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THE REVELATION AND DELEGATION PRINCIPLES IN COMMON AGENCY GAMES

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1. INTRODUCTION

THE USE OF THE REVELATION PRINCIPLE has become widespread in the optimal contracts literature,[2] in large part because of the convenient description it provides of the set of allocations that can be achieved when information is decentralized. Unfortunately, when contracting situations become more complex, the applicability and usefulness of the revelation principle comes into question.[3] In this note, we briefly address three questions that arise in the context of common agency games: (i) What is the set of allocations that is implementable as equilibria of games among competing mechanism designers for some exogenously given communication spaces and can this set be easily described? (ii) How does this set compare to truthful equilibrium allocations of the direct communication game in which agents report only their physical types? And, closely related, is the revelation principle still valid under decentralized contracting? Finally, (iii), can alternative approaches such as the extended-taxation (or delegation) principle be usefully applied?

One approach to this multi-principal implementation problem has been pioneered by Epstein and Peters (1999) who, in multi-principal-multi-agent games, demonstrate the existence of a universal message space for which the revelation principle is valid. Such a message space, in particular, must include a sufficiently rich language to incorporate the agent's underlying type *and* the market information (e.g., other principals' contracts). Of course, as McAfee (1993) and others have noted, there is a potential problem of infinite regress as each principal needs to enlarge the agent's type space to include messages about the other principal's contract offer. The remarkable contribution of Epstein and Peters (1999) is to show that such a sequence of enlargements converges to a universal type space. With such a type space, the revelation principle can be applied. Unfortunately, in practice it is quite difficult to characterize the universal type space for applications. We see the contribution of this note as providing a more practical alternative in the context of common agency games.

Our idea is to employ the "taxation principle" developed by Guesnerie (1981, 1995) and Rochet (1986). This principle holds that for any truth-telling, direct-revelation mechanism, there exists an associated schedule or menu of choices that can be offered to

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[2] The revelation principle has been stated by many researchers, including Gibbard (1973), Green and Laffont (1977), Dasgupta, Hammond, and Maskin (1979), Myerson (1979, 1982), and Harris and Townsend (1981).

[3] Green and Laffont (1986), for example, discuss the difficulties in applying the revelation principle in the single-principal environment when the set of messages available to the agent is type-dependent.

the agent and that implements the same equilibrium outcome through decentralization.[4] The extension of the taxation principle that we employ (to which we will hereafter refer as the "delegation principle" given the more general context) is essentially the reverse of the revelation principle. Take any indirect communication mechanism and replace it with the decentralized menus of payoff-relevant contracting choices as suggested by the original taxation principle. Providing that any size restrictions on the original message spaces are translated into corresponding restrictions on the spaces of decentralized menus, we show that the original equilibrium outcome remains an equilibrium in the new menu game. In this sense, when studying equilibria in multi-principal settings, it is without loss of generality to restrict attention to strategy spaces of decentralized menus of contracting variables. Because the traditional revelation principle is still available to a principal when determining best responses to a rival's conjectured strategy, care needs only to be taken to include important out-of-equilibrium menu offers when constructing equilibria in this menu game. In particular, if interest is restricted to the set of pure-strategy, deterministic communication equilibria, then the best-response correspondences are straightforward to calculate (just as in any single principal-agent setting), and Nash equilibria are found in the standard manner. The construction of universal message spaces is thereby avoided.

In Section 2, after we set up our notation for common agency games, we provide three basic reasons for a failure of the revelation principle, including a specific example for what we believe is the most economically significant failure—the presence of out-of-equilibrium messages. In Section 3, we develop the delegation principle by extending the taxation principle to multi-principal games, and in Section 4 we apply it to a few illustrative examples.

2. A FRAMEWORK FOR SIMPLE COMMON AGENCY GAMES

Consider a game with N principals, for $i = 1, \dots, N$, and a single agent. Each principal can contract with the agent over an allocation $d^i \in \mathcal{D}^i$, where $\mathcal{D}^i$ is finite. In a sense to be made precise, each principal's strategy space is the space of communication mechanisms defined over $\mathcal{D}^i$; the agent's strategy space consists of mappings from messages into choices available to the agent in each contract. It is important to note that principal i cannot contract over the set of allocations controlled by principal j. Let $\mathcal{D} \equiv \prod_{i=1}^{N} \mathcal{D}^i$ and $d \equiv (d^1, \dots, d^N) \in \mathcal{D}$ be the vector-space representation of contract actions.[5] The agent has a type drawn from a finite set, $\theta \in \Theta$, which is private information but whose probability distribution, $f(\theta)$, is common knowledge. The agent's vNM utility is given by $U(d, \theta)$ and each principal's utility is given by $V^i(d, \theta)$; the latter's preferences allow for a dependence upon the contract actions of the other principals.[6]

[4] Peters (2000) has recently provided a similar suggestion in order to bypass universal message spaces. His work is independent of our own and focuses on various notions of equilibrium robustness as one varies the mechanism space. Peters' Theorems 1 and 2 are closely related to our Theorem 1 and corollary, although the setting Peters explores includes hidden actions in addition to hidden information.

[5] Throughout, the superscript "$-i$" on a vector refers to the vector without the ith component. For example, $d^{-i} \equiv (d^1, \dots, d^{i-1}, d^{i+1}, \dots, d^N) \in \mathcal{D}^{-i} \equiv \prod_{j \neq i} \mathcal{D}^j$. It is worth noting that one could easily extend our analysis to consider issues of moral hazard or "obedience" in the sense of Myerson (1982) by appending an additional vector of imperfectly observable actions over which the principals make suggestions. The treatment of Peters (2000) is more general in this regard.

[6] The general case of direct payoff externalities among principals in common agency settings is examined in Martimort and Stole (1998).

We are interested in communication mechanisms (or contracts), which are functions from messages to probability measures over allocations controlled by each principal. Let $\mathcal{M}^i$ be a measurable message space available to principal i and denote individual messages as $m^i \in \mathcal{M}^i$. Let $\mathcal{M} \equiv \prod_{i=1}^N \mathcal{M}^i$ and $m \equiv (m^1, \ldots, m^N) \in \mathcal{M}$. Formally, a communication mechanism for principal i, $\pi^i(d^i|m^i)$, is a measurable mapping from messages to distributions over actions; i.e., $\pi^i : \mathcal{M}^i \to \Delta(\mathcal{D}^i)$.[7] We represent the set of mechanisms as $\Pi^i = (\Delta(\mathcal{D}^i))^{\mathcal{M}^i}$, assumed to be measurable. The corresponding product of these sets is denoted by $\Pi \equiv \prod_{i=1}^N \Pi^i$. Because the space of communication mechanisms, Π^i, depends fundamentally on the richness of the underlying message space, $\mathcal{M}^i$, which is of central interest in this note, we will often make this dependence explicit: $\Pi^i(\mathcal{M}^i)$. For the specific setting in which the communication mechanism is a degenerate probability distribution on $\mathcal{D}^i$ for every message, we say that the mechanism is *deterministic*.

Throughout this analysis, we take all of the primitives of our communication game as fixed, except for the message space, $\mathcal{M}$, which can be either finite or infinite. As a consequence, we will associate a common agency communication game, $\Gamma_{\mathcal{M}}$, with its message space. The timing of $\Gamma_{\mathcal{M}}$ is as follows. First, the agent draws its type from the distribution $f(\theta)$ on Θ. Second, each of the N principals simultaneously offers a contract, $\pi^i \in \Pi^i(\mathcal{M}^i)$, to the agent. Third, the agent chooses a vector of messages, m, reporting the ith component to principal i (and only to principal i). For our purposes, we suppress the participation decision of the agent, in effect assuming that participation is required.[8] Payoffs are awarded according to the contracts π and messages m, using public randomizing devices as necessary to generate each π^i. Each principal i chooses a strategy $\sigma_i(\pi^i) \in \Delta(\Pi^i)$, which is a probability distribution over Π^i and the agent's strategy is a mapping from type-contract space onto a distribution of messages; $\sigma_0 : \Theta \times \Pi \to \Delta(\mathcal{M})$, where we will represent the agent's conditional probability distribution over messages for each given (θ, π) by $\sigma_0(m|\theta, \pi)$. A strategy profile is represented as $\sigma = (\sigma_0, \ldots, \sigma_N)$. We denote by $\operatorname{supp} \sigma_0(\theta, \pi)$ the support of this strategy, i.e., the set of messages that are sent with a positive probability, $\sigma_0(m|\theta, \pi) > 0$, when the agent has type θ, has received the collective mechanism π, and follows the strategy σ_0. A similar notation applies to $\operatorname{supp} \sigma_i$. We consider Perfect Bayesian Equilibria (PBE) of $\Gamma_{\mathcal{M}}$.[9]

DEFINITION 1: A strategy profile σ^* is an *equilibrium of* $\Gamma_{\mathcal{M}}$ (i.e., $\sigma^* \in \text{PBE}(\Gamma_{\mathcal{M}})$) iff: (a) $\forall \theta \in \Theta$, $\forall \pi \in \Pi(\mathcal{M})$:

$$m \in \operatorname{supp} \sigma_0^*(\theta, \pi) \Longrightarrow m \in \arg\max_{\hat{m} \in \mathcal{M}} \sum_{d \in \mathcal{D}} U(d, \theta) \prod_{i=1}^N \pi^i(d^i|\hat{m}^i),$$

[7] Throughout, whenever characterizing probability distributions, we will use the first argument as an element of the support and any other arguments as conditionals; in an abuse of notation, we will also occasionally treat the distribution function as a vector, such as $\pi^i(\cdot|m) \in \Delta(\mathcal{D}^i)$. It should be understood that a communication mechanism is fully defined only when given a specific message space. When there is possible ambiguity about the associated message space, we will refer to the pair $\{\pi^i, \mathcal{M}^i\}$ for preciseness.

[8] Depending upon the nature of the game, we could model the agent's participation decision more generally by either requiring all principals to include a null contract in their offer, or by allowing the agent to reject the contract offers and walk away. In addition, we could allow the agent the option of either signing with all the principals or none of them (the case of *intrinsic* common agency) which may be appropriate in a regulatory context, or the option to sign with any subset of principals (the case of *delegated* common agency).

[9] To give any meaning to our analysis, we will assume that there exists such an equilibrium.

(b) $\forall i$: $\pi^i \in \operatorname{supp}\sigma_i^* \Longrightarrow$

$$\pi^i \in \arg\max_{\hat{\pi}^i \in \Pi^i} \int_{\pi^{-i} \in \Pi^{-i}} \int_{m \in \mathcal{M}} \sum_{\theta \in \Theta} \sum_{d \in \mathcal{D}} V^i(d, \theta) f(\theta) d\sigma_0^*$$

$$\times (m|\theta, \hat{\pi}^i, \pi^{-i}) \hat{\pi}^i(d^i|m^i) \prod_{j \neq i}^{N} d\sigma_j^*(\pi^j).$$

In short, a strategy profile is an equilibrium if (a) the agent places only positive weight on messages that are weakly optimal for any given set of offered contracts, and (b) principals choose only contracts with positive probability if the contracts are weakly optimal given the agent's communication strategy. We use the term *pure-strategy communication equilibrium* to mean a collection of pure strategies $\{m(\theta, \pi), \pi^1, \ldots, \pi^N\}$ that are jointly optimal. A pure-strategy equilibrium may still involve random (nondeterministic) mechanisms: $\pi^i \in \operatorname{int} \Delta(\mathcal{D}^i)$. For any strategy profile σ, we denote the equilibrium probability distribution over $\mathcal{D}$ for each $\theta \in \Theta$ by $\mu_\sigma(d|\theta)$. We refer to μ_σ as the *allocation* induced by the strategy profile σ. For reference as a benchmark, a *direct-revelation communication game* is a game in which each principal's message space is restricted to be the type space of the agent: $\mathcal{M}^i \equiv \Theta$ and $\mathcal{M} \equiv \Theta^N$. We denote such a specific communication game by Γ_{Θ^N}.

With this notation in hand, one is tempted to posit the revelation principle in its simplest (but incorrect) form: for each $\sigma^* \in \operatorname{PBE}(\Gamma_{\mathcal{M}})$, there exists $\tilde{\sigma}^* \in \operatorname{PBE}(\Gamma_{\Theta^N})$ such that (i) $\mu_{\sigma^*}(d|\theta) \equiv \mu_{\tilde{\sigma}^*}(d|\theta)$, $\forall (d, \theta)$, and (ii) $\tilde{\sigma}_0^*(\theta|\theta, \pi) = 1$, $\forall \theta \in \Theta$, $\forall \pi^i$ such that $\tilde{\sigma}_i^*(\pi^i) > 0$. In words, for any equilibrium σ^* of some communication game $\Gamma_{\mathcal{M}}$, there exists an equilibrium $\tilde{\sigma}^*$ of the direct-revelation communication game Γ_{Θ^N} that (i) gives rise to the same distribution over $\mathcal{D}$ as σ^* and (ii) has truth-telling as an equilibrium strategy profile.

3. DIFFICULTIES WITH THE REVELATION PRINCIPLE

In the standard proof of the revelation principle for the canonical principal-agent setting, one shows that the principal can prune any choices in the original game that are never chosen on the equilibrium path and restrict attention to the remaining set. Since the mapping from agent types into the pruned strategy set is surjective, the principal can index strategy choices by the agent's type, forming a direct mechanism. In the new direct game, the agent "reports" his type, thereby choosing the strategy that was optimal in the original indirect game. In the context of multi-principal games, this proof fails on three fronts.

First, when there are two or more principals, the agent may serve a new role as a correlating device in the indirect communication game. This role cannot be preserved in the direct communication game unless an appropriate randomizing device is appended to the agent's physical type space. This is a minor difficulty that can easily be addressed. The two remaining difficulties are more problematic.

Second, in the canonical principal-agent setting, the agent has no reason to be untruthful in the principal's direct mechanism when asked about his private information; it is assumed that the agent tells the truth to the principal when indifferent. In a multi-principal setting one principal may prefer to induce the agent to "lie" to the other principal, with the result that truthful equilibria may fail to exist in nonpathological market games. This nonexistence of truth-telling equilibria in multi-principal, exclusive-agent games was first demonstrated in an example by Myerson (1982), although the implications for the revelation principle were not explicitly noted. In a previous version of this note, Martimort and

	$d^2{=}A$	$d^2{=}B$	$d^2{=}C$
$d^1{=}A$	1,1,1	2,0,2	-1,5,10
$d^1{=}B$	0,2,2	1,1,1	0,0,0
$d^1{=}C$	5,-1,10	0,0,0	0,0,0

FIGURE 1.—Payoffs for a simple common-agency game.

Stole (1993), we provided a related counterexample in the context of common agency. A similar insight is also present in the exclusive agency game of Peck (1996), who independently arrived at related conclusions regarding the possibility that some mixed-strategy equilibria may not be truthfully implementable in direct mechanisms.

A third difficulty is that pruning out-of-equilibrium messages may destroy the associated equilibrium in the original game because the out-of-equilibrium messages may have critical strategic effects vis-à-vis the other principals. In a direct revelation game, such messages are not used (by definition). We find this failure more prevalent in our own research on common agency games because in many market, regulatory, and political situations it may be important that nonlinear schedules be extended beyond equilibrium choices.[10] In such settings, restricting attention to truthful equilibria in direct revelation communication games may eliminate some equilibria that were sustained by out-of-equilibrium messages and may (perhaps simultaneously) introduce other equilibria that could not be sustained in the indirect mechanisms. The essence of the problem is that the agent's strategy is no longer only a mapping from the agent's physical type space to the agent's choice set; now the agent's choice from principal i's offer depends upon principal j's offer. An indexation can only be accomplished if the index includes the market information (i.e., contracts offered by other principals) as well as types. This is the idea of the universal message space of Epstein and Peters (1999).

To illustrate this third failure more clearly, consider the simplest possible case where an agent's type is degenerate, $|\Theta| = 1$, thereby eliminating adverse selection issues for the moment. Initially, we also restrict each principal's strategy spaces to deterministic contracts; that is, we assume contracts cannot assign lotteries over actions,[11] even though principals may choose mixed strategies over the space of deterministic contracts. Consider a specific setting where each principal's contract space is $\mathcal{D}^i \equiv \{A, B, C\}$ and the payoffs to the three parties are represented as triplets, $\{V^1, V^2, U\}$, given in the payoff matrix in Figure 1.

It is useful to first consider a common agency menu-delegation game without explicit communication in which each principal can offer the agent any subset of $\mathcal{D}^i$ from which to choose; i.e., the principals offer the agent a menu of contract decisions from which to select and there is no direct communication. Note that in this delegation game, each principal i can only restrict the agent's choice over $\mathcal{D}^i$. In this simple example, the following contract offers form a perfect Bayesian-Nash equilibrium: Each principal offers the menu $\{B, C\}$, from which the agent chooses $d^i = B$. The resulting allocation yields a payoff vector of $\{1, 1, 1\}$. Note that C is offered by principal 1 as an off-the-equilibrium-path choice

[10] See, for example, Stole (1991), Martimort (1992, 1996), and Martimort and Stole (1998).

[11] In practice, such randomizations may be difficult to verify by a third-party or court of law.

for the agent to discourage principal 2 from offering A. If principal 1 were to offer only $\{B\}$ to the agent, principal 2's best response would be to offer $\{A\}$. When principal 1 offers $\{B, C\}$ in equilibrium, principal 2 will not offer A as then the agent would choose C from principal 1, yielding a payoff of -1 to principal 2; rather, it is a best response for principal 2 to offer $\{B, C\}$ as well, obtaining the payoff of $+1$.

Not surprisingly, any outcome of the menu-delegation game can be supported in an indirect communication mechanism game if the message space contains two messages because the principal can commit to choose from a subset of allowable actions according to the agent's request. For example, the equilibrium outcome of $\{B, B\}$ from the delegation game can be implemented as a perfect Bayesian-Nash equilibrium in the indirect common agency communication game with a two-message space, $\mathcal{M}^i \equiv \{m_b^i, m_c^i\}$. The contract offers of $\pi^i(B|m_b^i) = 1$ and $\pi^i(C|m_c^i) = 1$ by each principal characterize a PBE in the communication game in which the agent reports $m = \{m_b^1, m_b^2\}$ and the equilibrium allocation is $\{B, B\}$. It is also worth noting at this point that $\{C, C\}$ is an equilibrium outcome of the common agency game for any message spaces. Specifically, each principal implementing C regardless of messages from the agent is an equilibrium. Thus, the implementable set of outcomes within indirect common agency games with sufficiently rich message spaces includes $\{B, B\}$ and $\{C, C\}$.

Can the same equilibrium allocations $\{B, B\}$ and $\{C, C\}$ be implemented as equilibria in a direct mechanism communication game (i.e., where the message space is degenerate)? With only direct communication from the agent, each principal is restricted to choosing (perhaps randomly) a single $d^i \in \mathcal{D}^i$ to implement; there is no strategic role for the agent. Of the two outcomes above, only $\{C, C\}$ is truthfully implementable in the direct-mechanism communication game. Remarkably, the outcome $\{B, B\}$ cannot be implemented. The equilibrium outcome $\{B, B\}$ is unavailable in the direct revelation communication game because without effective communication between principal i and the agent, principal j will find it profitable to deviate to action A. With effective communication between principal i and the agent, $\{B, B\}$ can be sustained as an equilibrium outcome by allowing principal j's deviation to A to be communicated by the agent to principal i, resulting in choice C.

This example already suggests that enlarging the principals' strategy spaces and letting them offer any subset of the set of decisions they respectively control plays an important role in the description of the equilibrium set of a common agency game. Given the failure of direct-revelation communication games to replicate the equilibrium set of a communication game using fixed message spaces of a given complexity, we may instead look for a natural strategy space for the principals with the property that restricting those principals to offer subsets of this strategy space with a given complexity allows replication of the outcome of any communication game. This is precisely the purpose of the delegation principle that we propose below.

4. THE DELEGATION PRINCIPLE

We present a simple delegation principle for common agency games: the set of equilibrium outcomes obtainable in an indirect communication game with arbitrary message spaces can be replicated as equilibrium outcomes in a game in which the principals offer payoff relevant menus from which the agent chooses. With this principle, we can apply further simplifications to finding equilibria in the delegation game: (i) we can prune strategically dominated strategies, and (ii) use the revelation principle to calculate each principal's best response function. In many instances we can characterize the set of equilibria in a manner that is currently difficult using universal message spaces, since the latter approach is not amenable to optimization with simple incentive constraints.

The taxation principle in the one-principal context holds that any deterministic direct communication mechanism can be decentralized as a nonlinear tariff (or tax) by substitution. For example, in the context of nonlinear pricing by a monopolist facing a consumer with a one-dimensional private characteristic, the deterministic, direct communication mechanism $\{p(\theta), q(\theta)\}_{\theta \in \Theta}$ can be converted into the indirect nonlinear price, $P(q)$, by inverting the quantity function, $q(\theta)$, and substituting into the price function, $p(\theta)$: i.e., $P(q) \equiv p(\theta^{-1}(q))$. The taxation principle says that for any direct communication mechanism, there exists a nonlinear schedule that implements the same outcome. Hence, in applied work, an economist can use the revelation principle to justify optimizing over the set of truthful direct mechanisms, and then, upon finding an optimum, she can apply the taxation principle to convert the optimum into a more common contracting form. The insight of the taxation principle easily extends to random mechanisms, in which case the principal offers a menu of lotteries.

The intuition of devising a similar principle for multi-principal settings is particularly inviting in the context of common agency games: fundamentally, the agent and principals care only about communication insofar as it affects the final distribution over payoff-relevant variables. This is the basic motivation of the delegation principle in common agency games. Hence, while the revelation principle with simple type spaces is no longer available, the delegation principle still affords us some reduction in the complexity of the problem.

Consider a communication mechanism and a given message space, $\{\pi^i, \mathcal{M}^i\}$, for principal i. As the message m^i varies within $\mathcal{M}^i$, the mapping $\pi^i(\cdot|m^i)$ traces out a whole subset of payoff-relevant distributions over the action set, $\mathcal{D}^i$. We denote by $\pi^i(\cdot|\mathcal{M}^i)$ the image of this mapping over the message space, $\mathcal{M}^i$. Thus $\pi^i(\cdot|\mathcal{M}^i)$ is simply a subset—or "menu"—of distributions over actions. Of course, the size of this menu space depends upon any restrictions on communication. As an example, if $\mathcal{M}^i = \{m^i\}$, then no meaningful communication is possible and $\pi^i(\cdot|\mathcal{M}^i)$ consists of a single element that is a probability distribution over $\mathcal{D}^i$. If $\mathcal{M}^i = \{m_1^i, m_2^i\}$ with $m_1^i \neq m_2^i$, then $\pi^i(\cdot|\mathcal{M}^i)$ consists of (at most) two probability distributions over $\mathcal{D}^i$. More generally, $\pi^i(\cdot|\mathcal{M}^i)$ is a subset of $\Delta(\mathcal{D}^i)$, whose cardinality is no greater than the available communication space $\mathcal{M}^i$ because $\pi^i(\cdot|\mathcal{M}^i)$ is injective.

We say that an arbitrary menu of distributions, T^i, is *consistent* with a message space $\mathcal{M}^i$ if there exists a mechanism π^i defined on $\mathcal{M}^i$ such that $T^i = \pi^i(\cdot|\mathcal{M}^i)$. The set of all such menus consistent with $\mathcal{M}^i$ is given by $\mathcal{T}^i(\mathcal{M}^i)$.[12] Formally,

$$\mathcal{T}^i(\mathcal{M}^i) \equiv \{\pi^i(\cdot|\mathcal{M}^i) | \pi^i \in \Pi^i(\mathcal{M}^i)\}.$$

In words, $\mathcal{T}^i(\mathcal{M}^i)$ is the set of all subsets of $\Delta(\mathcal{D}^i)$ having cardinality at most that of $\mathcal{M}^i$. $\mathcal{T}^i(\mathcal{M}^i)$ can thus be viewed as the set of possible menus that principal i can offer when the communication space is $\mathcal{M}^i$.[13] We define $\pi(\cdot|\mathcal{M})$, $\pi^{-i}(\cdot|\mathcal{M}^{-i})$, $\mathcal{T}(\mathcal{M})$, and $\mathcal{T}^{-i}(\mathcal{M}^{-i})$ as the relevant products of these menus and sets of menus taken in the obvious way.

It is worth noting that while for any communication mechanism and message space, $\{\pi^i, \mathcal{M}^i\}$, the menu $\pi^i(\cdot|\mathcal{M}^i)$ is uniquely defined, this mapping is not one-to-one because

[12] We will use T^i to denote an arbitrary element of $\mathcal{T}^i(\mathcal{M}^i)$.

[13] Note that if $\mathcal{T}^i(\mathcal{M}^i)$ were defined instead to allow for redundant menu items, then the set of menus would have the same dimensions as $\Pi^i(\mathcal{M}^i)$. Because such redundancy has been pruned in the construction of the menu space, the principal's strategy space in the delegation game is less rich compared to the strategy space in the original communication mechanism game.

many distinct communication mechanisms give rise to the same set of lotteries over actions. As a simple example, any permutation of the message-lottery assignments will generate a distinct mechanism with an identical set of distributions. To take a more prosaic example, the physical nature of the contract (i.e., the lotteries it generates) may be the same regardless of whether the contract communication takes place in English, in French, or if delegation is used and no words are spoken. As such, the physical set of lotteries that a contract allows forms an equivalence class across the universe of possible modes of communication: i.e., two distinct communication mechanisms and message spaces, $\{\pi^i, \mathcal{M}^i\}$ and $\{\pi^{i'}, \mathcal{M}^{i'}\}$, belong to the same equivalence class if and only if their respective images are identical, i.e., $\pi^i(\cdot|\mathcal{M}^i) = \pi^{i'}(\cdot|\mathcal{M}^{i'})$.

The idea of the delegation principle is that it is without loss of generality to restrict attention to delegation games in which each principal's strategy space is coarsened to choices among the original equivalence classes, and the agent is allowed to choose which probability distribution to implement from the subset of offered equivalence classes. If true, this implies that the exact specification of the underlying communication spaces only matters to the extent that it affects the set of available equivalence classes from which the principal chooses. Moreover, because T^i is a subset of $\Delta(\mathcal{D}^i)$, one can begin to see the appeal of the delegation principle: For a contracting game between principals who have access to very rich message spaces for communication with the agent, the associated delegation game has strategies that are mixtures over subsets of $\Delta(\mathcal{D}^i)$.[14]

To define and prove the general version of the delegation principle for common agency games, we need to define an equilibrium for our menu game, $\Gamma^d_{\mathcal{T}(\mathcal{M})}$. In this game, principal i's strategy is represented as $\tilde{\sigma}_i(T^i)$, a mixed strategy over the space of menu offers $\mathcal{T}^i(\mathcal{M}^i)$; the agent's strategy is a measure, $\tilde{\sigma}_0(\tau|\theta, T)$, defined over selections, $\tau \in T$, from each possible menu, T. We define $\operatorname{supp}\tilde{\sigma}_0(\theta, T)$ as the subset of offered distributions that are chosen with positive probability by an agent with type θ when facing the collective menu of distributions, T, from the principals. A similar definition applies to $\operatorname{supp}\tilde{\sigma}_i$, which is the subset of menus from $\mathcal{T}^i(\mathcal{M}^i)$ chosen with positive probability by each principal i. A perfect Bayesian-Nash equilibrium in the delegation game is defined as follows.

DEFINITION 2: A strategy profile $\tilde{\sigma}^*$ is an *equilibrium of* $\Gamma_{\mathcal{T}(\mathcal{M})}$ (i.e., $\tilde{\sigma}^* \in PBE(\Gamma_{\mathcal{T}(\mathcal{M})})$) if and only if:

(a) $\forall\, \theta \in \Theta$:

$$\tau \in \operatorname{supp}\tilde{\sigma}_0^*(\theta, T) \Longrightarrow \tau \in \arg\max_{\hat{\tau}\in T} \sum_{d\in\mathcal{D}} \hat{\tau}(d)U(d,\theta),$$

(b) $\forall\, i: T^i \in \operatorname{supp}\tilde{\sigma}_i^* \Longrightarrow$

$$T^i \in \arg\max_{\widehat{T}^i\in\mathcal{T}^i(\mathcal{M}^i)} \int_{T^{-i}\in\mathcal{T}^{-i}(\mathcal{M}^{-i})} \int_{\tau\in T} \sum_{\theta\in\Theta}\sum_{d\in\mathcal{D}} V^i(d,\theta)f(\theta)\tau(d) \times d\tilde{\sigma}_0^*(\tau|\theta, \widehat{T}^i, T^{-i}) \prod_{j\neq i}^{N} d\tilde{\sigma}_j^*(T^j).$$

Defining the equilibrium distribution over $\mathcal{D}$ as $\mu_{\tilde{\sigma}^*}(d|\theta)$, the Delegation Principle can be stated succinctly.

[14] To be precise, the selection has cardinality no greater than that available in the underlying communication space.

THEOREM 1 (Delegation Principle for Common Agency Games): *For each* $\sigma^* \in \mathrm{PBE}(\Gamma_{\mathcal{M}})$, $\exists \tilde{\sigma}^* \in \mathrm{PBE}(\Gamma^d_{\mathcal{T}(\mathcal{M})})$ *such that* $\mu_{\sigma^*}(d|\theta) \equiv \mu_{\tilde{\sigma}^*}(d|\theta)$.

The complete proof is presented in the Appendix. There are a few technical difficulties involved in proving the delegation principle for common agency games. The most notable is that the choice of language (e.g., English or French in our example) may itself play a strategic role in the equilibrium of the indirect game, such as a randomization device for the agent. Any such role, however, can be maintained by allowing the agent to perform the randomization directly, thereby implementing the same distribution over actions as in the original equilibrium of the indirect game.

The Delegation Principle argues that focusing on communication per se is unnecessary to characterize the set of equilibrium allocations in communication games. All that matters is the restriction, embodied in $\mathcal{T}(\mathcal{M})$, that the size of the underlying communication spaces imposes. Whatever the initial constraints on communication imposed by each $\mathcal{M}^i$, there exist constraints on the sets of payoff relevant probability measures over actions that can be offered such that, within this constrained set, each principal finds it optimal to offer a menu of such probability measures, letting the agent choose within those menus, and the resulting equilibrium allocation is the same as in the original communication game. This last point is worth stressing: explicit communication between each principal and the agent is unnecessary.

5. APPLICATIONS OF THE DELEGATION PRINCIPLE

The extension of the delegation principle to common agency games provides the equivalence between equilibria in any given communication game for some given message space and the equilibria in an appropriately chosen menu game. Where the delegation principle has perhaps the greatest value, however, is precisely in those situations in which the underlying message space of the indirect game is very rich. Arguably, each principal should be able to choose larger message spaces if such a deviation is profitable. In a game with such unrestricted communication, the strategy space for a principal should be any communication space, $\mathcal{M}^i$, and a mixture σ_i over probability measures $\pi^i(\cdot|\cdot)$ that map $\mathcal{M}^i$ into $\Delta(\mathcal{D}^i)$. In this setting, it is difficult to characterize the set of all equilibria because it is hard to imagine the set of all mechanisms available in any deviation. The Delegation Principle says that it is enough to consider the class of unrestricted menu games.

For any message space $\mathcal{M}^i$, the set $\mathcal{T}(\mathcal{M}^i)$ remains a subset of $\Delta(\mathcal{D}^i)$. Hence as the complexity of $\mathcal{M}^i$ increases,[15] the cardinality of $\mathcal{T}(\mathcal{M}^i)$ cannot be greater than the cardinality of the power set of $\Delta(\mathcal{D}^i)$. This casts an upper bound on the complexity of the communication space useful to describe the equilibrium of common agency games when the principals face no constraints in designing their mechanisms. This communication space need not be more complex than $\Delta(\mathcal{D}^i)$.

COROLLARY 1: *Any equilibrium outcome of a communication game with unrestricted communication is an equilibrium outcome in the associated payoff-relevant menu game in which menus are unrestricted; i.e., each principal can offer any arbitrary subset* T^i *of* $\Delta(\mathcal{D}^i)$.

[15] $\mathcal{M}^{i'}$ is *more complex* than $\mathcal{M}^i$ if there exists an injective mapping from $\mathcal{M}^i$ into $\mathcal{M}^{i'}$ but not the reverse.

Hence, an economist interested in the set of equilibria to an unrestricted communication game needs only to investigate the set of equilibria in the decentralized menu game.[16] This characterization, while useful in showing how large the strategy spaces of the principals can be, is in general less useful in describing possible equilibrium allocations. Nevertheless, a few structured examples are useful to see the applicability of this corollary.

A SIMPLE EXAMPLE: Consider the following game between two principals and an agent who has type $\theta \in \{-1, +1\}$, which is drawn with equal probability. Each principal has a set of two allocations from which to choose: $\mathcal{D}^i \equiv \{d^i_A, d^i_B\}$ with $d^i_A = 0$ and $d^i_B = 1$. The utility of the agent over $\mathcal{D} \times \Theta$ is $U = \theta(d^1 + d^2)$; the utility of principal 1 is $V^1 = \theta(d^1 - d^2)$ and the utility of principal 2 is the reverse, $V^2 = \theta(d^2 - d^1)$. Hence, we have a very simple zero-sum game (between the principals) which can be thought of as a metaphor for product market competition between two wholesalers with a common retailer (agent), where θ represents the demand state. When the demand state θ is high, the agent prefers to implement the high actions for both principals; when the demand state is low, the agent prefers the low actions. Each principal prefers that the agent implement her own high action iff the state of demand is high, while each principal prefers the agent to take an action with respect to the other principal that is unprofitable for that principal.

Consider the case in which the indirect mechanism is a mapping from arbitrarily rich message space into the one-dimensional simplex. An associated menu is then a subset of the unit interval and the space of menus is the power set of [0, 1]. In the present case, we can now solve for the entire set of equilibrium outcomes in this indirect mechanism game. Because the structure of a pure-strategy menu offer from principal i is a (possibly infinite) collection of intervals of [0, 1], the joint offer received by the agent, $T = \{T^1, T^2\}$, can be represented as a collection of disjoint rectangles in $[0, 1]^2$. Given the agent's bilinear preferences, the $\theta = +1$ (resp., $\theta = -1$) agent will always choose the northeast (resp., southwest) corner of the most extreme northeast (resp., southwest) rectangle of $T \subset [0, 1]^2$. Because the equilibrium of the original indirect game must have allowed messages that generated a compact range in the neighborhood of the agent's choice, we can restrict attention to the closure of these rectangles. Figure 2 illustrates the game's geometry.

Consider any contract offer by principal 2. For any such offer, principal 1 will always benefit by increasing the range of her menu offers to include the deterministic choices d^1_A and d^1_B. This is because nothing that principal 1 does can affect the agent's choice over d^2, while the agent and principal 1's preferences are aligned for the choice over d^1. If principal 2 chooses a mixed strategy over menu offers, principal 1's best response still includes these extreme choices. An analogous argument establishes that principal 2 will always offer the agent the deterministic choices of d^2_A and d^2_B. With these offers, all other offers are irrelevant, and the agent always chooses $\{d^1_A, d^2_A\}$ when $\theta = -1$ and $\{d^1_B, d^2_B\}$ when $\theta = +1$. Therefore in all indirect communication equilibria, each principal makes zero profits and the agent makes an expected payoff of 1. Note that the example illustrates

[16] This is essentially Peters' (2000) idea of weak robustness presented in his Theorem 2. One could explore a stronger notion of robustness as in Peters (2000) and require that the addition of messages to a menu game not induce a different equilibrium outcome. If it is possible that the addition of messages allows a principal to change the equilibrium play to an alternative equilibrium, and if one believes that communication is an important determinant of equilibrium play, our corollary is not satisfactory and an explicit analysis of communication games seems warranted as noted by Peters (2000). Here, universal message spaces may have a particular appeal.

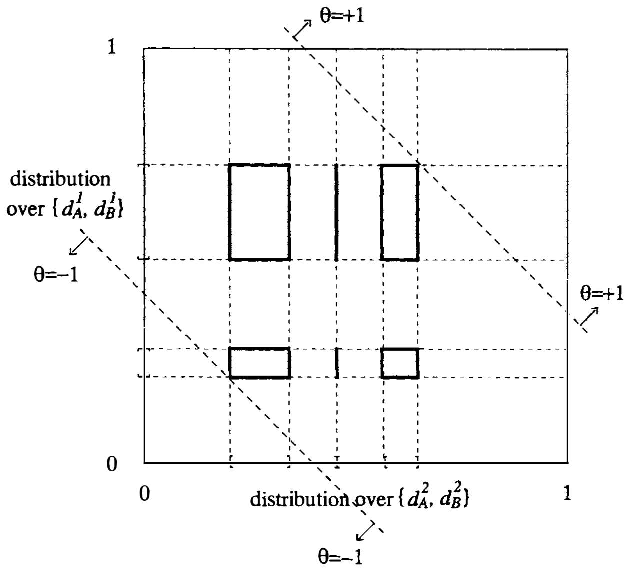

FIGURE 2.—Geometry of the example.

how techniques such as iterated dominance may be useful in menu-games with simple structures.

AN EXTENDED EXAMPLE: In previous papers on common agency in adverse selection settings (e.g., Martimort (1992, 1996) and Stole (1991)), the analysis was limited to equilibria in nonlinear pricing games (which may include out-of-equilibrium offers) without a clear sense of the loss of generality involved in restricting attention to those strategy spaces. The delegation principle makes clear that this equilibrium analysis is general.[17]

To see how the delegation principle may be useful in these more complex settings, consider the setting of a firm selling to a consumer with a one-dimensional private characteristic, but suppose now that the consumer buys a differentiated product from a rival firm. We consider the space of all indirect *deterministic* communication mechanisms.[18] A representative element of this space is $\pi^i \equiv \{p_i(m^i), q_i(m^i)\}_{m^i \in \mathcal{M}^i}$, where p_i is the price paid for q_i quantity of principal i's good. From this potentially complex mechanism, we construct the associated set of menus available to the agent offered π^i as

$$\pi^i(\cdot|\mathcal{M}^i) \equiv \{(p_i, q_i) \mid \exists\, m^i \in \mathcal{M}^i \text{ s.t. } p_i = p_i(m^i), q_i = q_i(m^i)\}.$$

Two mechanisms $\{\pi^i, \mathcal{M}^i\}$ and $\{\pi^{i'}, \mathcal{M}^{i'}\}$ are in the same equivalence class when they induce the same set of price-quantity pairs. In the delegation variation of the game, the

[17] As a recent example, Martimort and Stole (1998) use the delegation principle to explore the set of common agency equilibria when contracting externalities are present in the principal's payoff functions.

[18] This restriction on the original class of indirect mechanisms, of course, may be with loss of generality.

principal offers a menu of (p_i, q_i) pairs which, if the underlying communication space is unrestricted, can be arbitrarily large.

Because we are interested in the set of equilibria to the original indirect mechanism game, we can further prune away from the menu-delegation set all dominated strategies without affecting the equilibrium outcome. In the present context, this allows us to focus attention on all price schedules defined over a possibly restricted domain of outputs. Correspondences such that different messages yield the same price but different outputs or the same output but at different prices are easily ruled out because no dominated price-quantity pairs would ever be chosen by the consumer whose utility is strictly increasing in quantity and money. Hence, dominated price-quantity pairs cannot have a strategic effect. Only the lower envelope of offers is relevant in this context. With this fact in hand, the investigator interested in finding *all* pure-strategy, deterministic communication equilibria with arbitrarily large message spaces can posit a Nash equilibrium in price schedules, $\{P_1(q_1), P_2(q_2)\}$, and proceed using the revelation principle to check that the equilibrium choices of each principal form a best response given the choice of the rival.[19] The only difficulty that needs careful attention is with respect to considering all plausible out-of-equilibrium components of $P_i(q_i)$. Provided such out-of-equilibrium extensions are appropriately cared for, the entire set of deterministic pure-strategy contract equilibria outcomes can be determined by considering this simpler menu game. In Martimort (1992), the use of out-of-equilibrium actions was accounted for by allowing each principal to extend her price schedule over out-of-equilibrium choices; this extension can affect the equilibrium set in the case of economic substitutes in the agent's utility. In Stole (1991), the out-of-equilibrium issue was dealt with by making an additional assumption that underlying preferences are sufficiently concave; with such an assumption, out-of-equilibrium price schedules become economically irrelevant. Regardless of how one addresses out-of-equilibrium offers, a principal can determine her best response to the equilibrium strategy of the other principal (i.e., a nonlinear price schedule, $P_j(q_j)$ defined over a compact set Q^j) by considering the following indirect utility function:

$$\widehat{U}(q_i, p_i, \theta) \equiv \max_{q_j \in Q^j} U(q_i, p_i, q_j, P_j(q_j), \theta),$$

and then applying the standard paradigm of nonlinear pricing as if in a single-principal setting.[20]

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[19] Note that this result does not rely on any assumption on the type space. As an illustration, with two possible types, $\{\underline{\theta}, \bar{\theta}\}$, offering a menu $\{\underline{P}_i(q), \overline{P}_i(q)\}$ is no more useful for the principal than offering a single nonlinear price, $P_i(q) = \min\{\underline{P}_i(q), \overline{P}_i(q)\}$.

[20] Note each principal can use the revelation principle when constructing her best response given the offer of the rival, so the revelation principle retains considerable value. But because $\widehat{U}$ depends implicitly upon the shape of $P_j(q_j)$, both in and out of equilibrium, finding a Nash equilibrium in nonlinear prices requires some care to consider the effect of out-of-equilibrium tariffs.

APPENDIX

PROOF OF THE DELEGATION PRINCIPLE: The proof proceeds by construction of the equilibrium strategies of the principals and the agent in the delegation game, $\{\tilde{\sigma}_i^*\}_{i=0}^N$, given their equilibrium strategies in the original communication game, $\{\sigma_i^*\}_{i=0}^N$, with given message space, $\mathcal{M}$.

First, we construct the principals' strategies. For this purpose, fixing $\mathcal{M}$ we define $\Phi^i(T^i)$ to be the subset of communication mechanisms in $\Pi^i(\mathcal{M}^i)$ that forms the equivalence class for a given menu T^i having cardinality at most equal to that of $\mathcal{M}^i$:

$$\Phi^i(T^i) \equiv \{\pi^i \in \Pi^i(\mathcal{M}^i) | T^i = \pi^i(\cdot|\mathcal{M}^i)\}.$$

Implicitly, Φ^i depends upon $\mathcal{M}^i$ as well, but we suppress this for notational simplicity. We define $\Phi^{-i}(T^{-i})$ and $\Phi(T)$ as the obvious products of these sets. Note that $\pi^i \in \Phi^i(\pi^i(\cdot|\mathcal{M}^i)) \subseteq \Pi^i(\mathcal{M}^i)$ for all $\pi^i \in \Pi^i(\mathcal{M}^i)$, but, because two mechanisms π^i and $\pi^{i'}$ may have the same image, the set $\Phi^i(\pi^i(\cdot|\mathcal{M}^i))$ is strictly larger than $\{\pi^i\}$. Now, we can define for all $T^i \in \mathcal{T}^i(\mathcal{M}^i)$

$$\tilde{\sigma}_i^*(T^i) \equiv \int_{\pi^i \in \Phi^i(T^i)} d\sigma_i^*(\pi^i).$$

Because $\sigma_i^*(\pi^i)$ is well-defined over $\Pi^i(\mathcal{M}^i)$, and because the collection of subsets $\Phi^i(T^i)$ as one varies $T^i \in \mathcal{T}^i(\mathcal{M}^i)$ forms a partition of $\Pi^i(\mathcal{M}^i)$ by construction, $\tilde{\sigma}_i^*(T^i)$ is well-defined over $\mathcal{T}^i(\mathcal{M}^i)$. Notably, $\tilde{\sigma}_i^*$ generates the same probability distribution over the equivalence classes as does the original strategy, σ_i^*.

Second, we construct the agent's strategy in the menu delegation game. Here, care needs to be taken for two reasons. First, various distinct messages in $\mathcal{M}^i$ may generate the same distribution over $\mathcal{D}^i$ in the original communication game. This can easily be addressed by integrating the probability mass over the set of messages that gives rise to the same distribution over $\mathcal{D}^i$. A second problem arises because a principal may choose a mixed strategy over two distinct communication mechanisms from the same equivalence class (i.e., $\{\pi^i, \mathcal{M}^i\}$ and $\{\pi^{i'}, \mathcal{M}^i\}$ where $\pi^i(\cdot|\mathcal{M}^i) = \pi^{i'}(\cdot|\mathcal{M}^i)$), but where the realization of this mixture affects the agent's equilibrium choice over messages and the resulting distribution over allocations. Technically, this arises if the equilibrium has the agent using the principal's choice among strategically equivalent contracts as a randomizing device for communication. Reducing the principal's strategy space to menu offers eliminates this randomization device, but this loss can be addressed by building the corresponding randomization directly into the agent's menu-selection strategy. To this end, we need to preserve the agent's randomization over payoff relevant lotteries from the original game. The following construction of the agent's strategy in the game with menu offers accomplishes this objective along the equilibrium path:

$$\tilde{\sigma}_0^*(\tau|\theta, T) \equiv \int_{(\pi,m)\in\Psi(\tau)} d\sigma_0^*(m|\theta,\pi)\left(\frac{\prod_{i=1}^N d\sigma_i^*(\pi^i)}{\int_{\pi'\in\Phi(T)}\prod_{i=1}^N d\sigma_i^*(\pi^{i'})}\right),$$

where

$$\Psi(\tau) = \left\{(\pi, m) \,\middle|\, \prod_{i=1}^N \sigma_i^*(\pi^i) > 0 \text{ and } \tau(d) = \pi(d|m)\,\forall\, d \in \mathcal{D}\right\}.$$

The term in parentheses is the equilibrium probability distribution of π from the original communication game, conditional on T; it is used to calculate the agent's average randomization over strategically distinct choices from a given menu T, thereby preserving whatever mixture existed over available payoff-relevant decisions in the original game. Off the equilibrium path (i.e., for $\Psi(\tau) = \varnothing$), we can assign $\tilde{\sigma}_0^*(\tau|\theta, T) = \sigma_0^*(m|\theta, \pi)$ for some (π, m) such that $\tau \in \pi^i(\cdot|\mathcal{M}^i)$ and $\tau(d) = \pi(d|m)$ for all $d \in \mathcal{D}$. Because $\tau \in \pi(\cdot|\mathcal{M})$ for some mechanism, π, such a pair (π, m) exists.

We now demonstrate that these new strategies comprise an equilibrium in the menu game. First, consider a deviation by the agent from the proposed equilibrium. Suppose that there exists a $\tau' \in T$ that yields a greater expected payoff but for which $\tilde{\sigma}_0^*(\tau'|\theta, T) = 0$:

$$\sum_{d \in \mathcal{D}} \tau'(d) U(d, \theta) > \sum_{d \in \mathcal{D}} \tau(d) U(d, \theta).$$

Since $\tau' \in T$, there exists a π that is offered in the equilibrium of the original communication game such that $\tau' \in \pi(\cdot|\mathcal{M})$, and there exists an unsent message $m' \in \mathcal{M}'$ such that $\tau'(d) \equiv \pi(d|m')$ for all $d \in \mathcal{D}$. Substituting into the above inequality, we immediately obtain a contradiction with $\sigma_0^*(m'|\theta, \pi) = 0$ as the original strategies do not comprise an equilibrium in the communication game.

Next, consider a deviation by some principal, i. Suppose that $T^{i'}$ is a strictly preferred offer. Because $T^{i'} \in \mathcal{T}^i(\mathcal{M}^i)$, by our assumption of consistency between menus and message spaces, there exists a $\pi^{i'} \in \Pi^i(\mathcal{M}^i)$ such that $T^{i'} = \pi^{i'}(\cdot|\mathcal{M}^i)$. Moreover, all elements of this set are unsent in the original equilibrium given that $T^{i'}$ is unsent in the candidate equilibrium. But if $T^{i'}$ yields a higher expected payoff to principal i in the menu-delegation game, then so too must $\pi^{i'} \in \Pi^i$ in the communication game, which implies the original strategies were not an equilibrium in the communication game. Hence, $\tilde{\sigma}^*$ is an equilibrium to the delegation game.

Lastly, we establish the payoff equivalence between the equilibrium of the communication game and the equilibrium with menus offers. The equilibrium allocation under the original equilibrium is

$$\mu_{\sigma^*}(d|\theta) = \int_{\pi \in \Pi} \int_{m \in \mathcal{M}} \pi(d|m) d\sigma_0^*(m|\theta, \pi) \prod_{i=1}^{N} d\sigma_i^*(\pi^i).$$

Expanding the right-hand side, we obtain equivalently

$$\int_{T \in \mathcal{T}(\mathcal{M})} \int_{\pi \in \Phi(T)} \int_{m \in \mathcal{M}} \pi(d|m) d\sigma_0^*(m|\theta, \pi) \left(\frac{\prod_{i=1}^{N} d\sigma_i^*(\pi^i)}{\int_{\hat{\pi} \in \Phi(T)} \prod_{i=1}^{N} d\sigma_i^*(\hat{\pi}^i)} \right) \int_{\hat{\pi} \in \Phi(T)} \prod_{i=1}^{N} d\sigma_i^*(\hat{\pi}^i).$$

Using our equilibrium constructions in the new game and a change of variables, this becomes

$$\int_{T \in \mathcal{T}(\mathcal{M})} \int_{\tau \in T} \tau(d) d\tilde{\sigma}_0^*(\tau|\theta, T) \prod_{i=1}^{N} d\tilde{\sigma}_i^*(T^i),$$

which is the equilibrium allocation in the delegation game, $\mu_{\tilde{\sigma}^*}(d|\theta)$. *Q.E.D.*

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