



Investigating Risky Choices Over Losses Using Experimental Data

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Abstract

We conduct a battery of experiments in which agents make choices from several pairs of all-loss-lotteries. Using these choices, we estimate a representation of individual preferences over lotteries. We find statistically and economically significant departures from expected utility maximization for many subjects. We also estimate a preference representation based on summary statistics for behavior in the population of subjects, and again find departures from expected utility maximization. Our results suggest that public policies based on an expected utility approach could significantly underestimate preferences and willingness to pay for risk reduction.

Keywords: risky decision-making, loss domain, experiments

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1. Introduction

The common economic approach for addressing public hazards typically compares the expected costs and benefits of various policies, an approach which implicitly assumes people maximize expected utility (Viscusi, Magat, and Huber, 1991; Chichilnisky and Heal, 1993). However, a number of earlier studies, mainly based on experimental evidence, suggest that people make decisions inconsistent with expected utility theory.¹ While these

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¹The classic example is the Allais paradox, in which people frequently violate the independence axiom by making choices inconsistent with the notion of parallel linear indifference curves. Other examples of the Allais Paradox include the “common consequence effect,” “the certainty effect,” and the “Bergen Paradox.” Examples of papers that discuss earlier experimental results include Lichtenstein et al. (1978), Tversky and Kahneman (1979), Machina (1987), Baron (1992), Thaler (1992). Recall the expected utility model says that if a person’s preferences satisfy three axioms—ordering, continuity, and independence—we can model her behavior as if she is maximizing expected utility; see Marschak (1950), Machina (1982), Starmer (2000). See Machina (1982, 1987) or Thaler (1992) for a more in depth treatment of the literature involving independence violations.

earlier experimental results are intriguing and have shaped the literature, they are based on lotteries over gains. As there are important risks that entail lotteries over potential losses (i.e., homeland security issues, environmental problems), it is unclear one can directly translate results about risky gains to policies over risky losses.

Understanding how people react toward low-probability/high-loss risks remains under-researched.² The extant literature contains evidence showing that people seem to treat losses differently from equivalent gains (see, e.g., Tversky and Kahneman, 1981; Kahneman, Knetsch, and Thaler, 1990; Thaler, 1992; Camerer, 1995; Neilson and Stowe, 2002). Based on this evidence, one might speculate that more or different types of violations of the expected utility paradigm might emerge in an experimental design that focuses on potential losses.

In this paper, we report on the results from an experiment that is designed to identify choice behavior over lotteries based on potential losses. The paper starts off with a discussion of the details of our experimental design, in Section 2. Subjects choose between two lotteries, for a set of 40 pair-wise comparisons. After they complete their list of choices, one of the comparisons is selected at random, and the lottery they chose from that comparison is then played for real money. The data from subjects' choices is then analyzed to determine whether and to what degree people violate expected utility theory for all-loss-lotteries, and what this might imply for public policy.

We present two sets of results. The first set, which are reported in Section 3, are based on an analysis of the individual subjects' preferences. Here, we estimate parameters for individual preferences over losses under expected and non-expected utility specifications.³ We find departures in behavior from the expected utility paradigm, which are both statistically and economically important, for many subjects. Their behavior indicates the probability of the worst event enters into preferences in a non-linear fashion—indifference curves over lotteries are concave.

In Section 4, we discuss results at the population level. We use a mixed Logit model, which summarizes behavior for the entire sample of subjects by providing estimates that can be viewed as those of the "average" subject (McFadden and Train, 2000; Revelt and Train, 1998; Train, 1998, 1999). Since policy decisions are ultimately made for and based on the representative preferences of a group of agents, we believe this approach is an appropriate tool for policy decisions. Again our results suggest that important non-linearities emerge. These non-linearities, however, are not uniform across the probability space. For lotteries in which the best outcome is very likely and the two worst outcomes are extremely unlikely, expected utility organizes behavior for the subject pool reasonably well—indifference curves were nearly linear for such combinations. Yet for lotteries in which both the probabilities

²For a recent theoretical treatment see Kunreuther and Pauly (2004).

³Examples of econometric estimates of indifference curves under risk at the individual level can be found in Camerer (1989), Harless (1992), Harless and Camerer (1994), Hey (1995), Hey and Orme (1994), Hey and Carbone (1995). Also see the overviews by Camerer (1995) and Starmer (2000), and the citations therein. The general conclusion is that neither expected utility theory nor the non-expected utility alternatives best organize all observed behavior at the individual level. These papers do not address the aggregation-for-policy issues we explore.

of both the best and worst outcomes are relatively small, expected utility theory performs poorly. In this range, indifference curves are highly non-linear.

Such non-linearities imply a policy approach based solely on expected benefits and costs of the representative agent could significantly underestimate the population's real willingness to pay to reduce environmental risk within this range. We discuss these implications in Section 5, working out a numerical example based on the non-linear function we estimated in Section 4. We find that the implication of incorrectly assuming expected-utility maximizing behavior in this context could be to underestimate the true willingness-to-pay for a reduction in the probability of the worst event by nearly an order of magnitude. Concluding remarks are offered in Section 6.

2. Experimental design

In contrast to earlier work, our experiment confronts subjects with choices between a pair of risky choices, or “lotteries,” over potential losses. Our results allow us to infer subjects' preferences regarding risky outcomes that include the potential, with small probability, for a relatively large loss to occur.⁴ Assume three consequences can arise defined by three states of nature.⁵ Let y_1 , y_2 , and y_3 represent the monetary magnitudes of the events, where $y_1 < y_2 < y_3$. Let p_i reflect the probability that outcome y_i will be realized, for $i = 1, 2$, or 3 . The lottery \mathbf{p} is the vector of probabilities (p_1, p_2, p_3) . The expected utility hypothesis surmises that there is an increasing function $u(\bullet)$ over wealth, the von Neumann—Morgenstern utility function, such that the person prefers lottery \mathbf{p} to lottery \mathbf{q} if and only if $V(\mathbf{p}) > V(\mathbf{q})$, where

$$V(\mathbf{p}) = \sum_{i=1}^3 u(y_i)p_i. \quad (1)$$

The function $V(\bullet)$ is called the expected utility representation. Since the three probabilities sum to one, equation (1) can be simplified to

$$V(\mathbf{p}) = [u(y_1) - u(y_2)]p_1 + [u(y_3) - u(y_2)]p_3 + u(y_2). \quad (2)$$

⁴The observation that a lottery involves an outcome of “large consequence” does not tell us about the expected value of the lottery *per se*, nor does it imply that the difference between the expected values of two lotteries would be particularly large. It is the loss associated with an event, and not the expected loss, that is large. This interpretation of large-stakes events is in keeping with the traditional approach to modeling risky decision-making (Hirschleifer and Riley, 1992). One could argue that these events correspond to taking home \$20, \$70, or \$100 – and so represent gains instead of losses. We have two reactions. First, it is infeasible to design an experiment in which subjects are exposed to losses without initially endowing them so as to cover any potential losses, as subjects can not be expected to participate in an experiment where they anticipate giving the experimenter money at the end of the session. Second, a framing effect issue arises here. What matters is that subjects believe they are being exposed to losses. Anecdotal evidence, based on subjects' remarks after the end of the session, confirms that they felt that they really had been endowed with \$100 and they were being exposed to losses.

⁵The use of three states is largely motivated by Machina's (1982, 1987) writings. There is some debate as to whether one needs three states to elucidate risk preferences in welfare analysis. Freeman (1991, 1993) argues that a two-state model will suffice, while Shogren and Crocker (1991, 1999) argue that with more than two states of nature, preferences for risk disappear only under the restrictive presumption that public risk reduction actions are a perfect substitute for private risk reduction strategies.

The values $u(y_i)$ are constants, once the magnitudes of the outcomes are specified. Correspondingly, the representation $V(\bullet)$ is linear in the probabilities. Since $y_1 < y_2 < y_3$ and $u(\bullet)$ is increasing in y , the coefficient on p_1 is negative, while the coefficient on p_3 is positive. Recall the slopes of indifference curves are found by implicitly differentiating (2) to get

$$0 = dV = [u(y_1) - u(y_2)]dp_1 + [u(y_3) - u(y_2)]dp_3 \quad (3)$$

$$\Leftrightarrow dp_3/dp_1 = -[u(y_1) - u(y_2)]/[u(y_3) - u(y_2)].$$

We present each subject in the experiment with 40 pairs of lotteries, which we called “options.” Lotteries are defined as follows. Let x_1 , x_2 , and x_3 represent the magnitudes of the three consequences or losses, where $x_1 > x_2 > x_3$. The first possible outcome entails the largest loss, while the third outcome entails the smallest loss. With an initial endowment of y_0 , these events induce wealth levels $y_i = y_0 - x_i$, $i = 1, 2$, or 3. In our experiment, $x_1 = \$80$, $x_2 = \$30$, and $x_3 = \$0$; y_0 is the sum of the subject’s pre-existing wealth and \$100. Let p_i reflect the probability that outcome x_i will be realized, for $i = 1, 2$, or 3, and denote the vector of probabilities by $\mathbf{p} = (p_1, p_2, p_3)$. A person’s preference ordering over lotteries implies a representation, $V(\mathbf{p})$, with this being linear under expected utility.

We build the set of lotteries around three reference lotteries, which we selected to reflect specific risk scenarios. In lottery A, the ‘less bad’ outcome obtains with a small probability. This describes a situation in which both the worst outcome and the less bad outcome are not very likely to occur. In lottery B, the less bad outcome is more likely than the other events, but still is not highly probable. This corresponds to a situation with a substantial chance of medium size losses. In lottery C, losses are quite likely, but they are overwhelmingly more likely to be modest than large. These different scenarios are suggestive of different types of potential losses. For example, while oil spills are not rare, when they occur the damages are usually not enormous (as in lottery B). By contrast, one might argue that while large or enormous damages from global climate change are fairly possible, modest damages are a more likely outcome (as in lottery C).

Figure 1 illustrates our method for selecting lotteries. The three probabilities for lottery A in this example are $p_1 = .05$, $p_2 = .35$ and $p_3 = .6$. The three probabilities for B are $p_1 = .05$, $p_2 = .55$, and $p_3 = .4$. And, the three probabilities for C are $p_1 = .05$, $p_2 = .75$, and $p_3 = .2$. Notice that in each of these lotteries, the probability of the worst event (lose \$80) is quite small. Each of these reference lotteries was compared to twelve other points; four where p_1 was reduced to .01, four where p_1 was increased to .1, and four where p_1 was increased to .2. The decrease in p_1 from .05 to .01 was combined with a decrease in p_3 . Conversely, the increase in p_1 from .05 to either .1 or .2 was combined with an increase in p_3 . The decreases (and increases) in p_3 followed a specific path. For example, the four points where p_1 was increased from .05 to .1 are labeled as points B1 (.1,.49,.41), B2 (.1,.45,.45), B3 (.1,.4,.5), and B4 (.1,.3,.6) (the figure is not drawn to exact scale).

The experiment followed a five-stage procedure.

Stage #1: Starting the Experiment: We recruited subjects from classes at the University of Wyoming and from the city of Laramie. This allows us to gauge the influence of education level upon observed behavior. Subjects are asked to report to a specified room at a

Stage #3: The Tan Pitcher: After filling out the option sheet, all subjects wait until the monitor calls him or her to the front of the room. When called, the subject brings the waiver form, survey, and option sheet. There is a tan pitcher containing 40 chips on the front table, numbered from 1 to 40. The numbers on the chips correspond to the 40 options on the option sheet. The subject reaches into the tan pitcher without looking at the chips, and picks out a chip. The number on the chip determines which option he or she will play to determine his or her take-home earnings. For example, if he draws chip #23, he plays the option he circled for the pair #23 on his option sheet.

Stage #4: The Blue Pitcher: After the option to be played has been determined, the subject then draws a different chip from a blue pitcher. The blue pitcher has 100 chips, numbered 1 to 100. The number on the chip determines the actual outcome of the option—a loss of either \$80, \$30, or \$0. For example, suppose the option to be played has

$$p_1 = 10\%, p_2 = 50\%, p_3 = 40\%.$$

If the chip drawn by the subject is numbered between 1 and 10, event 1 obtains, so that the subject loses \$80; if he picks a chip between 11 and 60, he loses \$30; or if he picks a chip between 61 and 100, he loses \$0. If instead, the option to be played has

$$p_1 = 20\%, p_2 = 20\%, p_3 = 60\%$$

and the subject draws a chip numbered between 1 and 20, he loses \$80; if he draws a chip between 21 and 40, he loses \$30; if he draws a chip between 41 and 100, he loses \$0.

Stage #5: Concluding the experiment: After playing the option, each subject completes a tax form. After the monitor receives the tax form and the survey form, the subject is paid his or her take-home earnings in cash. The subject then leaves the room. All told, 53 subjects participated in our experiments, with the typical subject earning between \$70 and \$75.

3. Econometric results I: Behavior at the individual level

We begin our discussion of the econometric results by examining individual behavior. The preference ordering for a certain agent k is represented by a function $V_k(\mathbf{p})$, where \mathbf{p} is a probability distribution that places probability p_i on event $i = 1, 2$, and 3 . Agent k prefers lottery \mathbf{p} to lottery \mathbf{q} if $V_k(\mathbf{p}) > V_k(\mathbf{q})$. Allowing for decision errors, we regard this choice as probabilistic (Loomes, Moffat and Sugden, 2002): agent k chooses lottery \mathbf{p} over lottery \mathbf{q} if $V_k(\mathbf{p}) - V_k(\mathbf{q}) + \varepsilon > 0$, where ε reflects decision errors. Accordingly, the probability that agent k chooses lottery \mathbf{p} over lottery \mathbf{q} is

$$PR[\varepsilon > V(\mathbf{q}) - V(\mathbf{p})], \quad (4)$$

where ‘ $PR(E)$ ’ means “the probability that event E occurs”. Once a distribution for ε is specified and a parametric form for V_k is chosen, estimation of the parameters in V_k follows straightforward maximum likelihood techniques (Fomby, Hill, and Johnson, 1988).

Since we are interested in identifying the importance of non-linear effects, a natural approach to take is to specify V_k as a quadratic function.⁶ This may be regarded as a second-order Taylor's series approximation to a more general non-linear form. We parameterize the quadratic as:

$$V(\mathbf{p}) = \alpha + \beta_1 p_1 + \beta_2 p_3 + \beta_3 p_1^2 + \beta_4 p_1 p_3 + \beta_5 p_3^2. \quad (5)$$

Let $Y_1 = q_1 - p_1$, $Y_2 = q_3 - p_3$, $Y_3 = q_1^2 - p_1^2$, $Y_4 = q_1 q_3 - p_1 p_3$, and $Y_5 = q_3^2 - p_3^2$. Then, the agent selects option \mathbf{q} over option \mathbf{p} if

$$\varepsilon > \beta_1 Y_1 + \beta_2 Y_2 + \beta_3 Y_3 + \beta_4 Y_4 + \beta_5 Y_5. \quad (6)$$

Because the quadratic form may include approximation errors, the residual need not have zero mean. Correspondingly, we include a constant term in the regressions.

Before proceeding to a discussion of the econometric results, we briefly discuss the special case in which V is linear in the probabilities. In this case, indifference curves correspond to iso-expected utility curves. The slope of these curves is

$$dp_3/dp_1 = -\beta_2/\beta_1. \quad (7)$$

Recalling equation (3), we see that the coefficients β_1 and β_2 may be interpreted as differences in von Neumann—Morgenstern utilities at the various wealth levels: $\beta_1 = -[u(y_2) - u(y_1)]$ and $\beta_2 = u(y_3) - u(y_2)$. We therefore expect $\beta_1 < 0 < \beta_2$. If the agent is risk-neutral, these differences are proportional to the differences in wealth. In our design, $y_1 = y_0 - 80$, $y_2 = y_0 - 30$, and $y_3 = y_0$. We define the statistic

$$R = 5\beta_2 + 3\beta_1. \quad (8)$$

For a risk neutral agent, $R = 0$. If the agent is risk-averse ($u(\bullet)$ is concave), then $R < 0$. Alternatively, $R > 0$ represents a risk-seeker. For an expected-utility maximizer, we can obtain information concerning the agent's risk attitudes from a test of the hypothesis that $R = 0$. Since the parameters may reflect risk attitudes, we anticipate differences across agents, and we therefore perform separate regressions for each subject.

Empirical results are based on a Probit regression model; qualitatively similar results emerge from Logit regressions.⁷ We report parameter estimates and standard errors (shown

⁶Chew, Epstein, and Segal (1991) originally proposed the quadratic utility approach. They replace the independence axiom with the weaker mixture symmetry axiom that allows for indifference curves to be non-linear such that predicted behavior matches up reasonable well with observed behavior. The obvious advantage of this approach is parsimony; a disadvantage is that there is likely to be a fair bit of collinearity amongst the regressors. As our primary goal is to establish the combined statistical importance of the non-linear terms, we believe this is a relatively minor concern.

⁷The reader may wonder if there is any serial correlation in the disturbances. As subjects filled out the entire sheet prior to submitting it, and since there was no opportunity for feedback, our view is that the error structure is atemporal. As such, the issue of serial correlation in the error structure is moot.

in parentheses, below the parameter estimate to which it corresponds), for each of the 46 individuals for whom the Probit regression converged.⁸ Our primary goal is to determine whether agents' behavior is satisfactorily described by the expected utility hypothesis. This need not mean that agents purposefully act so as to maximize the weighted average of some utility function over wealth; rather, it means that the pattern of choices they exhibit cannot be statistically separated from those an expected utility maximizing agent would make. That is, one cannot reject the hypothesis that $V_k(\bullet)$ is linear. Because each subject made only 40 choices, using the functional form in equation (5) would yield a rather low level of degrees of freedom. Accordingly, the regressions we report below are based on a simplified version of equation (5), which contains only one non-linear term: $\beta_4 p_1 p_3$.⁹ The null hypothesis that subject behavior was consistent with the expected utility hypothesis then corresponds to the restriction $\beta_4 = 0$; it also requires that the intercept be zero.¹⁰

Table 1 contains the parameter estimates for the linear model, along with standard errors, in columns 2 and 3. The log-likelihood statistic for the linear model is presented in the sixth column, and the corresponding statistic for a corresponding quadratic model is in the seventh column (these are the columns $\ln L_1$ and $\ln L_2$). We also report the test statistic for the linear restriction on the parameters (in the column labeled χ_N^2). The main result we observe is the significance of this statistic for a substantial proportion of our subjects – half of the 46 subjects—at the 10% level.¹¹ This indicates a statistically important divergence from the expected utility model for many of our subjects.

Table 1 also includes estimated values of the statistic R (from equation (8)), along with the test statistic for the hypothesis that $R = 0$ (presented in the column labeled χ_R^2). For eight subjects (identified in the tables as subjects 4, 6, 13, 15, 28, 37, 41, and 43), the estimated coefficients β_1 and β_2 had the same sign. Such a representation would imply that the subject either regarded increased values of the probability of the worst event, or decreased probability of the best event, with favor. Accordingly, we do not compute R for these agents. Of the remaining 38 subjects, 12 had significantly positive values of R and

⁸Of the 53 subjects, seven (subjects 5, 9, 10, 11, 30, 50, and 52) made choices that did not vary sufficiently to allow our regressions to converge, so no estimates are listed for these individuals.

⁹Results from estimations based on the complete quadratic specification in equation (5) are available upon request. One qualification to our approach is the high level of collinearity in the exogenous variables. The right-hand side variables we use in this set of regressions are the difference in p_1 ; difference in p_3 ; difference in p_1^2 ; difference in $p_1 p_3$; and the difference in p_3^2 . One way to measure collinearity is by inspecting the variance inflation factor (VIF) for each regressor. For a particular regressor, one first regresses all other variables on the one of interest (i.e., difference in p_1 is related to the difference in p_3 ; difference in p_1^2 ; difference in $p_1 p_3$; and the difference in p_3^2). The VIF is then computed from the R^2 value in that regression as $VIF = 1/(1 - R^2)$; values larger than 10 are indicative of collinearity. In our application, there are five separate regressors to be analyzed; in every case, the VIF we obtain is greater than 20, suggesting that there is a high degree of collinearity amongst the variables. Perhaps because of the limited observations, we do not typically observe significance of more than one non-linear effect. The quadratic effect from the odds of the worst event or the interaction between probabilities of best and worst outcomes seems to be the more important effects.

¹⁰Recall we interpret the intercept as the result of approximation error. If the linear form is correct, there is no approximation and no role for the intercept to play.

¹¹Since there are four restrictions in this hypothesis, the test statistic (twice the difference in the value of the log-likelihood functions with and without the parameter restriction) would be distributed as a chi-squared variate with 4 degrees of freedom. The critical points are 7.78, 9.49, and 11.1 at the 10, 5, and 1% levels.

Table 1. Regression results, linear utility model.

Subject	β_1	β_2	R	χ^2_R	LnL_1	LnL_2	χ^2_N
1	-5.292 -2.958	5.362 -2.224	10.934	1.51	-23.458	-8.742	29.432**
2	-10.084 3.386	6.286 2.484	1.181	0.02	-20.794	-17.584	6.42
3	-1.818 3.408	11.157 4.199	50.332	9.53**	-16.957	-14.179	5.556
4	4.354 3.778	10.306 4.537	n.a.	n.a.	-13.198	-10.476	5.444
6	-14.973 4.488	-0.770 1.572	n.a.	n.a.	-15.075	-9.255	11.64**
7	-6.256 2.893	1.486 1.450	-11.337	1.86	-25.104	-20.745	8.718 ⁺
8	-6.879 3.290	7.931 2.819	19.017	3.38 ⁺	-21.034	-20.678	0.712
12	-7.171 3.054	5.150 2.175	4.238	0.23	-23.025	-21.528	2.994
13	-10.420 4.435	-4.619 2.538	n.a.	n.a.	-12.674	-8.207	8.934 ⁺
14	-13.676 4.438	9.672 3.249	7.333	0.51	-18.025	-14.537	6.976
15	-11.780 3.860	-0.760 1.541	n.a.	n.a.	-18.056	-13.652	8.808 ⁺
16	-0.486 2.608	4.250 1.733	19.791	5.59*	-23.471	-18.512	9.918*
17	-1.100 2.719	3.945 1.966	16.427	3.6 ⁺	-24.719	-23.182	3.074
18	-11.608 4.492	16.945 5.484	49.898	1.78	-15.038	-14.441	1.194
19	-15.350 5.073	12.386 4.122	15.881	5.21*	-16.636	-11.713	9.846*
20	-11.255 3.507	2.344 1.598	-22.046	0.17	-20.799	-19.669	2.26
21	-6.740 2.924	3.356 1.626	-3.441	2.99 ⁺	-24.132	-22.272	3.72
22	-39.225 15.256	29.471 11.257	29.681	0.49	-10.013	-5.409	9.208 ⁺
23	-8.708 3.016	4.010 1.945	-6.073	4.60*	-22.505	-14.761	15.488**

(Continued on next page.)

Table 1. (Continued.)

Subject	β_1	β_2	R	χ_R^2	LnL_1	LnL_2	χ_N^2
24	-9.848 3.291	2.117 1.574	-18.958	0.07	-22.040	-16.561	10.958*
25	-13.740 4.409	8.783 3.136	2.697	3.97*	-18.337	-10.851	14.972**
26	-99.570 48.493	24.364 11.823	-176.891	3.49 ⁺	-4.480	-1.910	5.14
27	-2.663 2.668	4.772 1.775	15.870	2.96 ⁺	-23.336	-20.501	5.67
28	2.694 2.585	1.129 1.474	n.a.	n.a.	-26.188	-24.535	3.306
29	-13.897 4.896	14.349 4.571	30.056	4.9*	-16.066	-11.310	9.512*
31	-0.858 2.516	0.473 1.431	-0.211	0.01	-27.649	-21.984	11.330**
32	-17.446 5.440	9.562 3.269	-4.524	0.18	-16.416	-10.640	11.552**
33	-33.308 12.589	24.009 9.304	20.122	1.58	-11.063	-8.133	5.86
34	-7.751 3.670	12.967 3.690	41.585	9.34**	-16.081	-0.0003	32.162**
35	-21.691 6.416	5.540 2.721	-37.373	8.06**	-14.121	-0.0004	28.242**
36	-15.880 4.581	0.021 1.580	-47.533	13.83**	-15.194	-5.557	19.274**
37	1.044 2.513	0.069 1.439	n.a.	n.a.	-27.602	-25.805	3.594
38	-21.066 6.959	13.438 4.781	3.992	0.11	-14.721	-8.847	11.748**
39	-5.057 3.247	8.729 3.189	28.477	5.74*	-20.520	-5.632	29.776**
40	-85.641 46.618	16.785 11.120	-173.001	3.97*	-4.304	-1.910	4.788
41	3.762 2.708	0.645 1.470	n.a.	n.a.	-25.921	-22.574	6.694
42	-17.833 5.443	11.474 4.020	3.870	0.12	-15.809	-12.529	6.56
43	6.055 3.368	5.111 2.389	n.a.	n.a.	-16.557	-11.532	10.05*

(Continued on next page.)

Table 1. (Continued.)

Subject	β_1	β_2	R	χ_R^2	LnL ₁	LnL ₂	χ_N^2
44	-23.493 7.530	11.151 3.897	-14.722	1.16	-14.051	-9.914	8.274 ⁺
45	-20.974 6.595	8.062 3.298	-22.613	3.46 ⁺	-15.050	-11.287	7.526
46	-5.371 2.720	2.656 1.571	-2.831	0.13	-25.280	-21.795	6.97
47	-7.602 4.988	28.380 10.319	119.095	8.51 ^{**}	-10.547	-9.051	2.992
48	-34.419 11.661	8.496 3.881	-60.777	7.59 [*]	-11.552	-6.530	10.044 [*]
49	-21.163 6.534	6.868 3.084	-29.146	5.41 [*]	-14.755	-5.686	18.138 ^{**}
51	-9.547 3.793	10.528 3.533	24.001	4.10 [*]	-19.007	-17.838	2.338
53	-5.976 2.778	0.611 1.472	-14.876	3.18 ⁺	-24.994	-24.000	1.988

⁺Significant at 10% level or better.

^{*}Significant at 5% level or better.

^{**}Significant at 1% level or better.

10 had significantly negative values of R (at the 10% level), which are consistent with risk-loving or risk-averse behavior. The estimate for R did not differ significantly from zero for the remaining 16 agents, consistent with risk-neutrality. Broadly speaking, these results are inconsistent with a view that agents are typically risk-loving with respect to losses. In addition, the hypothesis of linearity in the representation was rejected for 6 of the 12 apparent risk-lovers. This result suggests the potential for non-expected utility maximizing behavior to be confused with risk-loving behavior.

In Table 2 we also compute the critical value of p_1 where $\partial V/\partial p_3 = 0$ (presented in the column labeled \tilde{p}_1) and the critical value of p_3 where $\partial V/\partial p_1 = 0$ (the column labeled \tilde{p}_3). When $\beta_4 < 0$, indifference curves are convex when $p_1 < \tilde{p}_1$ and $p_3 > \tilde{p}_3$. Likewise, when $\beta_4 > 0$, indifference curves are concave when $p_1 > \tilde{p}_1$ and $p_3 < \tilde{p}_3$. We summarize this information in the final column, labeled “characteristic,” for those subjects whose choices indicated a rejection of expected utility. For such subjects, the characteristic is “NEU” (non-expected utility), along with the appropriate curvature statement. For some NEU subjects, the curvature is valid over the entire range of probabilities, or over the range used in the experiment ($0 \leq p_1 \leq .2; 0 \leq p_3 \leq .8$). For others, the restrictions on either p_1 or p_3 impinge on a large range of the probabilities used in the experiment. For such individuals, we conclude that choices are inconsistent with EU, but also imply downward sloping indifference curves over a substantial range of the probabilities used

Table 2. Regression results, simplified non-linear utility model.

Subject	β_0	β_1	β_2	β_4	lnL	\bar{p}_1	\bar{p}_3	Characteristic
1	2.765	23.217	13.557	-28.616 ⁺	-10.200	0.4738	0.8113	NEU
	1.062	10.64	5.957	17.062				convex ^{a,b}
2	-0.071	-15.746	5.598	11.046	-20.342	-0.5068	1.4255	EU, RN
	0.278	7.051	2.738	13.376				
3	0.338	13.42	14.471	-29.495 ⁺	-14.386	0.4906	0.4550	EU, RL
	0.350	7.86	5.249	15.977				
4	-0.246	-0.078	11.489	5.999	-12.846	-1.9152	0.0130	EU, fails
	0.345	9.084	5.465	19.148				dominance
6	-0.023	-19.814	-1.888	12.426	-14.756	0.1519	1.5946	NEU, fails
	0.311	7.694	2.487	16.067				dominance
7	-0.170	-5.691	2.333	-4.365	-24.866	0.5345	-1.3038	NEU,
	0.263	5.928	2.062	11.761				convex ^a
8	-0.043	-7.779	7.899	1.269	-21.013	-6.2246	6.1300	EU, RL
	0.281	6.915	3.074	13.515				
12	-0.239	-4.411	6.621	-10.530	-22.425	0.6288	-0.4189	EU, RN
	0.274	6.211	2.598	12.644				
13	0.117	-32.517	-9.942	43.51*	-10.047	0.2285	0.7474	NEU, fails
	0.381	11.955	4.394	21.41				dominance
14	0.199	-10.174	9.919	-4.737	-17.675	2.0939	-2.1478	EU, RN
	0.290	8.046	3.722	15.536				
15	-0.083	-2.654	1.250	-24.342	-17.130	0.0514	-0.1090	NEU, fails
	0.295	8.116	2.444	19.492				dominance
16	0.346	-2.772	2.896	10.866	-22.525	-0.2665	0.2551	NEU
	0.287	6.580	2.046	13.059				concave ^d
17	-0.264	3.912	6.060	-15.856	-23.632	0.3822	0.2467	EU, RL
	0.273	6.016	2.501	12.267				
18	-0.289	-11.804	18.845	-5.593	-14.648	3.3694	-2.1105	EU, RN
	0.340	8.313	6.459	16.505				
19	-0.216	-57.038	13.901	66.726*	-11.785	-0.2083	0.8548	NEU
	0.339	19.305	5.065	29.865				concave ^d
20	0.333	-18.037	0.471	18.709	-19.674	-0.0252	0.9641	EU, RN
	0.278	9.364	2.096	16.698				
21	-0.274	-12.609	3.461	7.753	-23.243	-0.4464	1.6263	EU, RA
	0.272	6.854	2.131	13.259				
22	0.461	-51.414	29.846	23.596	-9.000	-1.2649	2.1789	NEU
	0.394	21.651	12.464	25.400				concave

(Continued on next page.)

Table 2. (Continued.)

Subject	β_0	β_1	β_2	β_4	lnL	\bar{p}_1	\bar{p}_3	characteristic
23	-0.054	-14.622	3.354	11.42	-22.038	-0.2938	1.2809	NEU
	0.275	7.095	2.297	13.284				concave
24	0.288	-46.781	-0.979	67.295**	-16.637	0.0145	0.6952	NEU
	0.313	16.615	2.299	26.571				concave ^{c,d}
25	1.217	-43.191	7.940	60.003*	-10.932	-0.1323	0.7198	NEU
	0.393	19.952	4.269	30.666				concave
26	-0.355	-290.890	53.290	103.87	-3.040	-0.5130	2.8005	EU, RA
	10.410	2079.0	467.00	92.660				
27	-0.236	-6.630	5.160	4.447	-22.832	-1.1603	1.4909	EU, RL
	0.280	6.678	2.098	12.856				
28	-0.085	-2.422	0.600	9.892	-25.712	-0.0607	0.2448	EU, fails
	0.265	5.940	1.946	11.964				dominance
29	-0.342	-49.783	17.048	57.775 ⁺	-12.220	-0.2951	0.8617	NEU
	0.346	19.364	6.145	29.918				concave ^d
31	-0.966	-7.939	3.130	-1.907	-22.132	1.6413	-4.1631	NEU
	0.319	5.860	2.086	11.273				convex
32	-0.105	-64.810	10.821	71.81*	-11.832	-0.1507	0.9025	NEU
	0.342	23.362	4.069	33.309				concave ^d
33	0.600	-45.378	25.466	21.628	-9.535	-1.1775	2.0981	EU, RN
	0.382	19.871	11.255	24.397				
34	0.070	-17.376	12.376	20.936	-15.320	-0.5911	0.8300	NEU
	0.320	9.779	4.094	18.093				concave ^d
35	0.071	-109.031	8.379	114.816**	-6.912	-0.0730	0.9496	NEU
	0.475	41.237	5.332	47.104				concave ^d
36	-0.242	-1.100	5.320	-55.155 ⁺	-12.76	0.0965	-0.0199	NEU
	0.350	9.782	3.558	31.581				convex ^a
37	-0.095	6.891	1.635	-14.827	-26.684	0.1103	0.4648	EU, fails
	0.266	5.505	1.916	11.040				dominance
38	-1.145	-38.406	20.609	1.807	-10.213	-11.4051	21.2540	NEU
	0.467	17.672	7.312	21.509				concave
39	-0.704	-22.45	10.700	24.224	-16.341	-0.4417	0.9267	NEU
	0.343	9.572	4.000	18.151				concave ^d
40	-0.150	-267.80	42.300	103.86	-3.040	-0.4073	2.5785	EU, RA
	10.040	2003.0	451.00	92.660				
41	-0.053	4.004	0.910	-1.490	-27.274	0.6107	2.6872	EU, fails
	0.263	5.894	2.004	11.768				dominance

(Continued on next page.)

Table 2. (Continued.)

Subject	β_0	β_1	β_2	β_4	lnL	\bar{p}_1	\bar{p}_3	characteristic
42	0.669	-34.353	10.657	33.156	-13.151	-0.3214	1.0361	EU, RN
	0.337	15.813	4.537	22.676				
43	0.366	13.417	4.957	-9.59	-15.592	0.5170	1.3994	NEU convex ^a
	0.326	8.338	2.774	15.991				
44	0.324	-22.346	10.477	1.882	-13.427	-5.5670	11.8735	NEU concave
	0.306	14.716	4.001	24.316				
45	0.235	-62.145	7.964	63.081*	-11.992	-0.1263	0.9852	EU, RA
	0.347	22.416	3.892	29.965				
46	-0.052	-11.909	1.919	12.928	-24.609	-0.1484	0.9212	EU, RN
	0.268	6.501	1.962	12.408				
47	0.111	-0.022	30.959	-16.959	-10.135	1.8255	-0.0013	EU, RL
	0.424	9.600	11.578	19.850				
48	0.302	-88.210	8.090	87.459*	-8.603	-0.0925	1.0086	NEU concave
	0.433	33.486	4.732	41.479				
49	0.202	-143.98	11.812	162.89**	-5.727	-0.0725	0.8839	NEU concave ^d
	0.505	56.740	6.515	70.380				
51	0.054	-20.635	9.502	22.679	-18.053	-0.4190	0.9099	EU, RL
	0.290	10.155	3.740	17.675				
53	-0.289	-3.692	2.345	-11.066	-24.175	0.2119	-0.3336	EU, RA
	0.268	5.816	2.097	12.597				

Notes: [†] Significant at 10% level; * Significant at 5% level; ** Significant at 1% level.

^a: if $p_1 < \bar{p}_1$; ^b: if $p_3 > \bar{p}_3$;

^c: if $p_1 > \bar{p}_1$; ^d: if $p_3 < \bar{p}_3$.

in the experiment. We characterize these subjects as “NEU, fails dominance.” Similarly, we characterize those agents whose choices fail to reject linear indifference curves, but for whom the parameter estimates imply downward sloping indifference curves, as “EU, fails dominance.” The remaining subjects’ choices are consistent with the expected utility model. These subjects are identified as “EU;” we also indicate the apparent risk attitude, on the basis of the test of risk neutrality reported in Table 2. Subjects are labeled as “RA” (risk averse), “RN” (risk neutral), or “RL” (risk loving).

Table 3 summarizes the characteristics of the population of individuals based on the regressions. We see people’s risk preference range across predictable patterns. Nineteen individuals are characterized as expected utility maximizers, given the three classical definitions of risk attitudes. Twenty four people are non-expected utility maximizers, with either convex or concave indifference curves. Seven people failed the dominance tests. As we noted above, seven subjects made choices that did not vary sufficiently to allow estimation of their preferences. The most notable feature is the overall importance of concave, non-linear indifference curves (15 people). Such curves are consistent with “fanning in” in

Table 3. Characterization of individual subjects.

	Characterization	Number of subjects	Subject IDs
Expected utility	Risk-neutral	8	2, 12, 14, 18, 20, 33, 42, 46
	Risk-averse	5	21, 26, 40, 45, 53
	Risk-loving	6	3, 8, 17, 27, 47, 51
Non-expected utility	convex ICs over relevant range	3	7, 31, 43
	limited convex ICs	2	1, 36
	Concave ICs over relevant range	13	19, 22, 23, 25, 29, 32, 34, 35, 38, 39, 44, 48, 49
	limited concave ICs	2	16, 24
	EU, failed dominance	4	4, 28, 37, 41
	NEU, failed dominance	3	6, 13, 15
	Choices did not vary enough to allow estimation	7	5, 9, 10, 11, 30, 50, 52

the relevant range for our experiment (though they might be interpreted as “fanning out” in the region where p_1 is large and p_3 small). Concave indifference curves support Starmer’s (2000) caution that the less restrictive “betweenness” axiom still does not connect theory with behavior.¹² Our evidence supports the idea that a more descriptive theory would include mixed fanning with nonlinear indifference curves such as quadratic utility or models with decision weights.

We conclude this section by discussing those characteristics that might explain whether a subject’s behavior was consistent with expected utility maximization or not. We first created an indicator variable that equaled 1 for the 23 subjects whose behavior was consistent with expected utility maximization, and that equaled 0 for the other subjects. This latter group includes the seven individuals whose choices displayed insufficient variation to allow estimation. Because it was impossible to determine whether these individuals’ behavior was consistent with expected utility maximization, we also considered a version in which we left their observations out of the regression. For both sets, we estimated two Logit models of the indicator variable. In the first, we included five explanatory variables: *statistics* (which equaled 1 if the subject had taken a course in statistics or probability, and 0 otherwise); *gender* (which equaled 1 for males and 0 for females); *income* (the subject’s personal income, in thousands of dollars); *age*; and *high school* (which equaled 1 if the subject’s education did not proceed beyond high school). In the second regression, the last two variables were dropped.¹³

¹² Recall the betweenness axiom is a weaker form of the independence axiom. Betweenness says that preferences are such that any probability mixture of two lotteries will be ranked between the two. For further discussion, see Starmer (2000), Camerer and Ho (1994).

¹³All regressions excluded information from subject 18, who neglected to report her age.

Table 4. Logit analysis of expected utility characterization.

Explanatory variable	Model 1	Model 2	Model 3	Model 4
Statistics	1.5719* (0.7305)	1.5506* (0.7154)	1.1560 (0.7509)	1.2126+ (0.7259)
Gender	1.3765+ (0.7140)	1.4475* (0.7063)	1.3525+ (0.7386)	1.4551* (0.7290)
Income	-0.0677+ (0.0408)	-0.0616 (0.0394)	-0.0704+ (0.0437)	-0.0621 (0.0410)
Age	-0.0336 (0.0657)	-	0.0348 (0.0669)	-
High school	-0.1641 (0.7036)	-	-0.4213 (0.7340)	-
Constant	-1.8600 (1.6935)	-1.1491+ (0.6544)	-1.3757 (1.7310)	-0.8074 (0.6834)
Log-likelihood statistic	-28.324	-28.474	-25.833	-26.106
Pseudo- R^2	.1556	.1511	.1470	.1380
N	52	52	45	45

+ Significant at 10% level or better.

* Significant at 5% level or better.

Table 4 reports the results from these Logit regressions, which lists parameter estimates and standard errors (in parentheses), for each of the two lists of explanatory variables, and for both sets of observations. While the statistical strength of the various explanatory variables deviates slightly across the various regressions, we note three observations that appear consistently across specification. First, the variable *statistics* exerts a significant and positive effect in all regressions. The interpretation of this result is that those individuals who had been exposed to formal training in statistics were much more likely to display behavior consistent with expected utility maximization. Second, the variable *gender* exerts a significant and positive effect in all regressions. Apparently, male subjects were more likely to display behavior consistent with expected utility maximization, all else equal. The third result is that the variable *income* exerts a small and negative effect. This effect is of similar magnitude in all regressions, exerts a statistically significant effect (at the 10% level) in the two regressions with all variables, and just slightly fails to exert a statistically significant effect in the two other regressions. The interpretation is that those subjects with larger personal income levels were somewhat less likely to display behavior consistent with expected utility maximization. Statistical significance notwithstanding, the economic importance of this variable is apparently somewhat smaller than either *statistics* or *gender*.

We also investigated a multinomial Logit model, where we distinguished subjects on the basis of the classification in Table 3. For this analysis, we employed the same set of explanatory variables as in Table 4. The dependent variable used in this analysis equaled 0 for those subjects whose choices were inconsistent with expected utility maximization,

Table 5. Multinomial Logit analysis of risk posture.

Explanatory variable	Non EU vs. RN/RL	Non EU vs. RA	Non EU vs. RN/RL	Non EU vs. RA
Statistics	1.1780 (0.7795)	3.3325* (1.6326)	1.3815+ (0.7703)	2.3843* (1.2205)
Gender	1.2787+ (0.7645)	0.8937 (1.4434)	1.3434+ (0.7632)	1.7569 (1.1330)
Income	-0.0385 (0.0406)	-0.2631 (0.1780)	-0.0448 (0.0358)	-0.2188+ (0.1181)
Age	-0.0304 (0.0406)	0.1502 (0.1073)	-	-
High school	-0.5549 (0.7962)	0.5420 (1.3687)	-	-
Constant	-0.5298 (2.1224)	-5.9071* (3.0067)	-1.4898* (0.6839)	-1.8409 (1.1812)
Log-likelihood statistic	-35.191		-37.515	
Pseudo-R ²	.2034		.1508	

1 for those subjects whose choices were consistent with risk-loving or risk-neutral behavior, and equaled 2 for those subjects whose choices were consistent with risk-averse behavior.¹⁴

The results from the multi-nomial Logit analyses, which are reported in Table 5, are somewhat similar to those of the Logit analyses discussed above. First, *statistics* exerts a numerically large and positive effect that is statistically important in most of the regressions. It is noteworthy that this effect is more important for the comparison between non-expected utility maximizing and risk-averting behavior. Evidently, formal training in statistics is most closely related to choices that are consistent with risk-averse behavior. Second, *gender* exerts a positive impact, though here it seems to only be important in distinguishing between non expected utility behavior and risk-loving or risk-neutral behavior. One interpretation of this finding is that males who exhibit behavior consistent with expected utility maximization are less likely to be risk-averters, perhaps suggesting a more daring outlook on life. The third observation is that while *income* appears to exert a negative effect in all four comparisons, it is only statistically important in the comparison between non-expected utility maximizing and risk-averting behavior. While there is some evidence that the subject’s income may have an influence on his or her behavior, the effect is neither numerically nor statistically large in the majority of cases.

¹⁴We also ran a similar set of regressions using a variable that distinguished between risk loving and risk neutral subjects. Those results were qualitatively similar to the ones we report, except that the coefficients on the various variables for risk-loving and risk-neutral subjects were quite similar. To increase the power of our analysis, we therefore elected to re-run the regressions, combining these two sets of subjects into one cohort.

4. Econometric results II: Behavior by the population

Table 3 shows that while the majority of individual subjects (34 of 53) revealed behavior inconsistent with expected utility maximization, several subjects' behavior was consistent with expected utility maximization (19 of 53). With such variation in behavior, it is not immediately clear that one should reject the expected utility paradigm when modeling public policy problems which require cost-benefit analysis. One should ask whether expected utility does a reasonable job in organizing the behavior of the typical subject within a population of subjects. To this end, we analyze subjects' choices using a mixed Logit model.

The mixed Logit approach identifies summary information for the entire sample of subjects based on the average agent and regards each individual's taste parameters (the coefficients in our regression) as drawn from a population (McFadden and Train, 2000; Revelt and Train, 1998; Train, 1998, 1999). Under the mixed Logit approach, the econometrician identifies the sample mean of the coefficient vector. This mean vector then provides the summary information for the cohort, which can be used to identify behavior of a typical subject.

One obvious advantage of the mixed Logit approach is that we can use the entire dataset in the estimation procedure. There is a dramatic increase in the number of available observations, and this increase permits an expansion of the list of explanatory variables without significantly reducing degrees of freedom.¹⁵ We implemented the mixed Logit approach by using a third order Taylor's series approximation over probabilities, yielding the representation.¹⁶

$$V(\mathbf{p}) = \alpha + \beta_1 p_1 + \beta_2 p_3 + \beta_3 p_1^2 + \beta_4 p_1 p_3 + \beta_5 p_3^2 + \beta_6 p_1^3 + \beta_7 p_1^2 p_3 + \beta_8 p_1 p_3^2 + \beta_9 p_3^3 \quad (9)$$

Based on this specification, the agent prefers lottery \mathbf{p} over lottery \mathbf{q} if

$$\varepsilon > \beta_1 Y_1 + \beta_2 Y_2 + \beta_3 Y_3 + \beta_4 Y_4 + \beta_5 Y_5 + \beta_6 Y_6 + \beta_7 Y_7 + \beta_8 Y_8 + \beta_9 Y_9, \quad (10)$$

where Y_1 through Y_5 are as above, and $Y_6 = q_1^3 - p_1^3$, $Y_7 = q_1^2 q_3 - p_1^2 p_3$, $Y_8 = q_1 q_3^2 - p_1 p_3^2$, and $Y_9 = q_3^3 - p_3^3$.

The vector $(\beta_1, \dots, \beta_9)$ summarizes each agent's tastes, which we regard as a draw from a multi-variate distribution; our empirical analysis assumes the parameter vector is multi-normally distributed. Once the distribution for this vector is specified, the joint likelihood

¹⁵While one could increase the number of observations at the individual level by replicating the experiment with additional binary comparisons, our view is that such an experiment runs a considerable risk that the subjects would become fatigued or careless or bored.

¹⁶In principle, one would like to allow for an individual's endowed income to play a role in this representation. Since the mixed Logit approach requires any explanatory variable used in the regression to vary over options for the particular subject, and since income does not, we were unable to explicitly incorporate subject's personal income in the regressions. We do not perceive this as a shortcoming, however, since one can always interpret variations of parameter vectors across subjects as partially induced by income differences. Since the individual values of these parameter vectors is not observed—only the mean value is estimated—were are also unable to explain a subject's parameter vector by demographic characteristics, say in the manner of the regressions reported in Tables 4 and 5.

function may be written down. This likelihood function depends on the first two sample moments of the distribution over the parameters, and the stipulated distribution over the error term (e.g., extreme value for the Logit application). Estimates of the mean parameter vector are then obtained through maximum likelihood estimation.

Unfortunately, exact maximum likelihood estimation is generally impossible (Revelt and Train, 1998; Train, 1998). The alternative is to numerically simulate the distribution over the parameters, use the simulated distribution to approximate the true likelihood function, and to then maximize the simulated likelihood function.¹⁷ As there seems to be no *ex ante* reason to adopt a particular distribution over subjects' parameter vectors, we ran three versions of the mixed Logit model that assume the parameter vectors are (i) normally distributed, (ii) uniformly distributed, and (iii) a triangular distribution.¹⁸

Table 6 presents the results. For each of the three conjectured population distributions over the parameter vector, there are two columns. The first column lists the estimated population mean value of the parameter (β_1 , β_2 , and so on); the standard errors associated with that estimate is listed in parentheses below the parameter estimate. The second column lists the estimated population variance for the corresponding parameter (β_1 , β_2 , and so on); again, the standard errors are listed below in parentheses. There are two main points we wish to make in the context of these results. First, for all three of the conjectured population distributions, each of the population mean parameter values for all the coefficients β_3 through β_9 is statistically important; recall from equation (9) that these are the parameters corresponding to the non-linear effects. (In fact, only the non-linear effects are statistically important, and every one of these parameters is significant at the 1% level). Second, we see the estimated parameter vectors are numerically similar across the three regression models. The estimated parameter values, and their statistical significance, appear to be robust to the assumed distribution governing individual subject's parameter values. Taken together, we believe these observations provide overwhelming evidence of the statistical importance of non-linear effects in the typical subject's choice behavior. We now investigate the economic importance of this finding.

5. Implications

While we observe statistically significant coefficients on all of the non-linear terms in the mixed Logit model, whether these results are economically important is open for debate.

¹⁷We thank Kenneth Train for supplying the GAUSS program used to conduct our estimation. The mixed Logit model we employ regards the parameter vector as individual-specific. In light of the discussion in footnote 7 we expect the vector for a particular individual to be constant across choices. We therefore used the version of Train's estimation program that allows for variation in the parameter vector across agents but not across choices made by a given agent.

¹⁸The first specification might be regarded as a relatively conservative approach, in that no limits are imposed on the range of coefficients. The other two approaches assume a finite support, and so do not allow arbitrarily large or small values. In these two approaches, the range is not imposed by the econometrician; rather, it is implicitly generated as part of the estimation process. The main distinction between uniform and triangular distributions has to do with the relative weights put on parameter values close to the mean. All three approaches assume a symmetric parameter distribution, which we believe is reasonable as there is no *ex ante* reason to anticipate an asymmetric distribution.

Table 6. Results of mixed Logit analysis.

Parameter	Normally distributed		Coefficients assumed to be:			
	1st moment	2nd moment	uniformly distributed		Triangularly distributed	
	1st moment	2nd moment	1st moment	2nd moment	1st moment	2nd moment
β_1	10.978 (7.3031)	0.54072 (0.7434)	10.918 (7.2913)	0.03975 (0.82789)	10.984 (7.2957)	0.19762 (0.81414)
β_2	2.6989 (1.8223)	-0.2299 (0.29303)	2.6740 (1.8175)	-0.0488 (0.25059)	2.6948 (1.8191)	-0.27488 (0.55449)
β_3	-34.858** (8.7609)	0.64114 (0.51502)	-34.719** (8.7448)	-0.8463 (0.71081)	-34.825** (8.7505)	-1.3388 (1.3118)
β_4	35.808** (17.406)	1.0682 (1.0246)	35.781** (17.282)	-0.2879 (3.9400)	35.680** (17.295)	0.91837 (4.1759)
β_5	-11.011** (4.8629)	0.09809 (0.16055)	-10.965** (4.8418)	-0.0405 (0.33308)	-10.997** (4.8493)	-0.1471 (0.51636)
β_6	10.365** (3.3322)	-0.069 (0.1018)	10.312** (3.3207)	0.37523 (0.41059)	10.360** (3.3252)	0.44440 (0.48292)
β_7	19.956** (6.2676)	-0.1696 (0.38507)	19.940** (6.2289)	-1.0780 (1.4014)	19.904** (6.2457)	-1.6637 (1.5165)
β_8	-102.65** (20.587)	9.6542** (1.756)	-101.70** (20.525)	15.677** (3.9245)	-101.92** (20.554)	22.187** (4.8727)
β_9	16.578** (4.7984)	-0.0566 (0.18875)	16.514** (4.7738)	-1.2912 (0.91016)	16.539** (4.7856)	-1.4370 (1.2461)
Log-likelihood statistic	-1355.34		-1356.22		-1356.07	

Standard errors in parentheses; *Significant at 5% level or better; **Significant at 1% level or better.

To this end, we used the estimated mean parameter vector from the run based on normally distributed parameter vectors, as listed in the second column in Table 6 to identify numerically probability combinations that yield the same value of the value function $V(\mathbf{p})$; similar results obtain for the other two estimated parameter vectors. These combinations are then used to plot level curves for a “typical” subject within the Marschak-Machina triangle, which we do in figure 2. The key feature of this diagram is the striking non-linearity in the level curves in the heart of the triangle.

Figure 2 shows that these non-linearities are not uniform across the probability space. We see that when the probability of the best outcome is very likely and the two worst outcomes are extremely unlikely, the typical subject’s indifference curves were relatively linear. Expected utility seems to organize average behavior within the population reasonably well in this range. But for lotteries where the best and worst outcomes are each relatively unlikely, expected utility theory performs poorly. In this range, indifference curves are highly non-linear; evidently, aggregating individuals into a representative agent creates

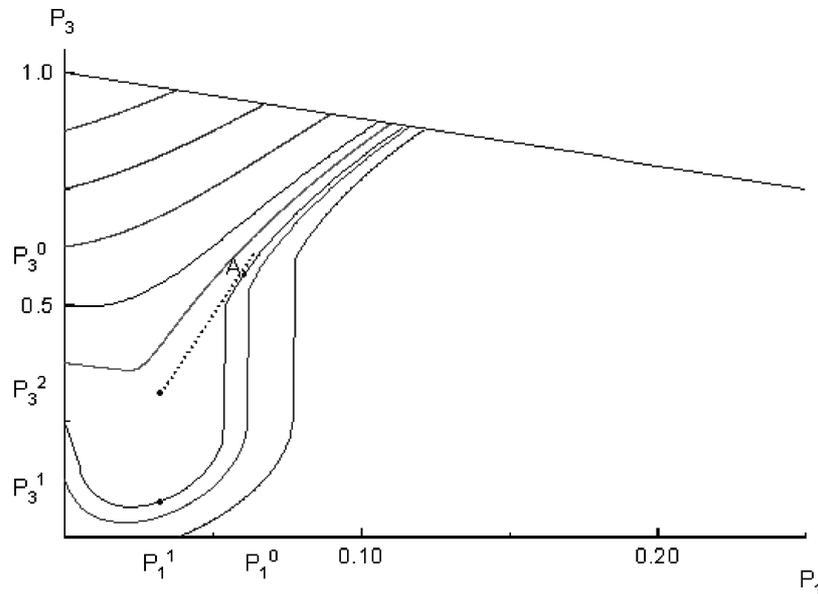


Figure 2. Level curves implied by cubic representation over lotteries.

indifferences curves that are risk-specific—they are neither linear nor non-linear throughout the probability space. For some risks, policymakers might not be that far off by following expected benefits estimates; for other risks, however, the policymaker could be far off the mark.

We illustrate the importance of these non-linearities from a policy perspective via the following thought experiment. Imagine the present situation implies a lottery such as the one marked *A* in figure 2. Consider now a policy that reduces the chance of the worst loss, event 1, from p_1^0 to p_1^1 . Within the Marschak-Machina triangle, a representative agent with level curves such as those we have plotted would be willing to accept a reduction from p_3^0 to p_3^1 of the probability that the good event (no loss) will occur.¹⁹ But a policy analyst who believed the representative agent to be (at least approximately) an expected utility maximizer would predict that the agent’s level curve was close to the tangent line at lottery *A*. The analyst would predict that the agent would only accept a reduction from p_3^0 to p_3^2 —an impressive underestimate of the representative agent’s willingness to pay (in terms of lower probability of no loss) to reduce the chance of the worst event. Such non-linearities imply that policies based on expected benefits could significantly underestimate willingness to pay to reduce risk.

We conclude our discussion of the mixed Logit results by investigating a functional form that allows us to infer willingness to pay for a specified change in a lottery faced by the

¹⁹This is akin to a risk-risk tradeoff (Viscusi, Magat, and Huber, 1991; Viscusi, 1992). Alternatively, one could determine the economic consequence of a reduction in p_1 by identifying the amount of cash an agent would pay to acquire the new lottery. We sketch out such an approach below.

average subject. This discussion is motivated by the following idea: suppose an agent's choices are consistent with the expected utility paradigm. Then we can use the data on his choices to estimate a linear representation over probabilities, and this linear form can be used to infer a Von Neuman—Morgenstern utility function over prizes. If the lotteries in question are defined over three prizes, as in our experiments, the inferred utility function is quadratic. This suggests an interpretation with non-linear representations over probabilities wherein the parameters on the various polynomial terms involving probabilities can be linked to some function of the associated prize. We can then use this link between parameters and prizes to estimate the representative agent's *ex ante* willingness to pay for a change in risk.²⁰

In our application, with a cubic representation over probabilities, there are 18 terms involving probabilities:

$$\begin{aligned} V(\mathbf{p}; \mathbf{y}) = & u_1 p_1 + u_2 p_2 + u_3 p_3 + u_4 p_1^2 + u_5 p_2^2 + u_6 p_3^2 + u_7 p_1 p_2 + u_8 p_1 p_3 \\ & + u_9 p_2 p_3 + u_{10} p_1^3 + u_{11} p_1^2 p_2 + u_{12} p_1^2 p_3 + u_{13} p_1 p_2^2 + u_{14} p_1 p_3^2 \\ & + u_{15} p_2^3 + u_{16} p_2^2 p_3 + u_{17} p_2^2 p_3 + u_{18} p_3^3, \end{aligned} \quad (11)$$

where the u_i 's are functions of the prizes y_i . We interpret prizes as the sum of monetary prize at the experiment summed with endowed income; as this application is based on the mean parameter vector for our subjects, we use average personal income for our subject pool in this calculation. Since the probabilities sum to one, we reduce this to a representation with nine parameters, as in equation (9). The resulting parameters (the β 's in equation (9)) are then tied to the original functions in a specific way. Next, we propose a functional relation between the parameters u_i in equation (11) and the associated prizes. The functional representation we propose is motivated by the observation that the highest-order function that can be employed with three prizes is quadratic, and by the constraint that there are only nine parameters estimated in the mixed Logit application; further details are provided in Appendix 4, which is available on request. The functional relations we assume are:

$$\begin{aligned} u_i &= \gamma_1 y_i + \gamma_2 y_i^2, \quad i = 1, 2, \text{ and } 3; \\ u_i &= \phi_1 y_{i-3} + \phi_2 y_{i-3}^2, \quad i = 4, 5, \text{ and } 6; \\ u_7 &= \eta y_1 y_2, u_8 = \eta y_1 y_3, \quad \text{and} \quad u_9 = \eta y_2 y_3; \\ u_{10} &= \omega_1 y_1 + \omega_1 y_1^2, u_{15} = \omega_1 y_2 + \omega_1 y_2^2, \quad \text{and} \quad u_{18} = \omega_1 y_3 + \omega_1 y_3^2; \\ u_{11} &= \xi_1 y_1 y_2 + \xi_2 y_1^2 y_2, u_{12} = \xi_1 y_1 y_3 + \xi_2 y_1^2 y_3, u_{13} = \xi_1 y_1 y_2 + \xi_2 y_1 y_2^2, \\ u_{14} &= \xi_1 y_1 y_3 + \xi_2 y_1 y_3^2, u_{16} = \xi_1 y_2 y_3 + \xi_2 y_2^2 y_3, \quad \text{and} \quad u_{17} = \xi_1 y_2 y_3 + \xi_2 y_2 y_3^2. \end{aligned}$$

Our goal is to obtain estimates of the parameters $\gamma_1, \gamma_2, \phi_1, \phi_2, \eta, \omega_1, \omega_2, \xi_1,$ and ξ_2 from

²⁰This approach is similar in spirit to that of Freeman (1991); it is also consistent with the approach suggested by Machina (1987), in that we focus first on the agent's representation over probabilities, $V(\mathbf{p})$, and then investigate the nature of the coefficients on the probability terms. An agent with a representation such that $\partial V/\partial p$ is concave in wealth corresponds to a risk-averse agent in the expected utility framework. An alternative approach would be to explicitly investigate the interrelation between probabilities and wealth; as discussed in footnote 16, this was not practical in our particular application.

Table 7. Implied coefficients on money in non-linear representation.

Parameter	Estimate	Asymptotic standard error
γ_1	748.54	79.459
γ_2	-1.2198	0.13154
ϕ_1	-0.18848	0.01824
ϕ_2	107.76	8.9830
η	-0.43364	0.04070
ω_1	-0.32267	0.03394
ω_2	0.00102	0.00012
ξ_1	-102.79	9.6790
ξ_2	0.32463	0.03397

the estimated parameters β_1 through β_9 . Such a process is tedious, involving substantial algebraic manipulation; in the interest of brevity we do not reproduce these calculations here (see Appendix 4 for further discussion). Table 7 lists the estimates of the nine new parameters of interest, based on the result of those manipulations and the parameter estimates from Table 6.

Armed with these values, we describe a monetary value of a policy change. For example, suppose a certain intervention could reduce the probability of the worst outcome from p_1 to p'_1 , with an offsetting increase in the probability of the middle outcome from p_2 to p'_2 . The monetary value of this intervention is the value of OP that solves

$$V(p_1, p_2, p_3; \mathbf{y}) = V(p'_1, p'_2, p_3; \mathbf{y} - \text{OP}). \tag{12}$$

The monetary value OP is the agent’s ex ante willingness to pay, irrespective of the ultimate state of nature that obtains, to effect the change in probabilities.

The following example illustrates the point. Suppose we start from the combination $(p_1, p_2, p_3) = (.06, .34, .6)$ and reduce p_1 by .03 (as in figure 2), thereby obtaining the new lottery $(p_1, p_2, p_3) = (.03, .37, .6)$, which adds \$1.50 to expected value. Using the parameters in Table 7, we calculate the ex ante monetary value of this change as $\text{OP} = \$1.358$. By contrast, suppose one mistakenly assumed this agent followed the expected utility paradigm, so that his representation was linear in the probabilities. The appropriate form to use would be the linear approximation at the starting point. This approximation is given by $v(y_i) = \partial V / \partial p_i, i = 1, 2, 3$, which one may derive in a straightforward manner from the cubic representation, based on the parameter estimates contained in Table 7. Based on these derivations, we find the monetary value of this shift in probabilities would be \$0.163 to the hypothetical expected utility maximizer. Evidently, adopting the expected utility assumption could lead to a substantial *underestimation* of the implied willingness to pay for the associated risk reductions, with the real willingness to pay on the order of five times the calculated value.

6. Discussion

Using a varying probability lottery experiment with a fixed set of three losses, we find that agents are quite heterogeneous and do not ubiquitously follow the expected utility hypothesis. The economic significance of these departures, however, is less easily evaluated. The key question is: what approximation errors must be accepted if one is to retain the expected utility model in public policy decisions? To answer this question, one must determine whether expected utility theory makes biased predictions about the choices a typical agent would make, and whether the bias is considerable. Expected utility uses net expected benefits to measure likely policy responses. If the appropriate measure was based upon a different value function, one that was non-linear in the probabilities, might a different policy be suggested? And if so, what would be the cost of the incorrect action? For one to conclude that the expected utility approach cannot be adequately applied in issues of public policy, an evaluation based upon the appropriate value function would have to show the potential for important policy errors, with attendant non-trivial opportunity costs to society.

Consider a scenario in which a decision-maker initially faces substantial risk. Suppose there are two possible outcomes: no loss or a very large loss. Such a combination corresponds to $p_2 = 0$ in our framework. Now imagine that an “insurance contract” is available, one that reduces the chance for the best event, but also lowers the chance of the worst event. Suppose also that such an arrangement lies below the tangent line to the indifference curve at the initial lottery. Under expected utility, such a policy would be regarded as unambiguously bad. But if the representative agent has concave indifference curves it is possible that such a policy leads to an improvement in well-being. Similar conclusions emerge if the agent’s indifference curves are locally concave, as with the model implied by our mixed Logit results. Placing this story in the context of a potential loss within a certain range of risk, our results suggest the potential for regulatory safeguards to raise social well-being, even when those safeguards have negative net expected benefits. Neglecting the potential for non-linear preferences, as described by the mixed Logit model, could result in the under-provision of risk reducing safeguards that are attractive from a collective perspective.

Appendix 1: Experimental instructions

Instructions

Welcome

This is an experiment in decision making that will take about an hour to complete. You will be paid in cash for participating at the end of the experiment. How much you earn depends on your decisions and chance. *Please do not talk* and do not try to communicate with any other subject during the experiment. If you have a question, please raise your hand and a monitor will come over. If you fail to follow these instructions, you will be asked to leave and forfeit any moneys earned. You can leave the experiment at any time without prejudice. Please read these instructions carefully, and then review the answers to the questions on page 4.

An overview

You will be presented with 40 pairs of options. For each pair, you will pick the option you prefer. After you have made all 40 choices, you will then play one of the 40 options to determine your take-home earnings.

The experiment

Stage #1: The Option Sheet: After filling out the waiver and the survey forms, the experiment begins. You start with \$100, and your choices and chance affect how much of this money you can keep as your take-home earnings.

You will be given an *option sheet* with 40 pairs of options. For each pair, you will circle the option you prefer. Each option is divided into 3 probabilities:

- P*1 is the probability you will lose \$80;
- P*2 is the probability you will lose \$30; and
- P*3 is the probability you will lose \$0.

For each option, the three probabilities always add up to 100% ($P1 + P2 + P3 = 100\%$). For example, if an option has $P1 = 20\%$, $P2 = 50\%$ and $P3 = 30\%$, this implies you have a 20% chance to lose \$80, a 50% chance to lose \$30, and a 30% chance to lose \$0.

On your option sheet, you circle your preferred option for each of the 40 pairs. For example, consider the pair of options, *A* and *B*, presented below. Suppose after examining the pair of options carefully, you prefer option *A* to *B*—then you would circle *A* (as shown below). If you prefer *B*, you would circle *B*.

- $P1 = 10\%, P2 = 20\%, P3 = 70\%$ (A)
- $P1 = 20\%, P2 = 20\%, P3 = 60\%$ (B)

Stage #2: The Tan Pitcher: After filling out your option sheet, please wait until the monitor calls you to the front of the room. When called, bring your waiver form, survey, and option sheet with you.

On the front table is a tan pitcher with 40 chips inside, numbered 1 to 40. The number on the chip represents the option you will play from your option sheet. You will reach into the tan pitcher without looking at the chips, and pick out a chip. The number on the chip determines which option you will play to determine your take-home earnings. For example, if you draw chip #23, you will play the option you circled for the pair #23 on your option sheet.

Stage #3: The Blue Pitcher: After you have selected the option you will play, you then draw a different chip from a second pitcher—the blue pitcher. The blue pitcher has 100 chips, numbered 1 to 100. The number on the chip determines the actual outcome of the option—a loss of either \$80, \$30, or \$0.

For example, if your option played has

$$P1 = 10\%$$

$$P2 = 50\%$$

$$P3 = 40\%,$$

then if you pick a chip numbered between 1 and 10, you lose \$80; if you pick a chip between 11 and 60, you lose \$30; or if you pick a chip between 61 and 100, you lose \$0.

If instead, your option played has

$$P1 = 20\%$$

$$P2 = 20\%$$

$$P3 = 60\%,$$

then if you pick a chip between 1 and 20, you lose \$80; if you pick a chip between 21 and 40, you lose \$30; or if you pick a chip between 41 and 100, you lose \$0.

Stage #4: Ending the experiment: After playing the option, you fill out a tax form. The monitor will then hand over your take-home earnings, and you can leave the room.

Now please read through the questions and answers on the next page.

Questions and Answers

1. When I make a choice, I will choose between how many options?
2
2. I will make how many choices?
40
3. My initial \$\$ endowment is how much?
\$100
4. P1 represents what?
The probability of losing \$80
5. P2 represents what?
The probability of losing \$30
6. P3 represents what?
The probability of losing \$0
7. For each option, the three probabilities sum to what?
100%
8. What does the number drawn from the tan pitcher represent?
The option (1 to 40) played from your option sheet
9. What does the number drawn from the blue pitcher represent?
The outcome (1 to 100) of the option played—determining whether you lose either \$80, \$30, or \$0

Are there any questions?

Appendix 2: The survey sheet

1. Social Security Number: _____
2. Gender: (circle) Male Female
3. Birthdate: _____ (month/day/year)
4. Highest Level of School Completed: (please circle)
 - Junior High School
 - High School or Equivalency
 - College or Trade School
 - Graduate or professional School
5. Courses Taken in Mathematics: (please circle all that apply)
 - College Algebra
 - Calculus or Business Calculus
 - Linear Algebra
 - Statistics or Business Statistics
6. Families' Annual Income: _____
7. Personal Annual Income: _____

Thank you

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