Supporting Information for "Joint modeling the frequency and duration of accelerometer-measured physical activity from a lifestyle intervention trial" by Juned Siddique, Michael J. Daniels, Gül Inan, Samuel Battalio, Bonnie Spring, and Donald Hedeker

A Accelerometer missingness/non-compliance

Below we report—by study phase and treatment condition—the number and proportion of MBC participants who wore the accelerometer for 10 or more hours a day on four or more days in a given week. These two characteristics are conventionally regarded as the minimum needed for validly estimating habitual PA levels [1, 2]

Table 1: Missing data patterns in MBC by treatment condition and overall. The first three columns indicate whether a participant wore the accelerometer for 10 hours on four or more days a week during each of the three study phases.

Baseline Data	Rx1 Data	Rx23 Data	Total (n=204) n (%)	iPA (n=95) n (%)	dSED (n=109) n (%)
O	O	O	174 (85.3)	81 (85.3)	93 (85.3)
M	O	O	14 (6.9)	5 (5.3)	9 (8.3)
O	M	M	7 (3.4)	3 (3.2)	4 (3.7)
O	M	O	4 (2.0)	2 (2.1)	2 (1.8)
O	O	M	4 (2.0)	4 (4.2)	0 (0.0)
M	M	O	1 (0.5)	0 (0.0)	1 (0.9)

Note: O=observed, M=missing

Over 85% of participants wore the accelerometer throughout the study. The second most common compliance pattern occurred among 6.9% of participants who were non-compliant at baseline only. This non-compliance occurred at the beginning of the study and was due to challenges experienced by study staff when using the new technology. In terms of dropout, only 7 of the 204 participants (3.4%) wore the accelerometer at baseline only.

B Descriptive Figures

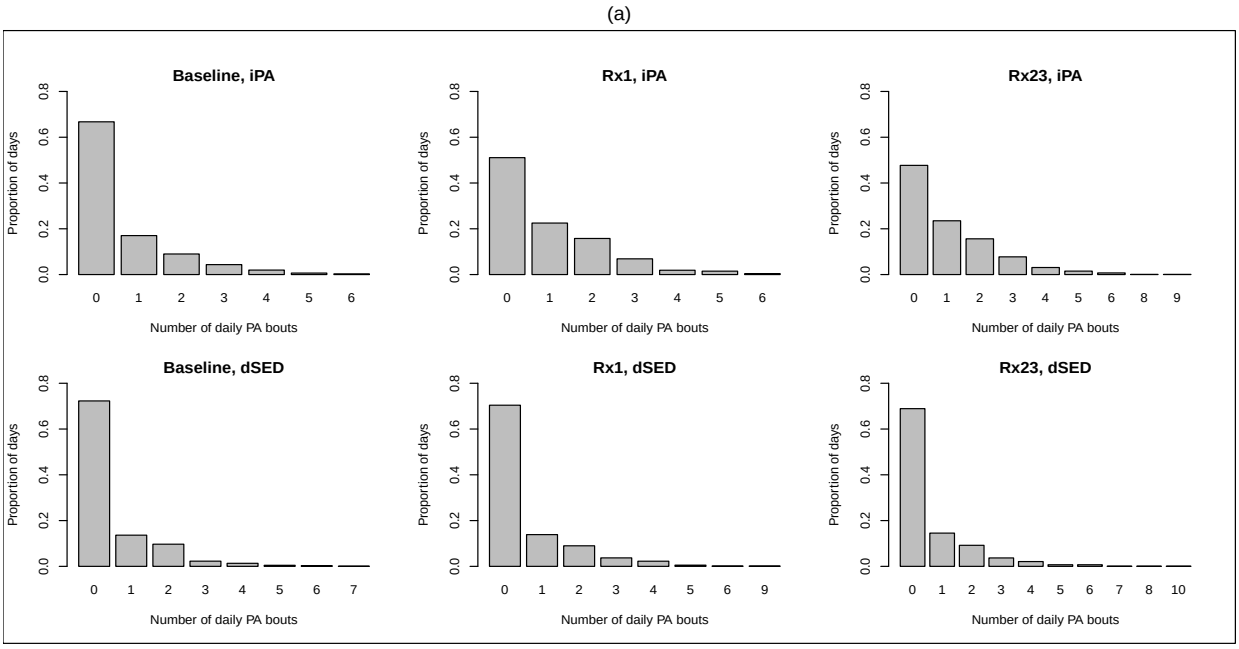

Figure 1: Barplots of number of daily exercise bouts by study phase (Baseline, Rx1, and Rx23) and treatment group in the MBC study. The top row are participants assigned to the increase physical activity condition (iPA). The bottom row are participants assigned to the decrease sedentary behavior (dSED) condition.

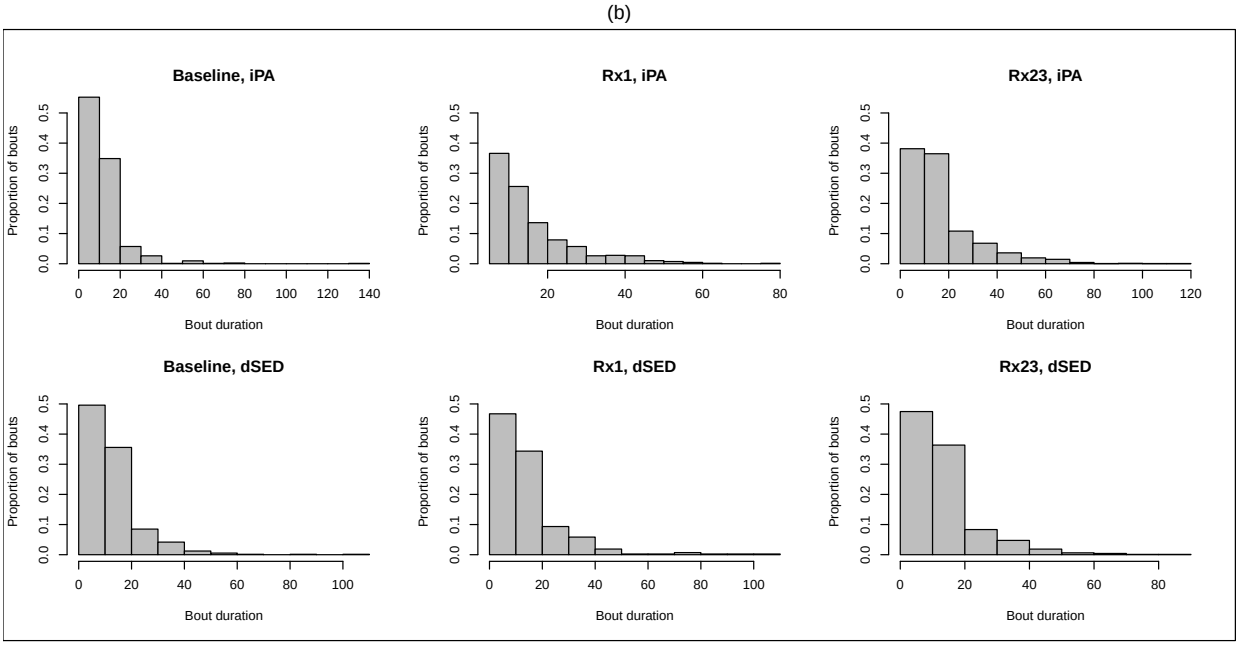

Figure 2: Histograms of bout duration by study phase (Baseline, Rx1, and Rx23) and treatment group in the MBC study. The top row are participants assigned to the increase physical activity condition (iPA). The bottom row are participants assigned to the decrease sedentary behavior (dSED) condition.

C Daily minutes of physical activity

In Section 2.4 of the manuscript, we describe how to calculate average daily minutes of physical activity across all study days. If the goal is to calculate total PA minutes on *exercise* days (i.e days with more than zero bouts), we replace Equation (9) in the manuscript with the mean of a truncated Poisson distribution, that is:

$$E(n_{ij} \mid n_{ij} > 0, \lambda_{ij}) = \frac{\lambda_{ij}}{1 - \exp(-\lambda_{ij})}, \tag{C.1}$$

so that the mean minutes of PA on exercise days is

$$E(n_{ij} \times \bar{y}_{ij} \mid \lambda_{ij}, \mu_{ij}) = \frac{\lambda_{ij}\mu_{ij}}{1 - \exp(-\lambda_{ij})}. \tag{C.2}$$

Plugging in (3), (4) and (6) from the manuscript into (C.2) we obtain

$$E(n_{ij} \times y_{ijk} \mid \mathbf{b}_i) = \frac{\exp(\mathbf{x}_{2ij}^T\boldsymbol{\beta}_2 + \mathbf{z}_{2ij}^T\mathbf{b}_{2i} + \mathbf{x}_{3ij}^T\boldsymbol{\beta}_3 + \mathbf{z}_{3ij}^T\mathbf{b}_{3i})}{\left[1 - \exp(-\exp(\mathbf{x}_{2ij}^T\boldsymbol{\beta}_2 + \mathbf{z}_{2ij}^T\mathbf{b}_{2i}))\right]}. \tag{C.3}$$

The constant 7.9 is added to (C.2) and (C.3) in order to put estimates back on the original duration scale. To obtain estimates of mean daily PA at the population level we integrate out the random effects in (C.3) and (11) in the manuscript in order to calculate marginal total duration by treatment group and time.

Integration of the random effects is performed at each MCMC iteration by averaging over 10 million draws of the random effects $\mathbf{b}_i \sim N(0, \Sigma^{(t)})$ where $\Sigma^{(t)}$ is the posterior draw of the random effects variance covariance matrix at MCMC iteration (t). The posterior distribution is obtained using 1000 MCMC iterations.

D JAGS Code

```
joint <- "
model{
C <- 10000
for (i in 1:n.bout) {
y[i] <- z.bout[i]
z[i]<-step(y[i]-1)  # d=I(y>0)

logit(p[i]) <- B[id.bout[i],1] + B[id.bout[i],5]*rx1.bout[i] + alpha1*rx1.bout[i]
            +  B[id.bout[i],9]*rx23.bout[i] + alpha2*rx23.bout[i]
            + alpha3*rx1.bout[i]*pa.bout[i]  + alpha4*rx23.bout[i]*pa.bout[i]

log(lambda[i]) <- B[id.bout[i],2] + B[id.bout[i],6]*rx1.bout[i] + beta1*rx1.bout[i]
               + B[id.bout[i],10]*rx23.bout[i] +beta2*rx23.bout[i]
               + beta3*rx1.bout[i]*pa.bout[i]  + beta4*rx23.bout[i]*pa.bout[i]

# Poisson hurdle log Likelihood
ll[i] <- (1-z[i])*log(1-p[i]) + z[i]*(log(p[i])+y[i]*log(lambda[i]) - lambda[i]
       - loggam(y[i]+1)-log(1-exp(-lambda[i])))

# zeros trick
zeros[i]~dpois(phi[i])
phi[i]<-  - ll[i] + C

# generate replicates for pp checking
any.new[i] ~ dbern(p[i])
numbout.new[i] ~ dpois(lambda[i])T(1,)
new.bouts[i] <- any.new[i] * numbout.new[i]
  } # end bout loop

# Duration sub-model
# gamma regression, log link
# mean parameterization, rate=shape/mu
for (i in 1:n.dur) {
y.hat[i] <- exp(B[id.dur[i], 3] + B[id.dur[i], 7]*rx1.dur[i] gam1*rx1.dur[i] +
         + B[id.dur[i], 11]*rx23.dur[i] + gam2*rx23.dur[i]
         + gam3*rx1.dur[i]*pa.dur[i]  + gam4*rx23.dur[i]*pa.dur[i])

# model the shape parameter
shape[i] <- exp(B[id.dur[i], 4] + B[id.dur[i], 8]*rx1.dur[i] + gs1*rx1.dur[i]
          + B[id.dur[i], 12]*rx23.dur[i] + gs2*rx23.dur[i]
```

```
          + gs3*rx1.dur[i]*pa.dur[i] + gs4*rx23.dur[i]*pa.dur[i])

y.dur[i] ~ dgamma(shape[i], shape[i]/y.hat[i])

# duration log likelihood
log_lik_dur[i]  <- logdensity.gamma(y.dur[i], shape[i], shape[i]/y.hat[i])

# generate replicates
y.new[i] ~ dgamma(shape[i], shape[i]/y.hat[i])
 } # end duration loop

# Log likelihoods
log_lik_poisson_sum <- sum(ll[])
log_lik_gamma_sum   <- sum(log_lik_dur[])
total_log_lik       <- sum(log_lik_poisson_sum, log_lik_gamma_sum)
mydev               <- -2 * total_log_lik

# Priors for regression coefficients
alpha1 ~ dnorm (0.0, .0001)
alpha2 ~ dnorm (0.0, .0001)
alpha3 ~ dnorm (0.0, .0001)
alpha4 ~ dnorm (0.0, .0001)
beta1 ~ dnorm (0.0, .0001)
beta2 ~ dnorm (0.0, .0001)
beta3 ~ dnorm (0.0, .0001)
beta4 ~ dnorm (0.0, .0001)
gam1 ~ dnorm (0.0, .0001)
gam2 ~ dnorm (0.0, .0001)
gam3 ~ dnorm (0.0, .0001)
gam4 ~ dnorm (0.0, .0001)
gs1 ~ dnorm (0.0, .0001)
gs2 ~ dnorm (0.0, .0001)
gs3 ~ dnorm (0.0, .0001)
gs4 ~ dnorm (0.0, .0001)

# Random effects
for (j in 1:J){
 B[j,1:K] ~ dmnorm (mu[], Tau.B[,])
}

# Priors for random effects
# random intercepts have non-zero mean
for (k in 1:4){
```

```
  mu[k] <- mu.raw[k]
  mu.raw[k] ~ dnorm (0, .0001)
}

# random time does have zero mean
# because we include their fixed effects in the model
for (k in 5:12){
  mu[k] <- 0
  mu.raw[k] ~ dnorm (0, .0001)
}

# Prior on random effects precision matrix
Tau.B[1:K,1:K] ~ dwish(W[,], df)
df <- K+1
Sigma.B[1:K,1:K] <- inverse(Tau.B[,])

# Calculate Random effect correlations and SDs
for (k in 1:K){
 for (k.prime in 1:K){
  rho.B[k,k.prime] <- Sigma.B[k,k.prime]/
   sqrt(Sigma.B[k,k]*Sigma.B[k.prime,k.prime])
  }
 sigma.B[k] <- sqrt(Sigma.B[k,k])
}
} #end model loop
"

# Function for starting values
inits <- function() {
  list(beta1=rnorm(1), beta2=rnorm(1),  beta3=rnorm(1), beta4=rnorm(1),
       alpha1=rnorm(1), alpha2=rnorm(1),  alpha3=rnorm(1), alpha4=rnorm(1),
       gam1=rnorm(1), gam2=rnorm(1),  gam3=rnorm(1), gam4=rnorm(1),
       gs1=rnorm(1), gs2=rnorm(1),  gs3=rnorm(1), gs4=rnorm(1),
       B=array(rnorm(J*K), c(J,K)), mu.raw=rnorm(K),
       Tau.B=rwish(K+1,diag(K)))

# Parameters to save
params <- c("alpha1", "alpha2", "alpha3", "alpha4", "beta1", "beta2", "beta3",
            "beta4", "gam1", "gam2", "gam3", "gam4", "mu", "Sigma.B", "rho.B")

fit <- run.jags(model=joint, data=jags.data, inits=inits, monitor=params,
               n.chains=5, method="parallel", adapt=5000, thin=20, sample=20000)
```

E Table of WAIC values

Table 2: Log pointwise predictive density (lppd), model complexity penalty (p_W), and widely applicable information criterion (WAIC) for four models with increasing numbers of random effects. The model with three random effects for duration shape has the lowest WAIC. Note that in models where no random effects were included for shape, the shape parameter was not modeled and was assumed constant across treatment condition and time.

Mean Random Effects	Shape Random Effects	Covar Params	lppd	p_W	WAIC (Δ^*)
Int.	None	6	-21190.8	285.0	42951.6 (—)
Int., Rx1, Rx23	None	45	-20878.6	490.8	42739.0 (212.6)
Int., Rx1, Rx23	Int.	55	-20764.9	553.4	42636.6 (102.4)
Int., Rx1, Rx23	Int., Rx1, Rx23	78	-20710.0	600.7	42621.2 (15.4)

*Change from previous model

F Calculating variance in the mixed-effects location scale gamma model

The mean-parameterized gamma distribution with mean μ and shape α has variance equal to μ^2/α. Therefore, the variance of PA duration is a function of the parameters in both (6) and (7) of the manuscript.

From (6) in the manuscript, the final model for the duration mean applied to the MBC study is:

$$\begin{aligned}\log(\mu_{ij} \mid \mathbf{b}_i) = &\beta_{30} + \beta_{31}\texttt{Rx1}_{ij} + \beta_{32}\texttt{Rx23}_{ij} + \beta_{33}\texttt{Rx1}_{ij} * \texttt{PA}_i + \beta_{34}\texttt{Rx23}_{ij} * \texttt{PA}_i \\ &+ b_{30i} + b_{31i}\texttt{Rx1}_{ij} + b_{32i}\texttt{Rx23}_{ij}, \end{aligned} \tag{F.1}$$

and the model for shape from Equation (7) in the manuscript is:

$$\begin{aligned}\log(\alpha_{ij} \mid \mathbf{b}_i) = &\beta_{40} + \beta_{41}\texttt{Rx1}_{ij} + \beta_{42}\texttt{Rx23}_{ij} + \beta_{43}\texttt{Rx1}_{ij} * \texttt{PA}_i + \beta_{44}\texttt{Rx23}_{ij} * \texttt{PA}_i \\ &+ b_{40i} + b_{41i}\texttt{Rx1}_{ij} + b_{42i}\texttt{Rx23}_{ij}. \end{aligned} \tag{F.2}$$

Then the subject-specific variance is:

$$\begin{aligned}\mathrm{Var}(y_{ijk} \mid \mathbf{b}_i)) = &\exp\big\{2(\beta_{30} + \beta_{31}\texttt{Rx1}_{ij} + \beta_{32}\texttt{Rx23}_{ij} + \beta_{33}\texttt{Rx1}_{ij} * \texttt{PA}_i + \beta_{34}\texttt{Rx23}_{ij} * \texttt{PA}_i \\ &+ b_{30i} + b_{31i}\texttt{Rx1}_{ij} + b_{32i}\texttt{Rx23}_{ij}) \\ &- (\beta_{40} + \beta_{41}\texttt{Rx1}_{ij} + \beta_{42}\texttt{Rx23}_{ij} + \beta_{43}\texttt{Rx1}_{ij} * \texttt{PA}_i + \beta_{44}\texttt{Rx23}_{ij} * \texttt{PA}_i \\ &+ b_{40i} + b_{41i}\texttt{Rx1}_{ij} + b_{42i}\texttt{Rx23}_{ij})\big\}. \end{aligned} \tag{F.3}$$

To obtain the marginal variance by time and treatment condition as well as the relative increase in variance from baseline to Rx1 between the iPA and dSED groups we integrate

Table 3: Marginal bout duration standard deviation by treatment condition and time.

Group/Time	Mean	Lower 95% CI	Upper 95% CI
Baseline	9.87	7.96	12.54
Increase PA, Rx1	15.30	12.06	20.24
Decrease SED, Rx1	10.70	8.42	13.85
PA/SED Ratio, Rx1	1.44	1.10	1.84
Increase PA, Rx23	18.34	14.67	22.96
Decrease SED Rx23	9.74	7.85	12.15
PA/SED Ratio, Rx23	1.89	1.50	2.34

out the random effects in (F.3). The marginal duration variance can be written as

$$\begin{aligned} \text{Var}(Y_i) &= \text{E}\{\text{Var}(Y_i|\mathbf{b}_i)\} + \text{Var}\{\text{E}(Y_i|\mathbf{b}_i)\} \\ &= \text{E}\{\text{Var}(Y_i|\mathbf{b}_i)\} + \text{E}\left[\{\text{E}(Y_i|\mathbf{b}_i)\}^2\right] - \left[\text{E}\{\text{E}(Y_i|b_i)\}\right]^2 \end{aligned} \tag{F.4}$$

where the outer expectation in each of the terms in (F.4) is with respect to the random effects distribution. To calculate these expectations, we use the same approach as in Section C where, at each MCMC iteration we average over 10 million draws from the random effects $\mathbf{b}_i \sim N(0, \Sigma^{(t)})$ where $\Sigma^{(t)}$ is the posterior draw of the random effects variance covariance matrix at MCMC iteration (t). As in Section C, we used 1000 MCMC iterations to obtain the posterior distribution.

Results are reported in Table 3 in terms of standard deviations. While duration variability stays the same in the dSED condition, variability increases in the iPA condition such that at Rx23, iPA variability is almost twice that of dSED variability.

G Model diagnostics

In addition to the posterior predictive checks reported in the manuscript, we performed additional tests of goodness of fit. Specifically, for both number of daily bouts and their duration, we generated replicates of these values from the posterior predictive distribution of our model and compared these values to the observed data using QQ plots. Figure 3 is a QQ plot of posterior predictive replicates of the daily bout values versus the observed values. Plotted are quantiles from 10 sets of replicates corresponding to 10 sets of parameter draws from our model. The distribution of the replicated data is similar to that of the observed data as indicated by the fact that values (with the exception of the maximum values) tend to fall on the 45° line (indicated in red). It is also worth noting that due to the highly skewed nature of the bout data, the 90th, 95th, and 99th percentiles of the observed data are 2, 3, and 5 bouts/day, respectively.

Figure 4 is a QQ plot of posterior predictive replicates of the duration values versus the observed values (minus 7.9 minutes). Plotted are quantiles from 10 sets of replicates corresponding to 10 sets of parameter draws from our model. The distribution of the replicated data is similar to that of the observed data as indicated by the fact that values (with the exception of the maximum values) tend to fall on the 45° line (indicated in red). As with the bout data, it is worth noting that due to the highly skewed nature of the duration data, the 90th, 95th, and 99th percentiles of the observed data are 21, 31, and 54 minutes, respectively.

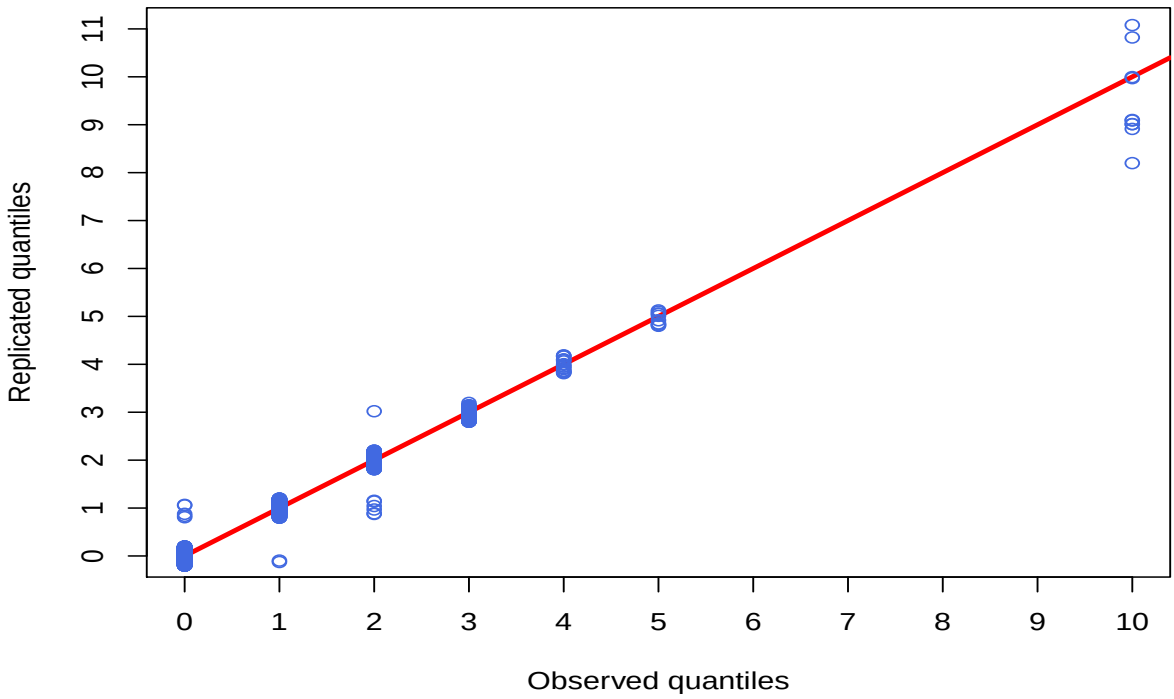

Figure 3: QQ plots of observed daily bout values versus quantiles from 10 sets of replicated values drawn from the posterior predictive distribution of our model. The distribution of the replicated data is similar to that of the observed data as indicated by the fact that values tend to fall on the 45° line (indicated in red). Due to the highly skewed nature of the bout data, the 90th, 95th, and 99th percentiles of the observed data are 2, 3, and 5 bouts/day, respectively.

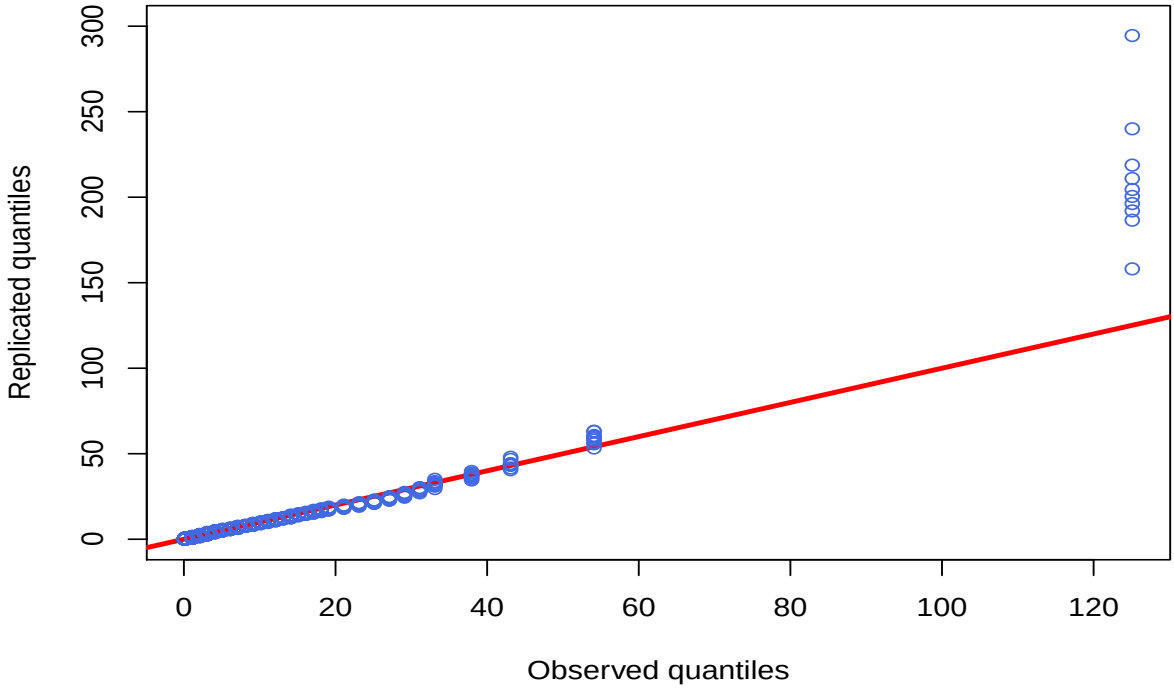

Figure 4: QQ plots of observed duration values (minus 7.9 minutes) versus 10 sets of replicated values drawn from the posterior predictive distribution of our model. The distribution of the replicated data is similar to that of the observed data as indicated by the fact that values tend to fall on the 45° line (indicated in red). Due to the highly skewed nature of the duration data, the 90th, 95th, and 99th percentiles of the observed data are 21, 31, and 54 minutes, respectively.

H Random effects variance-covariance matrix

Table 4: Lower diagonal of the random effect correlation matrix from the final model with 12 random effects. Values reported in the table are posterior means. Bolded values had 95% credible intervals that did not include 0. The prefix on the row and column names indicate the random effects timepoint. The suffix indicates the submodel: (1) Poisson hurdle; (2) Poisson truncated; (3) gamma mean; (4) gamma shape.

	Int_1	Int_2	Int_3	Int_4	Rx1_1	Rx1_2	Rx1_3	Rx1_4	Rx23_1	Rx23_2	Rx23_3
Int_2	**0.70**										
Int_3	-0.01	0.06									
Int_4	0.23	0.12	-0.23								
Rx1_1	0.00	0.05	0.3	0.05							
Rx1_2	0.03	-0.02	-0.08	0.16	0.27						
Rx1_3	0.08	-0.04	-0.22	-0.01	0.25	0.08					
Rx1_4	-0.3	-0.23	**0.45**	**-0.33**	0.08	-0.19	-0.2				
Rx23_1	-0.07	-0.02	0.24	0.01	**0.80**	0.3	0.37	0.05			
Rx23_2	0.07	-0.05	-0.06	0.18	**0.46**	**0.45**	0.24	-0.21	**0.55**		
Rx23_3	-0.01	-0.13	**-0.29**	-0.04	0.18	0.22	**0.62**	-0.22	0.35	0.28	
Rx23_4	-0.03	0.08	**0.44**	**-0.39**	**0.39**	-0.02	0.06	**0.35**	**0.37**	0.04	-0.03

I Separate models for bouts and duration

Table 5: Posterior estimates from the bouts only mixed-effects Poisson model with random effects for intercept, Rx1, and Rx23.

Variable	Mean	Lower 95% CI	Upper 95% CI
Logistic regression on probability of exercise day			
Intercept	-1.000	-1.191	-0.813
Rx1	-0.061	-0.339	0.216
Rx23	-0.020	-0.278	0.247
PA*Rx1	1.071	0.687	1.445
PA*Rx23	1.014	0.669	1.388
Loglinear regression on mean number of bouts			
Intercept	-0.060	-0.205	0.079
Rx1	-0.024	-0.272	0.221
Rx23	-0.028	-0.236	0.177
PA*Rx1	0.091	-0.148	0.328
PA*Rx23	0.161	-0.031	0.364
Random effect variances			
Sigma.B[1,1]	1.335	0.955	1.751
Sigma.B[2,2]	0.270	0.156	0.395
Sigma.B[3,3]	0.647	0.289	1.028
Sigma.B[4,4]	0.145	0.060	0.247
Sigma.B[5,5]	0.931	0.560	1.325
Sigma.B[6,6]	0.170	0.079	0.275

Table 6: Posterior estimates from the duration only mixed-effects Gamma location-scale model with random effects for intercept, Rx1, and Rx23.

Variable	Mean	Lower 95% CI	Upper 95% CI
Loglinear regression on duration mean			
Intercept	1.542	1.421	1.665
Rx1	0.136	-0.061	0.325
Rx23	-0.007	-0.183	0.163
PA*Rx1	0.394	0.157	0.628
PA*Rx23	0.625	0.417	0.843
Loglinear regression on duration shape			
Intercept	-0.389	-0.472	-0.303
Rx1	-0.027	-0.207	0.147
Rx23	-0.038	-0.172	0.091
PA*Rx1	0.145	-0.066	0.349
PA*Rx23	-0.004	-0.145	0.142
Random effect variances			
Sigma.B[7,7]	0.409	0.275	0.556
Sigma.B[8,8]	0.098	0.052	0.149
Sigma.B[9,9]	0.205	0.090	0.335
Sigma.B[10,10]	0.151	0.069	0.243
Sigma.B[11,11]	0.269	0.136	0.414
Sigma.B[12,12]	0.106	0.052	0.166

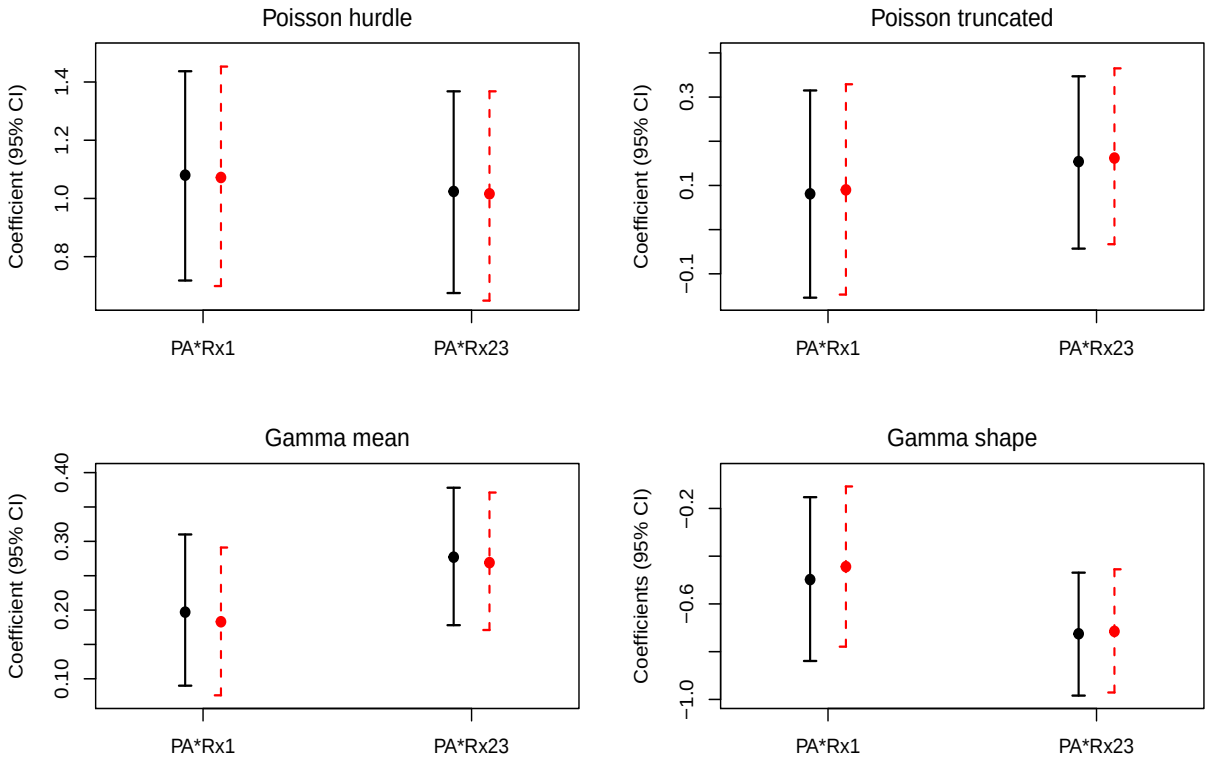

Figure 5: Parameter estimates and their 95% credible intervals for treatment effects from the four sub-models. Plotted are coefficients for the time by treatment interaction terms. The solid black lines are from a joint model of PA bouts and their duration. The red dotted lines are from a bouts-only model (top row) or a duration-only model (bottom row).

J Physical activity behavioral phenotypes

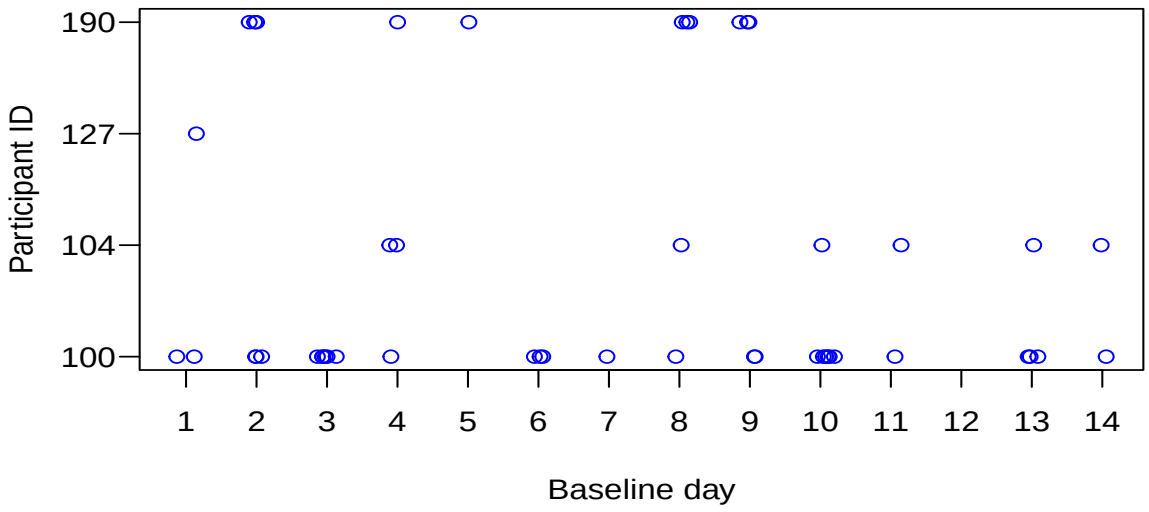

Figure 6: Strip plots of bouts per day during the two weeks of baseline from four individuals with varying non-zero bout days and bout frequencies. Participants were identified by looking at their random intercept terms in the Poisson hurdle sub-model. Participant 190 in the top row has only 5 non-zero bout days, but engages in multiple bouts on those days. Their probability of exercising on a given day was 0.40 and their mean number of bouts on exercise days was 1.24. Participant 127 has only a single non-zero bout day and engages in only one bout on that day. Their probability of exercising on a given day was 0.05 and their mean number of bouts on exercise days was 0.68. Participant 104 exercises about as frequently as Participant 190 (exercise probability=0.36), but only engages in a single bout on those days (mean number of bouts=0.83). Participant 100 is characterized by both a high frequency of non-zero bout days (exercise probability=0.72) and—like participant 190—multiple bouts per day (mean number of bouts=1.39).

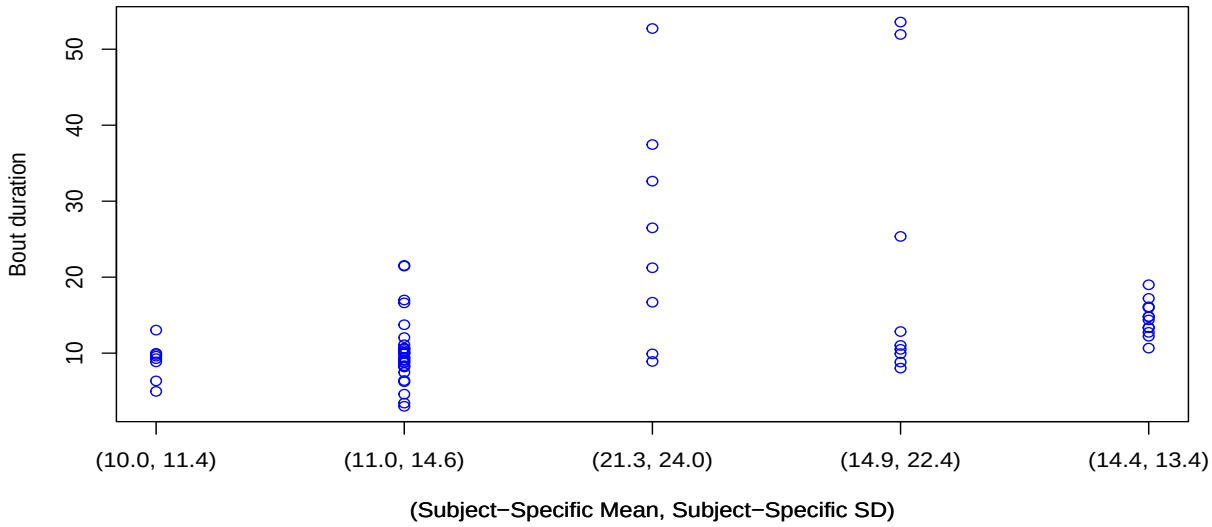

Figure 7: Strip plots of duration data at baseline from five individuals with different duration means and variances. The values on the x-axis are subject-specific means and standard deviations that were calculated using the random intercept terms in the duration mean model and the duration shape model. The first two participants have low duration means but the second participant has greater variability. The third and fourth participants have similar duration variances but the fourth participant has a lower duration mean. The last participant has a similar mean as the fourth participant but a lower variance. A model that treated the shape parameter as constant would not be able to distinguish between participants with similar mean duration values but different variances.

References

[1] Trost SG, Mciver KL, Pate RR. Conducting accelerometer-based activity assessments in field-based research. *Medicine and Science in Sports and Exercise* 2005; 37(11): S531.

[2] Migueles JH, Cadenas-Sanchez C, Ekelund U, et al. Accelerometer data collection and processing criteria to assess physical activity and other outcomes: a systematic review and practical considerations. *Sports Medicine* 2017; 47: 1821–1845.