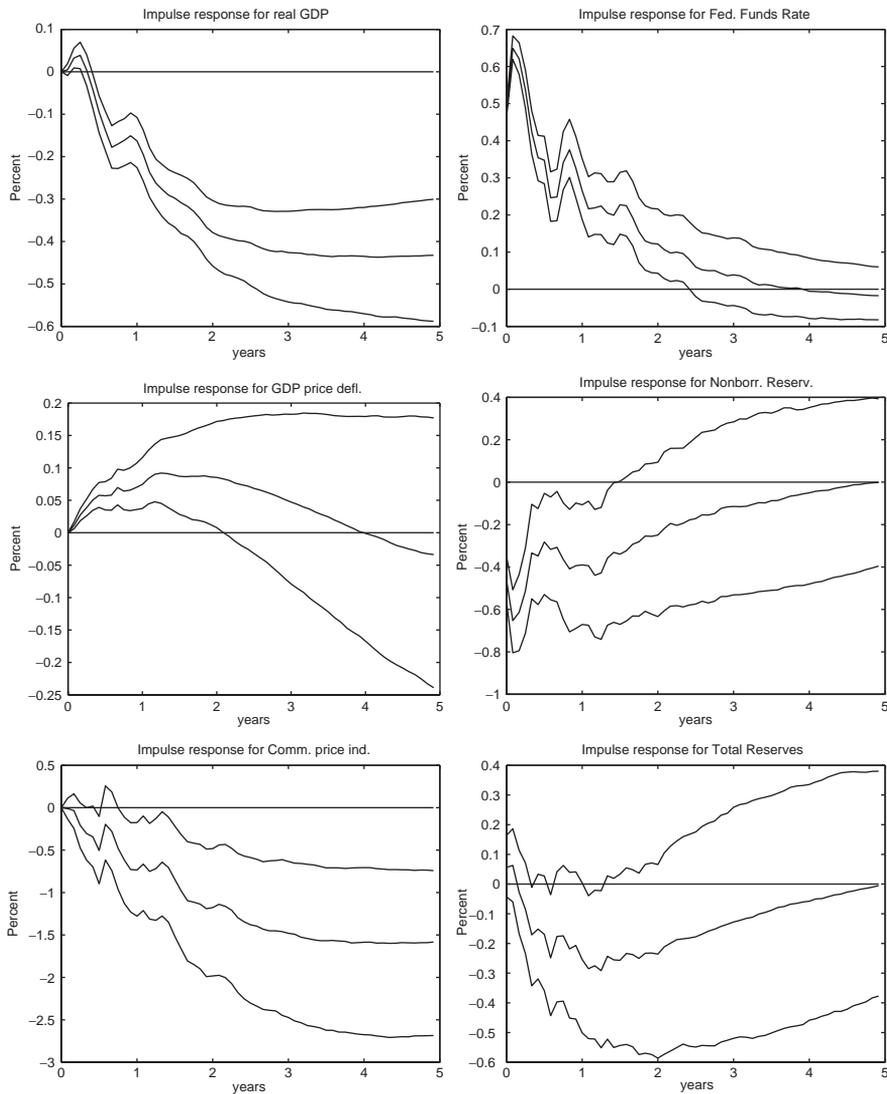


Fig. 5. Impulse responses to a contractionary monetary policy shock one standard deviation in size, identified as the innovation in the federal funds rate, ordered forth in a Cholesky decomposition before nonborrowed and total reserves. This conventional identification exercise is provided for comparison. The three lines are the 16% quantile, the median and the 16% quantile of the posterior distribution. The first column shows the responses of real GDP, the GDP deflator and the commodity price index. The second column shows the responses of total reserves, nonborrowed reserves, and the federal funds rate. This identification mostly generates “reasonable” results, but also the price puzzle: the GDP deflator rises first before falling.

variables and identifying a monetary policy shock, as well as a search over the time series to be included in the VAR in the first place.

One can also see a version of the “price puzzle” pointed out by Sims (1992): the GDP deflator moves somewhat above zero first before declining below zero after a monetary policy shock (see also the remarks in Appendix B). Eichenbaum (1992) has shown how the price puzzle can be mitigated with the inclusion of commodity prices in the VAR: they are included here, but do not lead to a resolution of the price puzzle now. Ordering the federal funds rate last helps in mitigating the price puzzle somewhat, but is less convincing as a conventional identification strategy: the results are not shown here. It may well be that the additional decade of data since 1992 has made this route to resolving the price puzzle more difficult. By contrast, the agnostic identification approach to be employed next avoids the price puzzle by construction.

### 3.1. Results for the pure-sign-restriction approach

Our benchmark result is contained in Fig. 6, showing the impulse responses from a pure-sign-restriction approach with  $K = 5$ . That is, the responses of the GDP price deflator, the commodity price index and nonborrowed reserves have been restricted not to be positive and the federal funds rate not to be negative for the 6 months  $k, k = 0, \dots, 5$  following the shock. The results can be described as follows:

1. The federal funds rate reacts largely and positively immediately, typically rising by 20 basis points, then reversing course within a year, ultimately dropping by 10 basis points.
2. With a 2/3 probability, the impulse response for real GDP is within a  $\pm 0.2\%$  interval around zero at any point during the first 5 years following the shock.
3. The GDP price deflator reacts very sluggishly, with prices dropping by about 0.1% within a year, and dropping by 0.4% within 5 years. The price puzzle is avoided by construction.
4. The commodity price index reacts swiftly, reaching a plateau of a 1.5% percent drop after about one year.
5. Nonborrowed reserves and total reserves both drop initially, with nonborrowed reserves dropping by more (around 1%) than total reserves (around 0.6%).

The initial 6-months response for most of these variables look rather conventional except for real GDP. Indeed, one may conclude from this figure that the reaction of real GDP can as easily be positive as negative following a “contractionary” shock. While this is consistent with the textbook view of gradually declining output after a monetary policy shock, the data does not seem to urge this view upon us. The answer to the opening question is: the effects of monetary policy shocks on real output are ambiguous. A one-standard deviation monetary policy shock may leave output unchanged or may drive output up or down by up to 0.2% in most cases, thus possibly triggering fairly sizeable movements of unknown sign.

The further course of all the responses looks perhaps less conventional, although not hard to explain. Here are some suggestions. Commodity prices react more

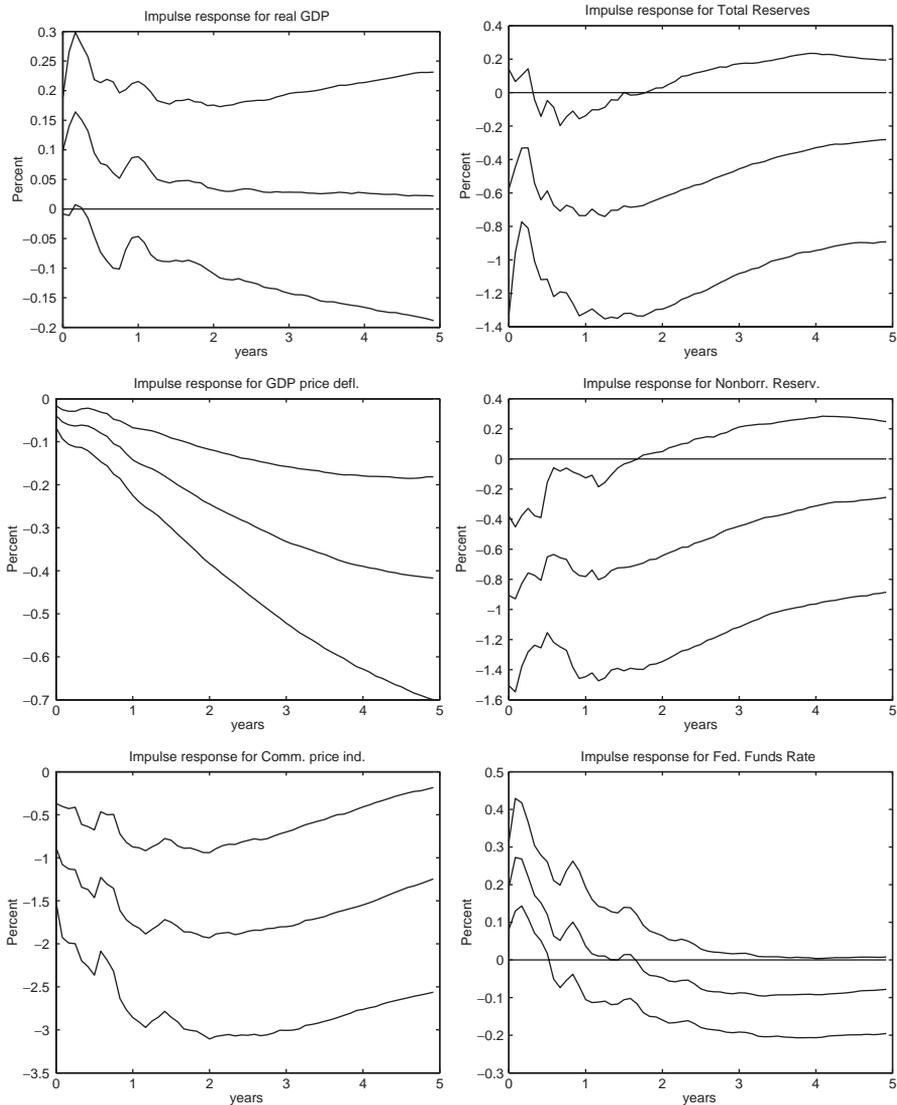


Fig. 6. Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach with  $K = 5$ . That is, the responses of the GDP price deflator, the commodity price index and nonborrowed reserves have been restricted not to be positive and the federal funds rate not to be negative for months  $k, k = 0, \dots, 5$  after the shock. The error band for the real GDP impulse response is a  $\pm 0.2$  interval around zero: while consistent with the textbook view of declining output after a monetary policy shock, it is also consistent with e.g. monetary neutrality.

quickly than the GDP deflator, since commodities are traded on markets with very flexible prices. As for reserves and interest rates, note that these impulse responses contain the endogenous reaction of monetary policy to its own shocks. The federal

funds rate reverses course and turns negative for perhaps one of the following two reasons. First, this may reflect that monetary policy shocks really arise as errors of assessment of the economic situation by the Federal Reserve Bank. The Fed may typically try to keep the steering wheel steady: should they accidentally make an error and shock the economy, they will try to reverse course soon afterwards. Second, this may reflect a reversal from a liquidity effect to a Fisherian effect: with inflation declining, a decline in the *nominal* rate may nonetheless indicate a rise in the *real* rate. Looking at the responses of reserves, I favor the first view. Obviously, other reasonable interpretations can be found.

This identification of the monetary policy shock seems appealingly clean to me as it only makes use of a priori appealing and consensual views about the effects of monetary policy shocks on prices, reserves and interest rates. There is one degree of choice here, though: the horizon  $K$  for the sign restrictions. How precisely does this horizon need to be specified, i.e. how sensitive are the results to changes in  $K$ ? The answer is provided in Fig. 7, showing the impulse response functions for real GDP, when imposing a variety of choices for  $K$ . The left column shows the results for a 3-months ( $K = 2$ ) and a 6-months ( $K = 5$ ) horizon, while the right column shows the results for a 12-months ( $K = 11$ ) and a 24-months ( $K = 23$ ) horizon. Essentially, all of these figures show again that the error band for the real GDP impulse response is a  $\pm 0.2$  range around zero. However, as one moves from shorter to longer horizons  $K$ , that band seems to move *up* somewhat, however, and starts to indicate a significant initial rise rather than fall of real GDP, following a “contractionary” shock. Definitely, a short-lived liquidity effect is better for the conventional view.

The results are not quite as sharp at the short end as for the Cholesky decomposition. This is to be expected: the Cholesky decomposition provides an exact identification, while the pure-sign-restriction approach does not. As the horizon increases, however, the degree of uncertainty about the response appears to be about the same. Apparently, the sign restrictions are about as restrictive as or even more restrictive than the Cholesky identification at horizons exceeding, say, 3 years after the shock. It is also interesting to note that the error bands in Fig. 6 are typically remarkably symmetric around the median.

Results for the penalty-function approach are in Section B.3.

### 3.2. How much variation do monetary policy shocks explain?

Having identified the monetary policy shock, it is then interesting to find out how much of the variation these shocks explain. What fraction of the variance of the  $k$ -step ahead forecast revision  $E_t[Y_{t+k}] - E_{t-1}[Y_{t+k}]$  in, say, real GDP, prices and interest rates, are accounted for by monetary policy shocks? These questions are answered by Fig. 8 for the benchmark experiment, i.e. using a pure-sign-restriction approach with a 6-months restriction ( $K = 5$ ). The variables are ordered as in Fig. 6.

According to the median estimates, shown as the middle lines in this figure, monetary policy shocks account for 5–10% of the variations in real GDP at all

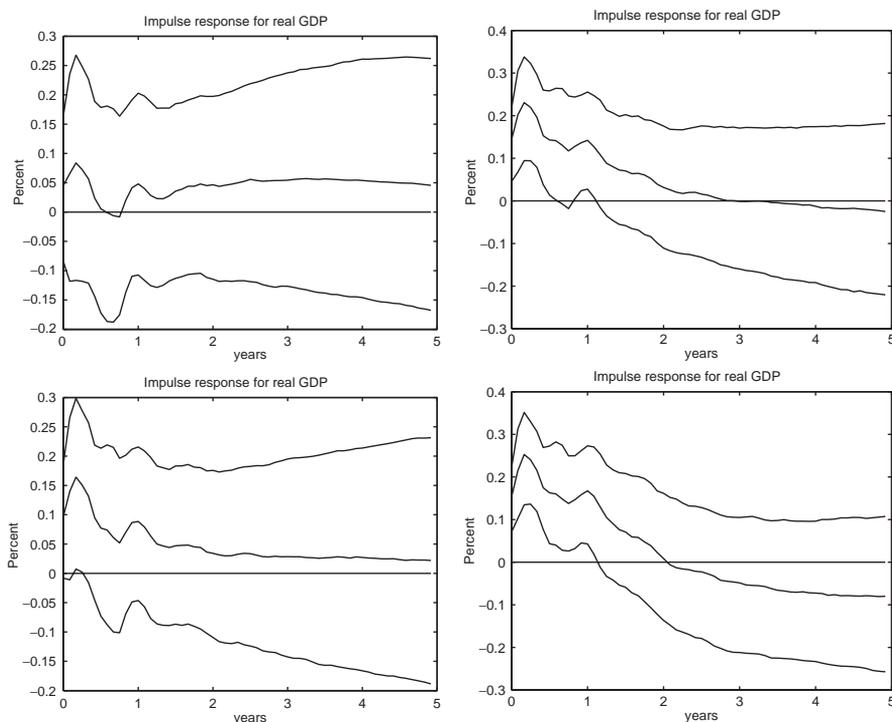


Fig. 7. Impulse responses of real GDP to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach. For the left column,  $K = 2$  and 5 were used, whereas  $K = 11$  and 23 have been used in the right column. Essentially, all of these figures show again the error band for the real GDP impulse response to be a  $\pm 0.2$  interval around zero. As one moves from shorter to longer horizons  $K$ , that band seems to move up. Overall, the evidence in favor of the conventional view of a fall in output after a “contractionary” monetary policy shock seems to weak at best.

horizons, for up to 20% of the long-horizon variations in prices and 15% of the variation in interest rates at the short horizon, falling off after that. Explaining just two or so percent of the real GDP variations at any horizon is within the 64% error band: it thus seems fairly likely, that monetary policy has practically no effect on real GDP. This may either be due to monetary policy shocks having little real effect, or due to a Federal Reserve Bank keeping a steady hand on the wheel, as argued by Cochrane (1994), Woodford (1994) or Bernanke (1996).

Among the six series, the largest fraction at the long end is explained for prices, which is somewhat supportive of the conventional view that in the long run, monetary policy only has effects on inflation and not on much else. For interest rates, the largest fraction of variation explained by monetary policy is at the short horizon, providing further support to the view that monetary policy shocks are accidental errors by the Federal Reserve Bank, which are quickly reversed. The remaining variations in prices and interest rates may still be due to monetary policy,

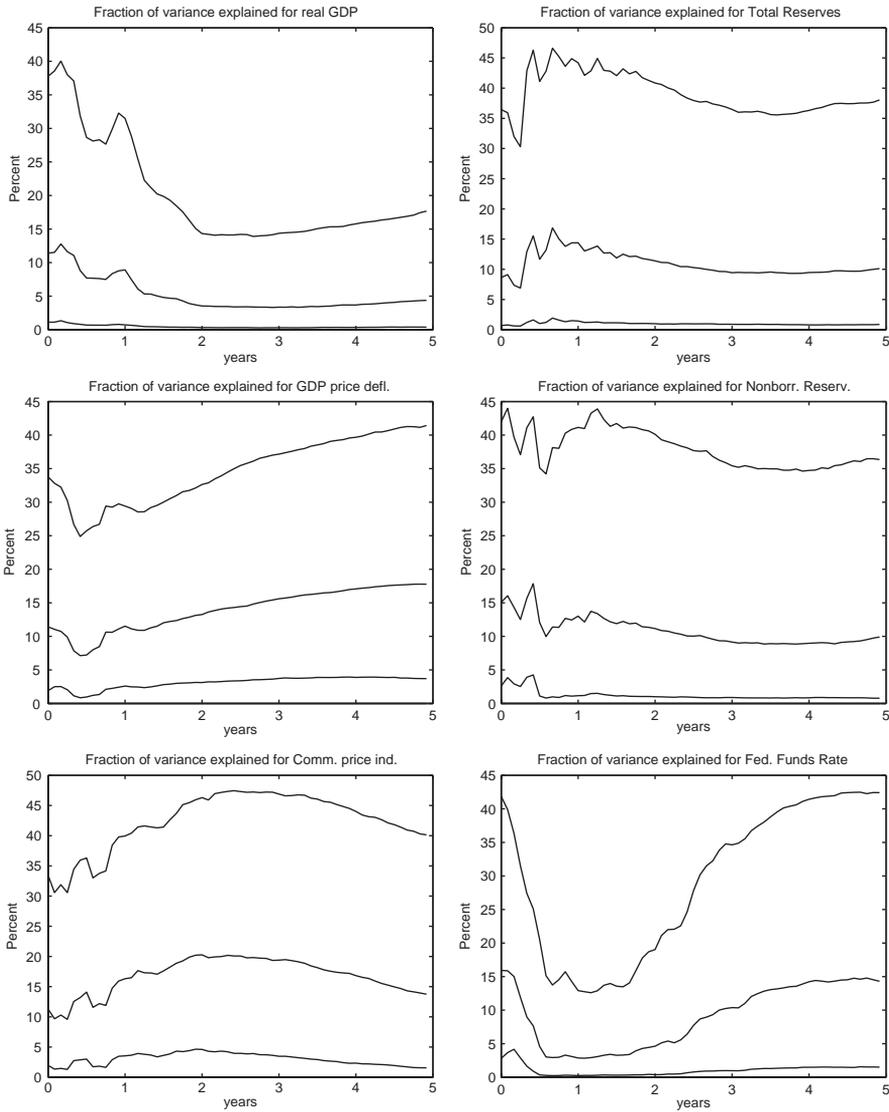


Fig. 8. These plots show the fraction of the variance of the  $k$ -step ahead forecast revision explained by a monetary shock, using a pure-sign restriction approach with  $K = 5$ . The three lines are the 16% quantile, the median and the 16% quantile of the posterior distribution. According to the median estimates, monetary policy shocks account for 10% of the variations in real GDP at all horizons, for up to 30% of the long-horizon variations in prices and for 25% of the variation in interest rates at the short horizon, falling off after that.

but then it needs to be due to the *endogenous* part of monetary policy: by systematically responding to shocks elsewhere, monetary policy may end up being responsible for 100% of the movements in prices. Only 30% of these movements can

directly be ascribed to shocks generated by monetary policy itself. These results are rather similar to the results found in the empirical VAR literature so far, see the surveys cited in the Introduction.

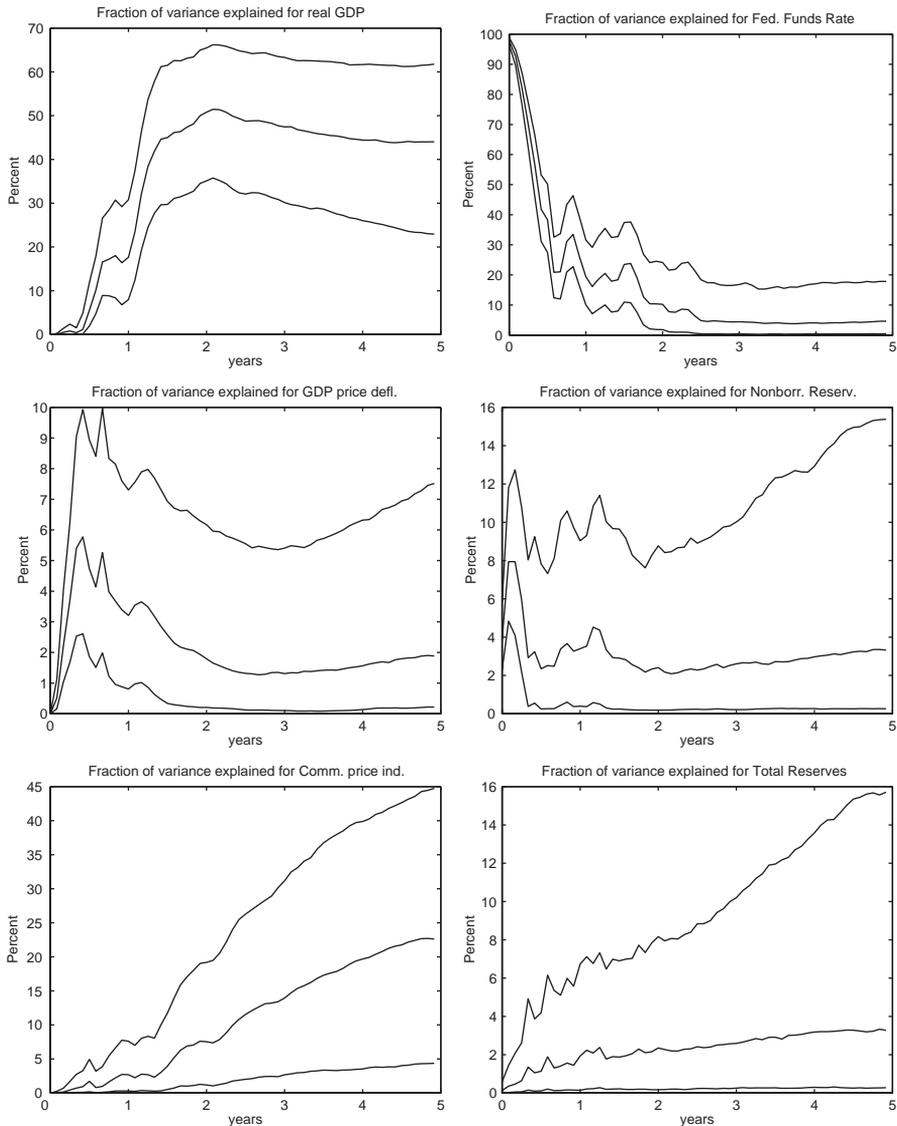


Fig. 9. These plots show the fraction of the variance of the  $k$ -step ahead forecast revision explained by a monetary shock identified via a Cholesky decomposition, see Fig. 5. The error bands are 68% error bands around the median, i.e. the upper and the lower line are the 84% quantile and the 16% quantile of the posterior distribution. The dash-dotted line is the estimate at the mean of the posterior, i.e. for the MLE estimates.

The results for the Cholesky decomposition are shown in Fig. 9 and are strikingly different in several important aspects. Most importantly, nearly half of all the variance in the 5-year ahead forecast revision for real GDP is explained as due to monetary policy shocks. This seems unplausibly large by standards of the conventional wisdom, as it would ascribe large long-lasting effects to observed monetary policy shocks. These results differ from other estimates in the literature, see e.g. Cochrane (1994).

### 3.3. Inflation and real interest rates

One can analyze the results shown further. For example, one can calculate the impulse response for inflation rates by calculating  $r_{\pi,a}(k) = r_{p,a}(k) - r_{p,a}(k - 12)$ , where  $r_{p,a}(k)$  is the horizon of the GDP deflator at horizon  $k$ , given the impulse vector  $a$ , and where we define  $r_{p,a}(k) = 0$  for  $k < 0$ . This in turn allows the calculation of a response of the *real* interest rate by subtracting the predictable change in inflation rates from the response of a 1-year T-bill rate, matching maturities:

$$r_{r,a}(k) = r_{T\text{-bill},a}(k) - r_{\pi,a}(k + 12).$$

To calculate this, I added a time series for the T-bill rate at constant maturity to the VAR specification above, increasing the number of variables from six to seven: the 1-year T-bill rate rather than the federal funds rate is the appropriate nominal interest rate from which to calculate annual real rates by subtracting the annual inflation rate. The data were obtained from the web site of the Federal Reserve Bank of St. Louis. I used the pure-sign-restriction approach with  $K = 5$  (and no restriction on the response of the 1-year T-bill rate) to identify the monetary policy shock. I calculated the implied response for inflation and the real rate. The results are in Fig. 10. What is perhaps somewhat striking is the fact that real rates are positive for up to 2 years, and then return to zero. The overshooting to the negative side, which is visible for both the response of the federal funds rate and the 1-year nominal T-bill rate, is also present in the response of the real rate.

### 3.4. What drives the conventional results?

A still skeptical reader might ask why the conventional Cholesky decomposition shown in Fig. 5 delivers such strikingly different conclusions regarding the response of output. There are three possible replies to this question. The first is that there is a pronounced price puzzle in Fig. 5. One can either proceed by accepting it, and building theories to explain it, see e.g. Altig et al. (2002), or one can suspect that an important ingredient has so far been left out in my agnostic identification approach. In particular, I have allowed prices and real GDP to react instantaneously within the period to monetary policy shocks. How much would it matter to fix a zero response?

Fig. 11 shows the results of fixing the initial GDP deflator response to zero (this restriction seems less plausible for commodity prices, and furthermore the data makes it hard to impose it in addition). Clearly, the results for the initial reaction of real GDP widen the gap to Fig. 5, starting from Fig. 6.

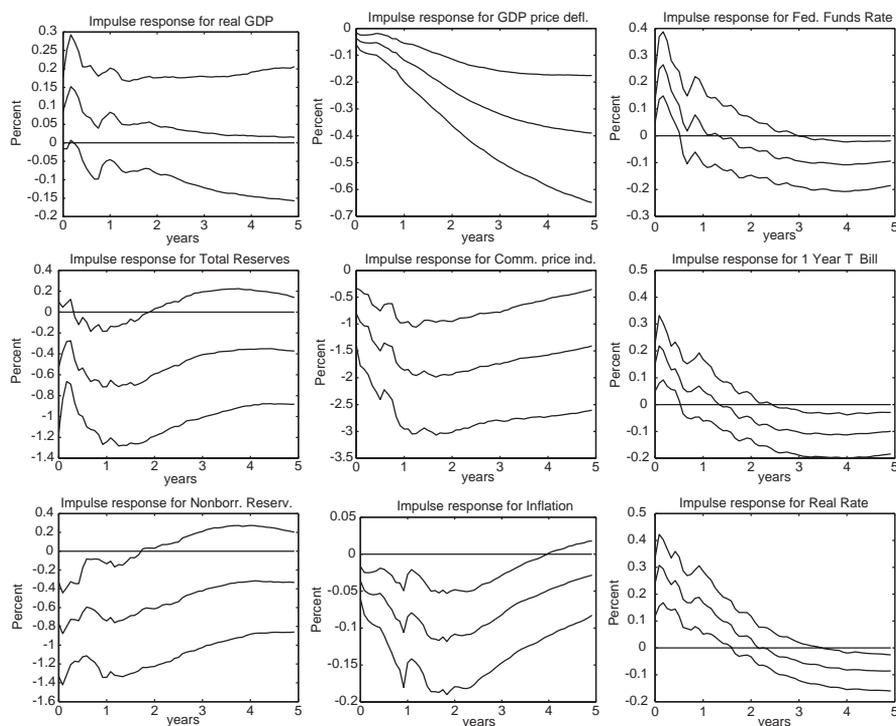


Fig. 10. Additional impulse responses for the 1-year treasury bill rate at constant maturity (added to the VAR), the inflation rate, calculated from the GDP deflator response and the implied real rate. The system has been estimated using a pure-sign-restriction approach with  $K = 5$ . The first column shows the responses of real GDP, total reserves and nonborrowed reserves. The second column shows the responses of the GDP deflator, the commodity price index and inflation. The third column shows the responses of the federal funds rate, the 1-year T-bill rate and the real rate.

Fig. 12 demonstrates, however, that fixing the initial response of real GDP to zero makes a substantial difference. Now, there appears to be considerable evidence that real GDP does indeed fall, following a surprise rise in interest rates. But why should it be plausible to restrict the initial reaction of real GDP? Investment and business plans may be likely to be quite sensitive to interest rates or even slight hints that the Fed might change them. Furthermore, the shocks eliminated in 12 compared to 6 are shocks which move up interest rates and real GDP, while moving down prices and nonborrowed reserves: it is hard to view them as the endogenous response of all other variables to, say, technology shocks or demand shocks. It seems that the life of the conventional wisdom hangs on the thin thread of a rather spurious identification restriction. Alternatively, the method here can be used to make this assumption, avoid the price puzzle and get conventional-looking results, if so desired.

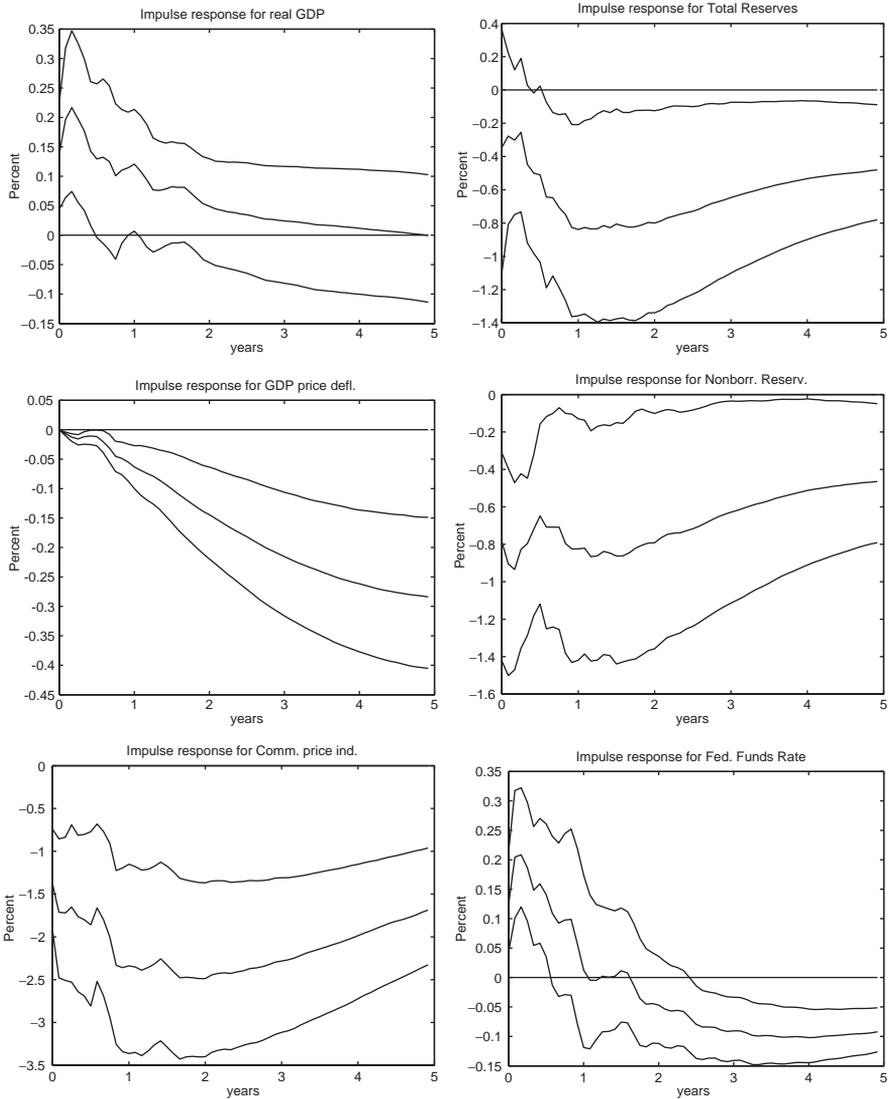


Fig. 11. Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach with  $K = 5$ , additionally imposing a zero response on impact for the GDP price deflator.

#### 4. Conclusions

This paper proposed a new agnostic method to estimate the effects of monetary policy, imposing sign restrictions on the impulse responses of prices, nonborrowed reserves and the federal funds rate in response to a monetary

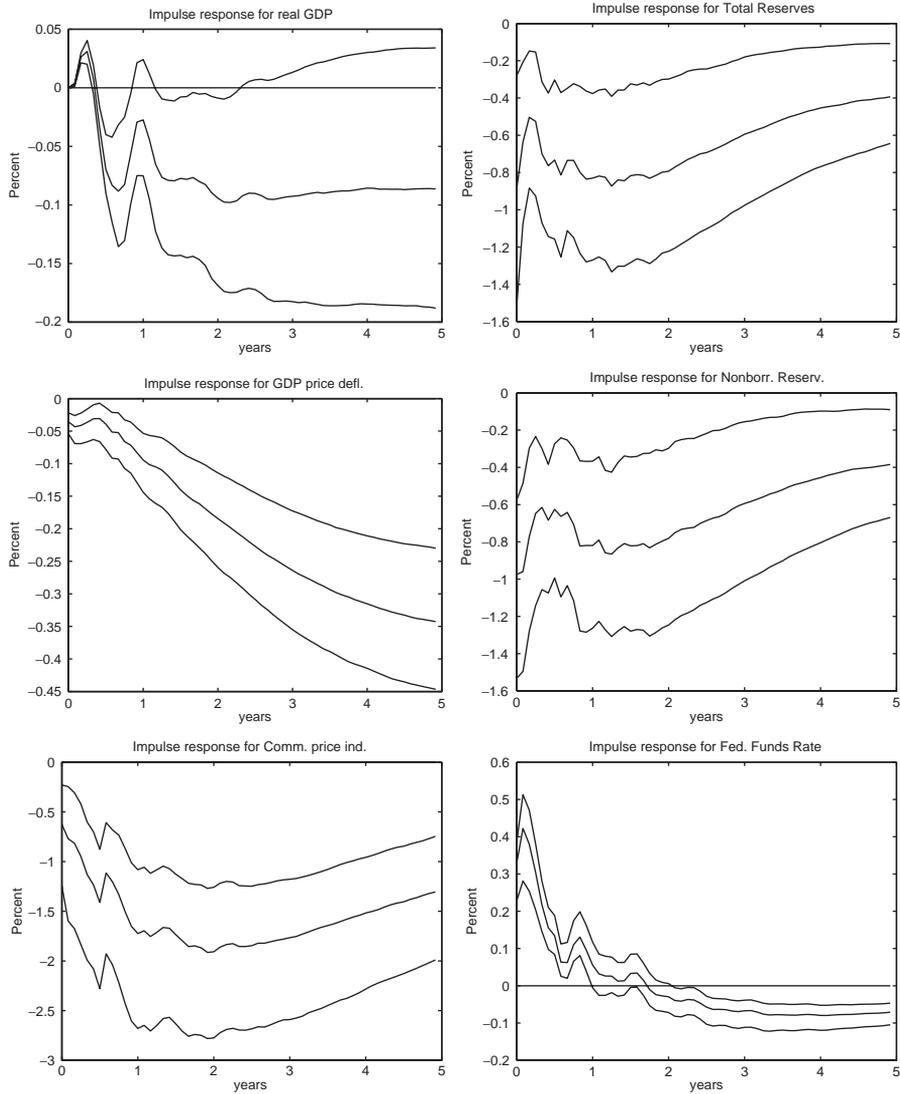


Fig. 12. Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach with  $K = 5$ , additionally imposing a zero response on impact for real GDP.

policy shock. No restrictions are imposed on the response of real GDP. It turned out that

1. “Contractionary” monetary policy shocks have an ambiguous effect on real GDP, moving it up or down by up to  $\pm 0.2\%$  with a probability of  $2/3$ . Monetary

policy shocks account for probably less than 25% of the  $k$ -step ahead prediction error variance of real output, and may easily account for less than 3%.

2. The GDP price deflator falls only slowly following a contractionary monetary policy shock. The commodity price index falls more quickly.
3. Monetary policy shocks account for only a small fraction of the forecast error variance in the federal funds rate, except at horizons shorter than half a year. They account for about one quarter of the variation in prices at longer horizons.

In sum, even though the general price level moves very gradually for a period of about a year, monetary policy shocks have ambiguous real effects and may actually be neutral. These observations largely confirm the results found in the empirical VAR literature so far, except for the ambiguity regarding the effect on output. This exception is, of course, a rather important difference. “Contractionary” monetary policy shocks do not necessarily seem to have contractionary effects on real GDP. One should therefore feel less comfortable with the conventional view and the current consensus of the VAR literature than has been the case so far. The key identifying assumption explaining the difference between my results and the results of, say, a conventional Cholesky decomposition appears to be that I do not restrict the on-impact response of real GDP to be zero.

The paper agrees with a number of other publications in the literature, that variations in monetary policy account only for a small fraction of the variation in any of these variables. Good monetary policy should be predictable policy, and should not rock the boat. From that perspective, monetary policy in the U.S. during this time span has been successful indeed.

## Appendix A. Characterizing impulse vectors

Let  $u$  be the one-step ahead prediction error in a VAR of  $n$  variables and let  $v$  be the vector of fundamental innovations, related to  $u$  via some matrix  $A$ ,

$$u = Av.$$

Let  $\Sigma$  be the variance–covariance matrix of  $u$ , assumed to be nonsingular, while the identity matrix is assumed to be the variance–covariance matrix of  $v$ . If  $v = e_1$ , i.e. the vector with zeros everywhere except for its first entry, equal to unity, then  $u = Ae_1$  equals  $a_1$ , the first column of  $A$ . Hence, the  $j$ th column of  $A$  describes the  $j$ th impulse vector, i.e. the representation of an innovation in the  $j$ th structural variable as a one-step ahead prediction error. Put differently, the  $j$ th column of  $A$  describes the immediate impact on all variables of an innovation in the  $j$ th structural variable. Our aim is to characterize all possible impulse vectors. One can do so, using the observation that any two decompositions  $\Sigma = AA'$  and  $\tilde{A}\tilde{A}'$  have to satisfy that

$$\tilde{A} = AQ \tag{6}$$

for some orthogonal matrix  $Q$ , i.e.  $QQ' = I$ , see also Faust (1998) and Uhlig (1998). I find the following proposition useful, which I shall prove directly. I follow the general convention that all vectors are to be interpreted as columns.

**Proposition A.1.** *Let  $\Sigma$  be a positive definite matrix. Let  $x_i, i = 1, \dots, m$  be the eigenvectors of  $\Sigma$ , normalized to form an orthonormal basis of  $\mathbb{R}^m$ . Let  $\lambda_i, i = 1, \dots, m$  be the corresponding eigenvalues. Let  $a \in \mathbb{R}^m$  be a vector. Then, the following four statements are equivalent:*

1. *There are coefficients  $\alpha_i, i = 1, \dots, m$  with  $\sum_{i=1}^m \alpha_i^2 = 1$ , so that*

$$a = \sum_{i=1}^m (\alpha_i \sqrt{\lambda_i}) x_i.$$

2.  *$\tilde{\Sigma} = \Sigma - aa'$  is positive semidefinite and singular.*
3. *The vector  $a$  is an impulse vector, i.e., there is some matrix  $A$ , so that  $AA' = \Sigma$  and so that  $a$  is a column of  $A$ .*
4. *Let  $\tilde{A}\tilde{A}' = \Sigma$  for some matrix  $\tilde{A} = [\tilde{a}_1, \dots, \tilde{a}_m]$ . Then there are coefficients  $\alpha_i, i = 1, \dots, m$  with  $\sum_{i=1}^m \alpha_i^2 = 1$ , so that*

$$a = \sum_{i=1}^m \alpha_i \tilde{a}_i.$$

Note that there are  $m - 1$  degrees of freedom in picking an impulse vector, and that impulse vectors cannot be arbitrarily long: the Cauchy–Schwarz inequality implies that

$$\|a\| \leq \sqrt{\sum_{i=1}^m |\lambda_i| \|x_i\|^2},$$

for example.

**Proof.** First, I show that the third statement implies the second statement. To that end, write  $A = [a_1 \dots a_m]$  in form of its columns, and note that

$$\Sigma = AA' = \sum_{i=1}^m a_i a_i'.$$

Assume w.l.o.g., that  $a$  is the first column,  $a = a_1$ . Then,  $\tilde{\Sigma} = \sum_{i=2}^m a_i a_i'$ , which is positive semidefinite and singular, since each of the matrices  $a_i a_i'$  are of rank 1.

Next, I show that the second statement implies the third. Find the nonzero eigenvalues  $\tilde{\lambda}_i, i = 2, \dots, m$  and its corresponding eigenvectors  $\tilde{x}_i, i = 2, \dots, m$  for the positive semidefinite matrix  $\tilde{\Sigma} = \Sigma - aa'$ , noting that  $\tilde{\Sigma}$  must be of rank  $m - 1$ , since  $\Sigma$  is of rank  $m$ . Let

$$A = \left[ a, \quad \sqrt{\tilde{\lambda}_2} \tilde{x}_2, \quad \sqrt{\tilde{\lambda}_3} \tilde{x}_3 \quad \dots \quad \sqrt{\tilde{\lambda}_m} \tilde{x}_m \right].$$

A simple calculation shows that indeed  $\Sigma = AA'$ .

To see that the third statement implies the last, note that  $A = \tilde{A}Q$  for some matrix  $Q$  with  $QQ' = I$ , see Eq. (6). The coefficients  $\alpha$  can now be found in the first column of  $Q$ . Conversely, given any such vector  $\alpha$  of unit length, complement it to an orthogonal basis to form the matrix  $Q$ . Then, let  $A = \tilde{A}Q$ .

To see the equivalence between the third and the first statement, follow the same argument, noting that

$$\tilde{A} = [x_1 \quad \dots \quad x_m] \begin{bmatrix} \sqrt{\lambda_1} & 0 & \dots & 0 \\ 0 & \sqrt{\lambda_2} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \sqrt{\lambda_m} \end{bmatrix}$$

is simply a particular decomposition  $\tilde{A}\tilde{A}' = \Sigma$ .

This finishes the proof.  $\square$

Given an impulse vector  $a$ , one would like to calculate the part of the one-step ahead prediction error  $u_t$  which is attributable to shocks proportional to that vector. If the entire matrix  $A$  was available and  $a$  was the, say, first column, one would simply calculate  $v_t = A^{-1}u_t$  and use  $v_{t,1}$  as the scale of the shock attributable to  $a$ . Motivated by this reasoning, define:

**Definition A.2.** Given an impulse vector  $a$  and a one-step ahead prediction error  $u \in \mathbb{R}^m$ ,  $v^{(a)} \in \mathbb{R}$  is called the scale of a shock attributable to  $a$ , if there exists a matrix  $A$  with  $A'A = \Sigma$ , of which  $a$  is the  $j$ th column for some  $j$ , so that  $v^{(a)} = (A^{-1}u)_j$ .

It turns out that this ties down the scale uniquely, provided  $\Sigma$  is not singular.

**Proposition A.2.** Given an impulse vector  $a$  and a one-step ahead prediction error  $u$ , the scale of the shock  $v^{(a)}$  attributable to  $a$  is unique and can be calculated as follows. Let  $b \in \mathbb{R}^m$  solve the two equations

$$\begin{aligned} 0 &= (\Sigma - aa')b, \\ 1 &= b'a. \end{aligned}$$

The solution  $b$  exists and is unique. Then,

$$v^{(a)} = b'u.$$

**Proof.** Suppose  $A$  was available and assume w.l.o.g., that  $a$  is its first column. Thus,  $A$  can be partitioned as  $A = [a|A_2]$ . Likewise, partition  $B = A^{-1}$  into

$$B = \begin{bmatrix} b' \\ B_2 \end{bmatrix}$$

as well as  $v = A^{-1}u$  into

$$v = \begin{bmatrix} v^{(a)} \\ V_2 \end{bmatrix}.$$

Clearly,  $v^{(a)} = b'u$ : thus, the task is to characterize  $b$ . Note first that

$$\Sigma = AA' = aa' + A_2A_2'. \quad (7)$$

Next, note that

$$I_m = BA = \begin{bmatrix} b'a & b'A_2 \\ B_2a & B_2A_2 \end{bmatrix}.$$

Hence,  $b'a = 1$  and  $b'A_2 = 0$ . The latter equality implies together with Eq. (7)

$$0 = b'A_2A_2'b = b'(\Sigma - aa')b.$$

Since  $\Sigma - aa'$  is symmetric, this is equivalent to  $(\Sigma - aa')b = 0$ . Note that there is unique one-dimensional subspace of vectors  $b$  satisfying  $(\Sigma - aa')b = 0$ , since  $\Sigma$  is assumed to be regular. Also, because  $\Sigma$  is regular,  $a'b \neq 0$  for any  $b \neq 0$  which satisfies this equation. Thus, there is a unique  $b$ , which also satisfies  $b'a = 1$ .  $\square$

With  $v^{(a)}$  it is now furthermore clear that the part of  $u$  which is attributable to the shock proportional to the impulse vector  $a$  is given by  $v^{(a)}a$ .

## Appendix B. Estimation and inference

For convenience, I collect here the main tools for estimation and inference, see also Uhlig (1998). I use a Bayesian approach since it is computationally simple and since it allows for a conceptually clean way of drawing error bands for statistics of interest such as impulse responses, see Sims and Zha (1999) for a clear discussion on this point. Note that draws from the posterior are candidate truths. Thus, if e.g. the true impulse response for prices should not violate the imposed sign restriction, then this should also literally be true for any draw from the posterior. Thus, the price puzzle in Fig. 5 is a violation by candidate truths, and worrisome. With a classical approach, by contrast, considerations of significance would enter: a violation may be considered as consistent with the sign restriction if it is insignificant, requiring further judgement. Put differently, a Bayesian approach is more convenient to implement and cleaner to justify. The reader who rather wishes to pursue a classical approach and inference regarding impulse response functions in vector autoregressions is referred to the work by Mittnik and Zdrozny (1993), Kilian (1998a, b) and Berkowitz and Kilian (2000).

Using monthly data, I fixed the number of lags at  $l = 12$  as in Bernanke and Mihov (1998a, b). Stack system (1) as

$$\mathbf{Y} = \mathbf{XB} + \mathbf{u} \quad (8)$$

where  $X_t = [Y'_{t-1}, Y'_{t-2}, \dots, Y'_{t-l}]'$ ,  $\mathbf{Y} = [Y_1, \dots, Y_T]'$ ,  $\mathbf{X} = [X_1, \dots, X_T]'$ ,  $\mathbf{u} = [u_1, \dots, u_T]'$  and  $\mathbf{B} = [B_{(1)}, \dots, B_{(l)}]'$ . To compute the impulse response to an impulse

vector  $a$ , let  $\mathbf{a} = [a', 0_{1,m(l-1)}]'$  as well as

$$\Gamma = \begin{bmatrix} \mathbf{B}' \\ I_{m(l-1)} & 0_{m(l-1),m} \end{bmatrix}$$

and compute  $r_{k,j} = (\Gamma^k \mathbf{a})_j, k = 0, 1, 2, \dots$  to get the response of variable  $j$  at horizon  $k$ . The variance of the  $k$ -step ahead forecast error due to an impulse vector  $a$  is obtained by simply squaring its impulse responses. Summing again over all  $a_j$ , where  $a_j$  is the  $j$ th column of some matrix  $A$  with  $AA' = \Sigma$ , delivers the total variance of the  $k$ -step ahead forecast error. I assume that the  $u_t$ 's are independent and normally distributed. The MLE for  $(\mathbf{B}, \Sigma)$  is given by

$$\hat{B} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}, \hat{\Sigma} = \frac{1}{T}(\mathbf{Y} - \mathbf{X}\hat{B})'(\mathbf{Y} - \mathbf{X}\hat{B}). \tag{9}$$

Our prior and posterior for  $(\mathbf{B}, \Sigma)$  belongs to the Normal–Wishart family, whose properties are further discussed in Uhlig (1994), extending the standard treatment in Zellner (1971). A proper Normal–Wishart distribution is parameterized by a mean coefficient matrix  $\bar{B}$  of size  $ml \times m$ , a positive definite mean covariance matrix  $S$  of size  $m \times m$  as well as a positive definite matrix  $N$  of size  $ml \times ml$  and a degrees-of-freedom real number  $v \geq 0$  to describe the uncertainty about  $(\mathbf{B}, \Sigma)$  around  $(\bar{B}, S)$ . The Normal–Wishart distribution specifies, that  $\Sigma^{-1}$  follows a Wishart distribution  $\mathcal{W}_m(S^{-1}/v, v)$  with  $E[\Sigma^{-1}] = S^{-1}$ , and that, conditionally on  $\Sigma$ , the coefficient matrix in its columnwise vectorized form,  $\text{vec}(\mathbf{B})$ , follows a Normal distribution  $\mathcal{N}(\text{vec}(\bar{B}), \Sigma \otimes N^{-1})$ . To draw from the Wishart distribution  $\mathcal{W}_m(S^{-1}/v, v)$ , an easily implementable method is to calculate  $\Sigma = (R * R')^{-1}$ , where  $R$  is an  $m \times v$  matrix with each column an independent draw from a Normal distribution  $\mathcal{N}(0, S^{-1}/v)$  with mean zero and variance–covariance matrix  $S^{-1}$ .

Proposition A.1 on p. 670 in Uhlig (1994) states that if the prior is described by  $\bar{B}_0, N_0, S_0$  and  $v_0$ , then the posterior is described by  $\bar{B}_T, N_T, S_T$  and  $v_T$ , where

$$\begin{aligned} v_T &= T + v_0, \\ N_T &= N_0 + \mathbf{X}'\mathbf{X}, \\ \bar{B}_T &= N_T^{-1}(N_0\bar{B}_0 + \mathbf{X}'\mathbf{X}\hat{B}), \\ S_T &= \frac{v_0}{v_T}S_0 + \frac{T}{v_T}\hat{\Sigma} + \frac{1}{v_T}(\hat{B} - \bar{B}_0)'N_0N_T^{-1}\mathbf{X}'\mathbf{X}(\hat{B} - \bar{B}_0). \end{aligned}$$

I use a weak prior, and  $N_0 = 0, v_0 = 0, S_0$  and  $\bar{B}_0$  arbitrary. Then,  $\bar{B}_T = \hat{B}, S_T = \hat{\Sigma}, v_T = T, N_T = \mathbf{X}'\mathbf{X}$ , which is also the form of the posterior used in the RATS manual for drawing error bands, see e.g. 10.1 in Doan (1992).

No attempt has been made to impose more specific prior knowledge such as the no-change-forecast of the Minnesota prior, see Doan et al. (1984), special treatments of roots near unity, see the discussion in Sims and Uhlig (1991) as well as Uhlig (1994), or to impose the more sophisticated priors of Leeper et al. (1996) or Sims and Zha (1998) or Kadiyala and Karlsson (1997). Also, I have not experimented with regime switching as in Bernanke and Mihov (1998a, b) or with stochastic volatility as in Uhlig (1997).

### B.1. The pure-sign-restriction approach

Two rather technical remarks are in order. First, by parameterizing the impulse vector, i.e. by formulating the prior as a product with an indicator variable in  $(B, \Sigma, \alpha)$ -space rather than  $(B, \Sigma, a)$ -space, an undesirable scaling problem is avoided. Consider some  $(B, \Sigma)$  as well as  $(B, \lambda\Sigma)$  for some  $\lambda > 0$ . Rescaling  $\Sigma$  induces rescaling of  $\mathcal{A}(B, \Sigma, K)$ . As a result, a prior with an indicator variable in  $(B, \Sigma, a)$ -space would assign  $\lambda^{(m-1)/2}$  as much weight to the  $\varepsilon$ -ball around  $(B, \lambda\Sigma)$  as to the  $\varepsilon$ -ball around  $(B, \Sigma)$  beyond the weights given by the Normal–Wishart prior. With the formulation in  $(B, \Sigma, \alpha)$ -space, the weight of these two balls is given by the Normal–Wishart prior alone.

Second, it should be noted that all decompositions  $\Sigma = \tilde{A}\tilde{A}'$  together with a uniform prior for  $\alpha$  result in the *same* prior on the impulse vectors  $a$  and thus the same inference, because two different decompositions differ by an orthogonal rotation  $Q$ , see Eq. (6). Therefore, changing to a different decomposition is equivalent to rotating the distribution for  $\alpha$  with the appropriate  $Q$ . Since an orthogonal rotation of a uniform distribution on the unit sphere will leave that distribution unchanged, there is no change in the implied prior on impulse vectors. In particular, reordering and choosing a different Cholesky decomposition in order to parameterize impulse vectors will not yield different results. In sum, any smooth, matrix-valued function of  $\Sigma$ , satisfying  $\tilde{A}(\Sigma)(\tilde{A}(\Sigma))' = \Sigma$  will lead to the same inference, because two such functions differ only in an orthonormal transformation and thus by a Jacobian equal to unity.

Finally, the flat prior is appealing, as the likelihood function is uninformative about the appropriate choice of  $\alpha$ , i.e., using Jeffreys prior would also result in the choice of a flat prior in  $\alpha$ . This is not true in  $(B, \Sigma, a)$ -space due to the rescaling issue described above. By change of variable, the prior chosen in  $(B, \Sigma, \alpha)$ -space can be transformed in a prior in  $(B, \Sigma, a)$ -space, obviously. Alternatively, one could calculate the implied prior in the space of impulse responses, which provide another means of parameterization. Dwyer (1997) pursues that route.

To draw inferences from the posterior for the pure-sign-restriction approach, I take  $n_1$  draws from the VAR posterior and, for each of these draws,  $n_2$  draws  $\alpha$  from the  $m$ -dimensional unit sphere. A draw  $\alpha$  from the  $m$ -dimensional unit sphere is easily obtained by drawing  $\tilde{\alpha}$  from the  $m$ -dimensional standard normal distribution, and then normalizing its length to unity,  $\alpha = \tilde{\alpha}/\|\tilde{\alpha}\|$ . From  $\Sigma$  and  $\alpha$ , I construct the impulse vector, using characterization (3) or, alternatively, some other characterization in Proposition A.1.

For each draw, I calculate the impulse responses, and check, whether the sign restrictions are satisfied. If they are, I keep the draw. If not, I proceed to the next. Finally, error bands, etc. are calculated using all the draws which have been kept. For the calculations, I have chosen  $n_1 = n_2$  and high enough, so that a couple of hundred joint draws satisfied the sign restriction. It turned out that I could use  $n_1 = n_2 = 200$  in all cases.

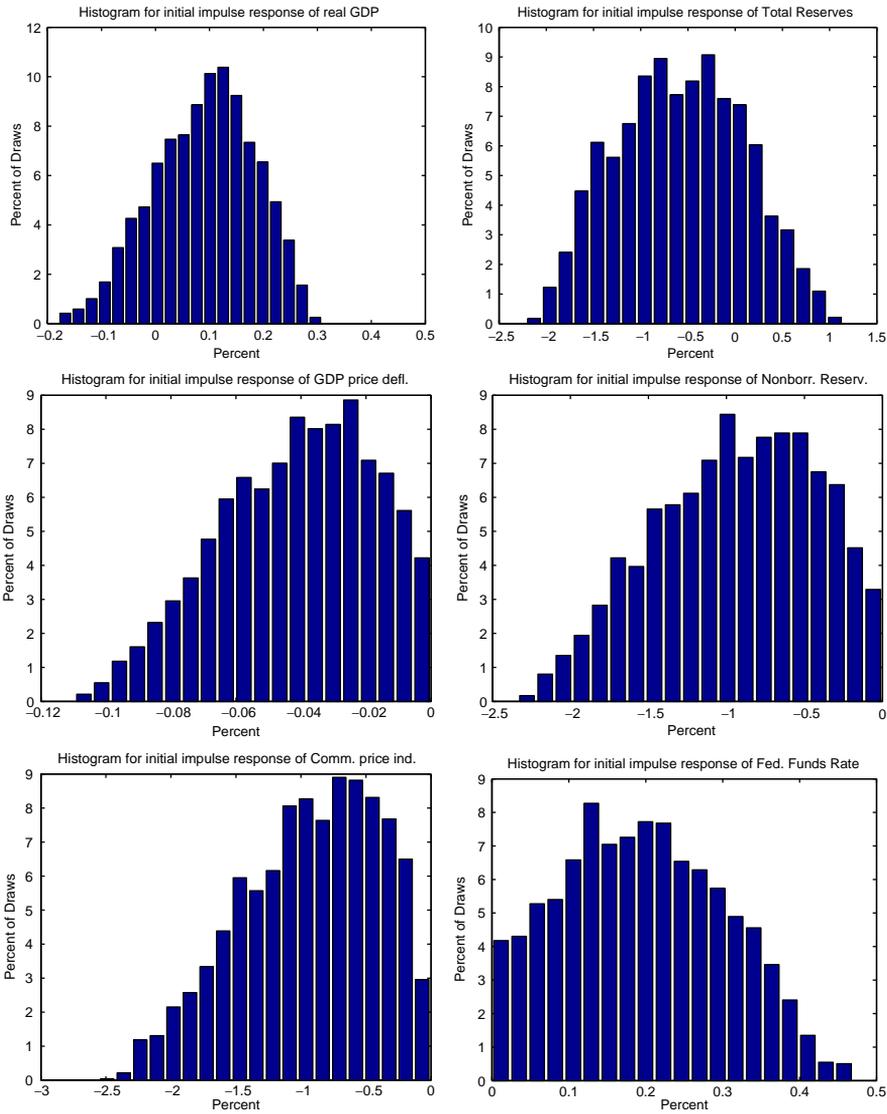


Fig. 13. This figure shows the posterior distribution of the impact impulse response (i.e. at horizon 0), when imposing the sign restrictions for  $K = 5$ , using the pure-sign-restriction approach.

Fig. 13 shows the posterior of the initial responses of all variables as histogram, similar to 3. While there are some subtle differences, they do not appear to be large. This is comforting. The Bayesian approach allows for clean posterior inference, but does not produce strong and thus potentially hard-to-explain deviations from a simple OLSE-with-sign-restrictions analysis.

B.2. The penalty-function approach

With the penalty-function approach, assumption A.1 is replaced with

**Assumption B.1.** A monetary policy impulse vector is an impulse vector  $a$  minimizing a given criterion function  $f(\cdot)$  on the space of all impulse vectors, which penalizes positive impulse responses of prices and nonborrowed reserves and negative impulse responses of the federal funds rate at horizons  $k = 0, \dots, K$ .

Assumption *ass:puresign* is replaced with

**Assumption B.2** (for the penalty-function approach). The parameters  $(B, \Sigma)$  are drawn from a Normal–Wishart prior. The monetary policy impulse vector  $a$  is identified, using assumption B.1.

To compare this approach to the pure-sign-restriction approach, it is instructive to consider the case of overidentification, i.e. if the set  $\mathcal{A}(B, \Sigma, K)$  is empty. In that case, the first approach will consider the particular  $B$  and  $\Sigma$  impossible, i.e. the posterior will be constrained to be zero there. This is not a problem in principle, as long as there are *some*  $B$  and  $\Sigma$ , for which  $\mathcal{A}(B, \Sigma, K)$  is nonempty: the first method will only permit these for drawing inferences. By contrast, the second approach will always find a best impulse vector  $a$  for any given  $(B, \Sigma)$ . If the set  $\mathcal{A}(B, \Sigma, K)$  is empty, the second approach will find an impulse vector  $a$  which comes as close as possible to satisfying the sign restrictions by minimizing a penalty for sign restriction violations.

Numerically, I implement the penalty-function approach as follows. Define the penalty function

$$f(x) = \begin{cases} x & \text{if } x \leq 0, \\ 100 * x & \text{if } x \geq 0 \end{cases} \tag{10}$$

which penalizes positive responses in linear proportion and rewards negative responses in linear proportion, albeit at a slope 100 times smaller than the slope for penalties on the positive side.

Make Assumption B.2. For the true VAR coefficients, let  $r_{j,a}(k), k = 0, \dots, K$  be the impulse response of variable  $j$  and  $\sigma_j$  be the standard deviation of the first difference of the series for variable  $j$ . Let  $i_j = -1$ , if  $j$  is the index of the federal funds rate in the data vector, and else, let  $i_j = 1$ . Define the monetary policy impulse vector as that impulse vector  $a$ , which minimizes the total penalty  $\Psi(a)$  for prices, nonborrowed reserves and (after flipping signs) the federal funds rate at horizons  $k = 0, \dots, K$ ,

$$\Psi(a) = \sum_{j \in \left\{ \begin{array}{l} \text{“ GDP deflator ”} \\ \text{“ Comm. price index ”} \\ \text{“ Nonborr. reserves ”} \\ \text{“ Federal funds rate ”} \end{array} \right\}} \sum_{k=0}^K f\left(i_j \frac{r_{j,a}(k)}{\sigma_j}\right).$$

The rescaling by  $\sigma_j$  is necessary to make the deviations across different impulse responses comparable to each other. Note that the sign of the penalty direction is flipped for the federal funds rate. Since the true VAR is not known, find the monetary policy impulse vector for each draw from the posterior. This requires numerical minimization. Keep all draws and accordingly calculated monetary policy impulse vectors, and calculate statistics based on these.

Perhaps the most controversial aspect of the penalty function in (10) is the *reward* given to responses satisfying the restriction. Numerically, this feature is needed in order to (generically) exactly identify a best impulse vector, if  $\mathcal{A}(B, \Sigma, K)$  is not empty. But does this also make economic sense? I believe it does for the following reason. At any point in time, many shocks hit the economy. In isolation or together, some nonmonetary shocks may trigger minor responses in interest rates, prices and nonborrowed reserves which satisfy the sign restrictions. On the other hand, it is plausible that a monetary policy shock moves all these variables quite substantially. Given a choice among many candidate monetary impulse vectors in  $\mathcal{A}(B, \Sigma, K)$ , it might therefore be desirable to pick the one, which generates a more decisive response of the variables, for which sign restrictions are imposed: this is what the penalty-function approach does. The drawback of this feature should also be clear: one is, in effect, imposing somewhat more than just the sign restrictions. While I have treated all sign restrictions symmetrically, one could alternatively modify the penalty-function approach so that rewards are only given for those variables or at those horizons, for which a large response in the correct direction seems a priori most plausible. For example, one may expect monetary policy shocks to move interest rates a lot in the first few periods, but one may be less sure about a strong reaction of prices or a strong reaction of interest rates further out, compared to other shocks hitting the economy.

A few remarks should be made in defense of the particular functional form used for the penalty function in (10). First, because I wish to impose sign restrictions, the penalty function should be asymmetric, punishing violations a lot more strongly than rewarding large and correct responses. Second, a continuous penalty function is needed in order to make standard minimization procedures work properly. Some minimization procedures even require differentiability: this can be accommodated fairly easily by smoothing out the kink at zero, modifying the function in a small neighborhood around zero. Third, I do want to punish even small violations—which is why e.g. a quadratic functional form is a less appealing choice than a linear one—but at the same time, I do want to punish larger deviations more than small ones and not treat them as equally bad—which is why e.g. a square-root function form is also less appealing. A square-root specification would also generate infinite slopes at zero, which may create numerical problems. Nonetheless, to check the robustness of my results, I have also experimented with a square-root specification, replacing  $x$  by  $\sqrt{|x|}$  in the calculation of the penalty on the right-hand side of Eq. (10) as well as with a square specification, similarly replacing  $x$  with  $x^2$ .

To draw inference from the posterior for the penalty-function approach, I take  $n$  draws from it, employing a Monte-Carlo method: because optimizing over the shape of the impulse responses is time consuming, I usually took  $n = 100$ . For each of these

draws, I calculate the impulse responses and the variance decomposition and collect them. Thus, after 100 draws, I have 100 draws for each point on an impulse response function I may wish to estimate: it is now easy to calculate their median and their 68% error band.

To do the numerical minimization of the criterion function  $\Psi$  for each draw from the posterior, I needed to parameterize the space of vectors  $(\alpha_j)_{j=1}^6$  of unit length: I found the parameterization

$$\alpha = \begin{bmatrix} \cos(\gamma_1) \cos(\gamma_2) \cos(\gamma_3) \\ \cos(\gamma_1) \cos(\gamma_2) \sin(\gamma_3) \\ \cos(\gamma_1) \sin(\gamma_2) \\ \sin(\gamma_1) \cos(\gamma_4) \cos(\gamma_5) \\ \sin(\gamma_1) \cos(\gamma_4) \sin(\gamma_5) \\ \sin(\gamma_1) \sin(\gamma_4) \end{bmatrix}, \quad (\gamma_j)_{j=1}^5 \in \mathbb{R}^5$$

particularly convenient. I have coded all my routines in MATLAB, and used its general purpose minimizer `fmins.m` to perform the minimization task numerically. It turned out that `fmins` sometimes stopped the search before converging to the optimal solution: I thus performed `fmins.m` three times in a row, starting it first at a randomly selected  $(\gamma_j)_{j=1}^5 \in \mathbb{R}^5$  and then starting it successively at the previously found optimum. Now, the minimization seemed to miss the minimum in safely less than 5% of all cases. To achieve near-certain convergence, I did this procedure twice, starting it from two different initial random vectors  $(\gamma_j)_{j=1}^5 \in \mathbb{R}^5$ , and selecting the best of the two minimas found. That way, the chance of missing the optimum was safely below 0.3%. To calculate this for a single draw from the posterior took around four minutes on a Pentium-based machine. I used a sample of 100 draws from the posterior for inference.

### B.3. Results from the penalty-function approach

Fig. 14 provides some of the results for the penalty-function approach and the 6-months horizon,  $K = 5$ . This figure should be compared to Fig. 6 for the pure-sign-restriction approach. The results look qualitatively largely the same. The magnitudes are slightly larger, and the error bands considerably sharper, in particular immediately after the shock, compared to the pure-sign-restriction approach. The greatest difference is obtained for the impulse response for real GDP, i.e. for my central question. Here, one can perhaps see some evidence for the conventional view: real output now seems to clearly stay above zero for most of the first year at 0.1% above the no-shock scenario.

The differences between these two approaches in Figs. 6 and 14 are easy to explain. While the pure-sign-restriction approach is agnostic about the size of the impulse response away from the sign restriction, larger responses are rewarded by the penalty-function approach at least as long as this does not generate sign-violations elsewhere. Instead of a range of impulse vectors consistent with the sign

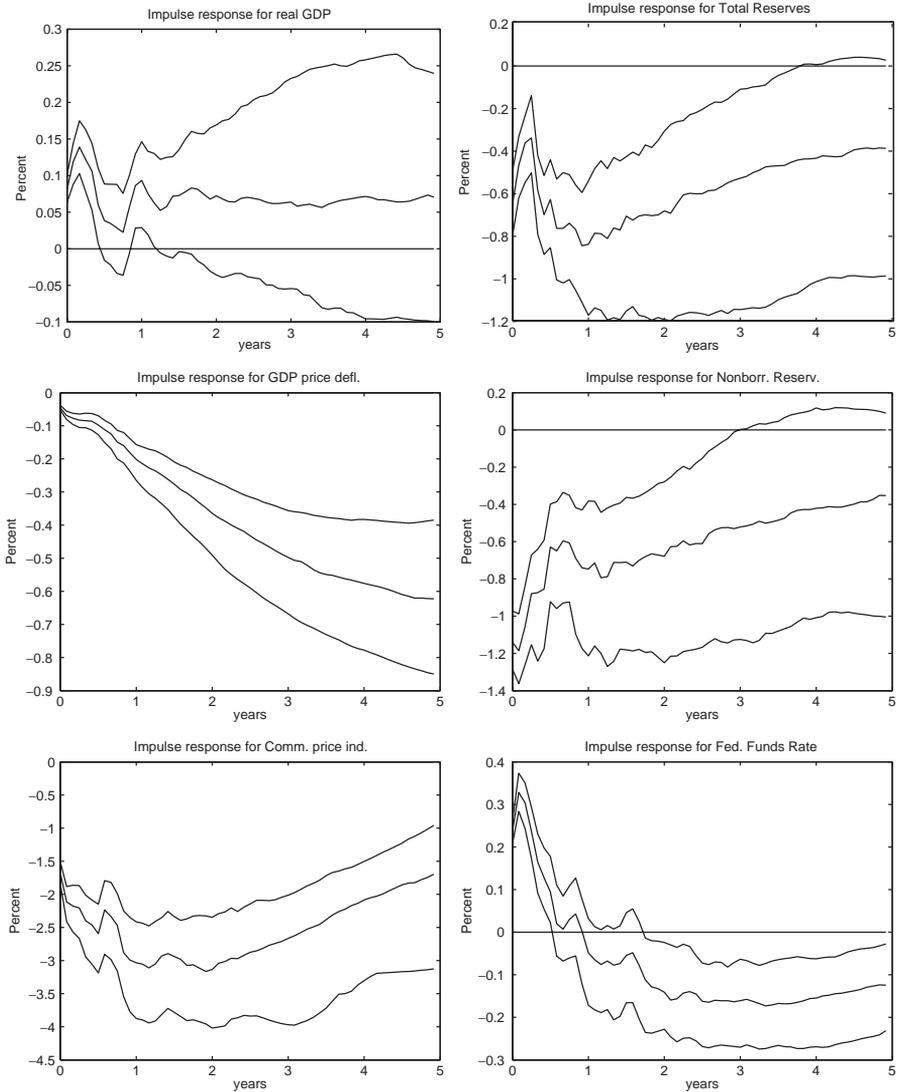


Fig. 14. Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the penalty-function approach with  $K = 5$ . That is, the responses of the GDP price deflator, the commodity price index, nonborrowed reserves and the negative of the federal funds rate have been penalized for positive values and slightly rewarded for negative values in the months  $k, k = 0, \dots, 5$  following the shock: the shock was identified by minimizing total penalties. The error bands are now much sharper. While the real GDP response is still within the  $\pm 0.2$  interval around zero estimated before, there now seems to be a piece between 1 and 12 month, showing a conventional response.

restriction, the penalty-function approach seeks a unique monetary policy impulse vector by searching e.g. for a large initial reaction of the federal funds rate. Indeed, this reaction is now fairly sharply estimated to be about 30 basis points, quickly

rising by another 10 basis points. One obtains similar sharp error bands elsewhere. The monetary policy impulse vector uniquely identified by the penalty function is an element in the set of the vectors admitted by the pure-sign-restriction approach, given a draw for the VAR coefficients, provided that set is not empty. One would therefore expect the range of impulse responses of the penalty-function approach to be contained in the range of impulse responses of the pure-sign-restriction approach. Indeed, this seems to be the case: with 64% posterior probability, the response for the real GDP response, for example, never seems to venture outside the  $\pm 0.2\%$  interval around zero calculated similarly for the pure-sign-restriction approach.

One can thus either view the results in Fig. 14 as a *sharpening* of the results in Fig. 6, due to additionally desirable properties imposed on the restricted impulse responses, or as a *distortion* of the results in Fig. 6 due to additional ad hoc restrictions. Since the aim is to impose the sign restrictions and nothing else, I find the pure-sign-restriction approach to be more appealing. The results of the second approach are nonetheless informative in that they show the additional mileage obtained from additional, potentially desirable restrictions, opening the door to more detailed investigations.

Additional figures (not shown) demonstrate that the results of the penalty-function approach are also more sensitive to the choice of the restriction horizon  $K$ . Likewise, additional calculations show that the results are not affected much by the specific functional form of the penalty functions.

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