

Nonlinear Pricing with Average-price Bias

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Empirical evidence suggests that consumers facing complex nonlinear prices often make choices based on average (not marginal) prices. Given such behavior, we characterize a monopolist's optimal nonlinear price schedule. In contrast to the textbook setting, nonlinear prices designed for "average-price bias" distort consumption downward for consumers with the highest marginal utility and typically feature quantity premia rather than quantity discounts. These properties arise because the bias replaces consumer information rents with "curvature rents." Whether or not a monopolist prefers consumers with average-price bias depends upon underlying preferences and costs.

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The theory of optimal screening with privately-informed agents has been fruitfully applied to many settings, including optimal taxation (e.g., James Mirrlees (1971)), second-degree price discrimination (e.g., Michael Mussa and Sherwin Rosen (1978), Eric Maskin and John Riley (1984)), and regulation (e.g., David Baron and Roger Myerson (1982)). In each case, a principal (e.g., tax authority, monopolist, regulator) designs an optimal menu of choices (e.g., a nonlinear schedule or tariff) from which the agent selects. Importantly, it is assumed that the agent rationally equates marginal benefit with marginal cost.

Suppose instead the agent has difficulty discerning the relevant margin from the offered nonlinear schedule and wrongly focuses on average price. What is the optimal contract in light of this bias? We specialize this question to the setting of monopoly nonlinear pricing, supposing that buyers wrongly consume where marginal benefit equals average price – an error we call *average-price bias*. This paper's purpose is to understand how a strategic monopolist would exploit such a bias, and to explore the welfare implications of such behavior.

Several papers document the presence of average-price bias in decision making. In Koichiro Ito (2014), spatial discontinuities in electricity service areas in California are used to identify consumers' perceived prices; the paper finds that average price explains short-run demand variations far better than marginal price.

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Blake Shaffer (2018) finds additional supporting evidence in the British Columbia electricity market.¹ Laboratory evidence also documents a bias toward average prices. Using a sample of MBA students instructed to choose between two investment decisions (one tax-free and one taxed), Charles de Bartolome (1995) finds that most students fail to compute the correct marginal tax rate from the provided tables, relying instead on the average rate.

The evidence to date of average-price bias is provocative, if not persuasive.² While we provide a simple justification for such behavior based on the consumer's misspecification of the price schedule, our primary goal is not to rationalize average-price bias. Rather, our contribution is to take average-price bias as an exogenous feature of consumer decision-making and examine the implications. Combining the textbook model of monopoly nonlinear pricing with the assumption of average-price bias, we find the mechanics of optimal nonlinear pricing with average-price biased consumers are remarkably simple and in stark contrast to the standard model. First, in a world with biased consumers, the monopolist no longer faces a tradeoff between information-rent extraction and efficient consumption. Instead, because consumers effectively perceive the monopolist's total price as a linear function of consumption, private information does not generate information rents. That said, average-price biased consumers still earn consumer surplus through what we call *curvature* rents. The monopolist's desire to extract curvature rents rather than information rents leads to significant differences in the shape of optimal nonlinear prices. For example, in a simple quadratic preference model, the quantity discounts arising in Maskin and Riley (1984) are replaced with quantity premia. Additionally, because average-price bias represents both a cost and a benefit to the monopolist (i.e., curvature rents have replaced information rents), it is unclear whether a monopolist will prefer to face rational consumers or those with bias. Indeed, with constant unit costs, quadratic utility, and uniformly distributed preferences, we show that expected monopoly profits and consumer surplus are equal across the regimes, although the quantity allocations and price schedules are remarkably different. We then introduce cost nonlinearities to identify sources of variation in profit and surplus across the two settings, illuminating the consequences of average-price bias on cost pass through.

Literature review

Several papers incorporate models of decision-making from behavioral economics and psychology into standard contracting paradigms.³ Juan Carlos Carbajal and Jeffrey Ely (2016) and Jong Hee Hahn, et al. (2018), for example, introduce loss aversion and reference-dependent preferences (as in Botond Köszegi and Matthew Rabin (2006)) into a nonlinear pricing model. Our paper is in the same spirit but focuses on average-price bias. More loosely related, the literature

¹Shaffer (2018) includes a richer model of consumer heterogeneity, allowing for multiple behavioral types, and estimates 85% of behavioral types exhibit average-price bias.

²Jeffrey Liebman and Richard Zeckhauser (2004) provide additional evidence.

³Botond Köszegi (2014) provides a survey.

on inattention and salience provides evidence that consumers are inattentive to the correct prices: e.g., ignoring taxes that are not salient (Raj Chetty, Adam Looney and Kory Kroft (2009)), treating shipping costs on eBay differently from product prices (Tanjim Hossain and John Morgan (2006)), ignoring fixed-price offers that are dominated by eBay auctions (Elrike Malmendier and Young Han Lee (2011)), and for consumers who routinely purchase large sized products in grocery stores, ignoring dominating price promotions on small-sized packages of the same product (Sofronis Clerides and Pascal Courty (2017)).⁴

Two early papers are closely related to our investigation. Joel Sobel (1984) considers naive “price-taking” buyers who are identical to our average-price biased consumers, but assumes constant average costs and, importantly, that demand becomes weakly more elastic as consumption increases. In contrast, our focus is on the more natural setting where demand becomes less elastic moving downward along the demand curve and where average costs are not necessarily constant. Unlike Sobel (1984), whether bias raises or lowers profit, for example, depends upon the economics of utility and costs. The working paper of Liebman and Zeckhauser (2004) is also closely related.⁵ In their model, a monopolist sells to a consumer with one of two possible types, but is constrained to use piecewise-linear price schedules that originate from the origin. Given this restriction, the monopolist always prefers selling to biased consumers relative to unbiased ones. Such restricted schedules are suboptimal in the unbiased setting, however, as they leave rent to the low-type consumer. We emphasize that neither Sobel (1984) nor Liebman and Zeckhauser (2004) identify the key implication of average-price bias, that consumer *curvature rents* are substituted for information rents.⁶ Illuminating this substitution and its consequences for the firm’s optimal price schedule and consumer allocations are the primary contributions of this paper.

I. The model

A. Preferences

We adopt a textbook model of nonlinear pricing by a monopolist. The firm’s per-consumer cost is $C(q)$ is convex, continuously differentiable with $C(0) = 0$.

⁴Stefano Della Vigna (2009) surveys the evidence of limited attention, salience, and suboptimal heuristics in field experiments. A related literature considers “unawareness” in contracts where the agent may be unaware of contingencies and the principal may leave contracts incomplete, lest attention be drawn to the unconsidered dimensions. See Emel Filiz-Obay (2012) for an example in the context of insurance. In our setting, the consumer is unaware that the price schedule is nonlinear, but we do not consider the possibility the price schedule could be used to alter this belief.

⁵Liebman and Zeckhauser refer to manifestations of average-price bias as “ironing.” Emmanuel Farhi and Xavier Gabaix (2018) develop a general framework of optimal taxation with behavioral agents which is sufficiently general that it embeds the model of Liebman and Zeckhauser (2004), as well as a list of other well-studied heuristics and biases. We emphasize the specific implications of average-price bias, for which the analysis in Liebman and Zeckhauser (2004) is more directly related.

⁶In an uncirculated working paper that was discovered late in the editorial process, Phuong Hô (2017) takes a similar approach as ours to examine the implications of bias for nonlinear Ramsey-pricing. Phuong (2017) does not note the replacement of information rents for curvature rents, as we have emphasized.

The type- θ consumer's payoff is

$$u(q, \theta) - P(q),$$

where $q \in \mathcal{Q} \equiv [0, \bar{q}]$ is the quantity (or quality) consumed, $\theta \in \Theta \equiv [\theta_0, \theta_1]$ is the consumer's type which is distributed according to a differentiable, log-concave distribution, F , with density f , and $P(q)$ is the total payment made to the firm for q units.⁷ We make standard assumptions that u is thrice continuously differentiable, strictly concave in q , $u_\theta(q, \theta)$ is nonnegative and bounded, and $u_{q\theta}(q, \theta) > 0$. The familiar single-crossing property implies higher type consumers have a higher marginal value of consumption. We normalize the consumer's outside option to zero, $u(0, \theta) = 0$.

Given unit price p , we denote the type- θ consumer's Marshallian demand function as

$$D(p, \theta) \equiv \arg \max_{q \in \mathcal{Q}} u(q, \theta) - pq,$$

and the corresponding inverse-demand function as $p = u_q(q, \theta)$. Throughout we assume that the implied type- θ consumer's elasticity of demand,

$$\varepsilon(q, \theta) \equiv -\frac{u_q(q, \theta)}{u_{qq}(q, \theta)q} > 0,$$

is nonincreasing in q .⁸

In what follows, it is useful to consider the artificial setting in which a firm faces an unbiased consumer of known type θ , but is forced to offer a linear price. We will refer to this as a case of *perfect third-degree price discrimination* because the firm can perfectly condition its linear price on θ and offer $P(q|\theta) = p(\theta)q$ to a consumer of type θ . The firm's type- θ revenue in this case is

$$R(q, \theta) = qu_q(q, \theta),$$

and the assumption that $\varepsilon(q, \theta)$ is non-increasing in q implies that the firm's type- θ profit, $R(q, \theta) - C(q)$, is single-peaked.⁹ For simplicity, we assume throughout

⁷We assume \bar{q} strictly exceeds the efficient level of consumption for all types.

⁸A variety of demand functions satisfy this assumption including linear demand, log-linear demand, $p = a - bq^\delta$, and $p = a - b \ln q$. The reverse assumption that elasticities weakly increase in q implausibly requires that the demand curve is everywhere elastic and nowhere intersects either axis.

⁹Specifically, $\varepsilon_q(q, \theta) \leq 0$ implies

$$u_{qqq}q \leq -u_{qq} \left(1 + \frac{1}{\varepsilon(q, \theta)} \right),$$

hence

$$R_{qq}(q, \theta) = 2u_{qq} + u_{qqq}q \leq u_{qq} \left(1 - \frac{1}{\varepsilon(q, \theta)} \right),$$

which is negative for $\varepsilon(q, \theta) > 1$. Because $C'(q) \geq 0$, the unique solution to the first-order condition occurs in the strictly concave region.

that marginal revenue is increasing in θ , $R_{q\theta}(q, \theta) > 0$.¹⁰

B. Optimal nonlinear pricing without bias

As a benchmark, consider the textbook model of nonlinear pricing. The firm offers the consumer a nonlinear price schedule, $P : \mathcal{Q} \rightarrow \mathfrak{R}_+$, in order to maximize expected profits,

$$E_{\theta} [P(q(\theta)) - C(q(\theta))],$$

where

$$q(\theta) \in \arg \max_{q \in \mathcal{Q}} u(q, \theta) - P(q).$$

Define the consumer surplus for the unbiased setting as

$$U^*(\theta) \equiv \max_{q \in \mathcal{Q}} u(q, \theta) - P(q).¹¹$$

As is well known, an allocation $q(\theta)$ is implementable by some nonlinear price schedule if and only if $q(\cdot)$ is nondecreasing and $U^{*'}(\theta)$ is equal to $u_{\theta}(q(\theta), \theta)$ almost everywhere. The latter implies the consumer's expected surplus is

$$E_{\theta} [U^*(\theta)] = U^*(\theta_0) + E_{\theta} \left[u_{\theta}(q(\theta), \theta) \frac{1 - F(\theta)}{f(\theta)} \right],$$

which allows us to write expected profit (total surplus minus consumer surplus) as

$$E_{\theta} [\Pi^*(\theta)] = E_{\theta} \left[u(q(\theta), \theta) - C(q(\theta)) - \frac{1 - F(\theta)}{f(\theta)} u_{\theta}(q(\theta), \theta) - U^*(\theta_0) \right].$$

Assuming the standard regularity conditions that the integrand is single-peaked in q and exhibits increasing differences over (q, θ) , the pointwise maximum of q over θ is nondecreasing and represents the firm's optimal allocation:

$$(1) \quad q^*(\theta) = \arg \max_{q \in \mathcal{Q}} u(q, \theta) - C(q) - \frac{1 - F(\theta)}{f(\theta)} u_{\theta}(q, \theta).$$

The corresponding optimal nonlinear tariff, $P^*(q)$, is uniquely determined from an envelope condition and a participation constraint. The solution exhibits three properties:

- 1) *No surplus at the bottom.* The lowest-type consumer obtains zero surplus, $U^*(\theta_0) = 0$.

¹⁰Given $u_{q\theta} > 0$, it is sufficient for $R_{q\theta} > 0$ that $u_{qq\theta} \geq 0$.

¹¹We use * to indicate variables related to the unbiased consumers and an overline to denote the corresponding variables for biased consumers (e.g., biased consumer surplus is denoted $\bar{U}(\theta)$).

2) *No distortion at the top.* The highest-type consumes efficiently:

$$u_q(q^*(\theta_1), \theta_1) = C'(q^*(\theta_1)).$$

3) *Quantity discounts.* As noted by Maskin and Riley (1984), under mild assumptions (e.g., $C(q) = cq$ and u is quadratic) the price schedule exhibits quantity discounts.

These properties are typically absent when consumers suffer from average-price bias.

II. Average-price bias

A. Behavioral Assumption of Average-Price Bias

We assume that consumers with average-price bias equate marginal utility with the average price. Formally, for a given price schedule, $P(q)$, a biased consumer chooses output to satisfy

$$(2) \quad q \in \arg \max_{\tilde{q} \in \mathcal{Q}} u(\tilde{q}, \theta) - \frac{P(q)}{q} \tilde{q},$$

when such a choice exists, and $q = 0$ otherwise.¹²

We have in mind that the consumer searches via trial and error to find a choice of q for which her marginal utility equals her perception of the marginal price, $P(q)/q$, leading to a demand which satisfies (2). Suppose, for example, that the average-price biased consumer chooses $q = D(\tilde{p}, \theta)$ given a perceived unit price of \tilde{p} . If the resulting average price, $P(q)/q$, is greater than (resp., less than) the consumer's marginal utility, $u_q(q, \theta)$, we suppose that the consumer revises their belief to a slightly higher (resp., lower) unit price and reduces (resp., increases) consumption accordingly. This process repeats itself until reaching a consistent consumption level that satisfies (2) or consumption is reduced to zero. Formally, we can model such a tâtonnement by a process with steps indexed by t , requiring at each stage of consideration for which $q(t) > 0$,

$$\frac{dq(t)}{dt} = \lambda \left(u_q(q(t), \theta) - \frac{P(q(t))}{q(t)} \right) = \frac{\lambda}{q(t)} (R(q(t), \theta) - P(q(t))), \quad \lambda > 0,$$

and $\frac{dq(t)}{dt} = 0$ if $q(t) = 0$. For this simple adjustment rule, wherever the price schedule $P(q)$ intersects the revenue function $R(q, \theta)$ from below, the solution to (2) is asymptotically stable. Given that $P(\cdot)$ is nondecreasing and $R(q, \theta)$ is decreasing in the inelastic region, at least one such stable solution exists if

¹²We assume that if there are multiple solutions to (2), then the monopolist's preferred solution is selected. This is inconsequential because the firm's optimal price schedule never exhibits multiple, positive solutions.

$u_q(0, \theta) > P'(0)$. Moreover, if only one such locally stable solution exists, then the solution is globally stable when starting from any $q > 0$.¹³ If the price schedule $P(q)$ exceeds the revenue function $R(q, \theta)$ for all $q > 0$, then the globally stable solution to the adjustment process is $q = 0$.

A similar adjustment process can be founded as the result of misspecification bias by an otherwise rational consumer. For example, suppose the consumer repeatedly makes choices, using past, noisy consumption-price data to estimate a misspecified model of linear pricing to inform the current choice. In a previous version of this paper, we employed a simple normal-learning model as in Bengt Holmström (1999) in which the consumer incorrectly believes $P(q_t) = (p + \eta_t)q_t$, where η_t is normally-distributed noise. Each period the consumer uses past data to estimate p and then chooses q_t to equate marginal utility to the current estimate of unit price. Under some mild assumptions, consumption converges to a choice satisfying (2). More generally, this is an example of a Berk-Nash equilibrium as developed in Ignacio Esponda and Demian Pouzo (2016).¹⁴ In a Berk-Nash equilibrium, the agent chooses an optimal strategy given their beliefs, and the agent's beliefs put probability one on the (possibly misspecified) set of subjective distributions over consequences that are closest (in terms of Kullback-Leibler divergence) to the true distribution. Thus, the consumer's belief is required to be the best estimate given the linear misspecification error. Esponda and Pouzo (2016) provide a more general learning foundation for Berk-Nash equilibria which is in the spirit of the normal-learning model previously sketched. We find the notion that in the presence of misspecification bias, a consumer's choice should ultimately conform to a Berk-Nash equilibrium to be persuasive, and are unable to find an alternative behavioral rule that both encapsulates the empirical evidence of average-price bias and that is internally consistent.

B. Implementability and curvature rents

Given (2) is satisfied, we first determine which quantity allocations, $q : \Theta \rightarrow \mathcal{Q}$, are implementable and the price schedules that implement them. Compared to the setting with unbiased consumers, the answer is surprisingly straightforward. The key is to note that for any type θ , $q(\theta) > 0$ can be implemented by offering a price $P(q(\theta))$ such that

$$P(q(\theta)) = u_q(q(\theta), \theta)q(\theta).$$

The type- θ consumer can be induced to choose any $q > 0$, in isolation, by choosing a price schedule whose average price $P(q)/q$ is equal to marginal utility, $u_q(q, \theta)$, at q . Similarly, the monopolist can implement $q(\theta) = 0$ for any type θ by fiat.

¹³When there is a single $\bar{q} > 0$ for which $P(\bar{q}) = R(\bar{q}, \theta)$, it is necessarily the case that P intersects R from below at \bar{q} . Thus, $R(q, \theta) > P(q)$ for all $q \in (0, \bar{q})$ and $R(q, \theta) < P(q)$ for all $q > \bar{q}$. Consequently, convergence to \bar{q} arises for any starting point $q > 0$.

¹⁴Such an equilibrium is also known as a *restricted perceptions* learning equilibrium from the macroeconomics literature; see George Evans and Seppo Honkapohja (2001).

Not every allocation mapping, $q(\cdot) : \Theta \rightarrow \mathcal{Q}$ is implementable because it is not possible to implement the same $q > 0$ for two different types of consumers: e.g., $P(q)/q = u_q(q, \theta)$ implies $P(q)/q \neq u_q(q, \theta')$ for $\theta \neq \theta'$. Except for the infeasibility of implementing pooling allocations for some $q > 0$, however, there are no other restrictions. An allocation $\bar{q}(\cdot)$ is implementable under average-price bias if and only if it is fully separating over positive quantities. In such a case, the implementing price schedule is

$$(3) \quad \bar{P}(q) = \begin{cases} u_q(q, \bar{\vartheta}(q))q & \text{for all } q \in \bar{q}([\theta_0, \theta_1]) \text{ and } q > 0 \\ \infty & \text{otherwise} \end{cases}$$

where $\bar{\vartheta}(q) = \bar{q}^{-1}(q)$ for $q > 0$.

Because $u(q, \theta)$ is strictly concave, a consumer that equates marginal utility to average price necessarily obtains surplus. To illustrate, define the difference between a consumer's average utility and marginal utility by

$$\Delta(q, \theta) \equiv \frac{u(q, \theta)}{q} - u_q(q, \theta).$$

Because u is strictly concave, $\Delta(q, \theta) > 0$ for $q > 0$. Consumer surplus, given (3), is therefore

$$CS(q, \theta) = u(q, \theta) - P(q) = u(q, \theta) - u_q(q, \theta)q = \Delta(q, \theta)q.$$

We refer to $\Delta(q, \theta)q$ as the consumer's *curvature rent* as it reflects the surplus a consumer obtains due to diminishing marginal utility.¹⁵ Because of this curvature rent, a consumer's participation constraint is trivially satisfied. Indeed, if $\bar{q}(\theta_0) > 0$, then the lowest-type agent will earn a positive rent, unlike the standard nonlinear pricing model.

C. Optimal nonlinear pricing with average-price bias

Given (3), finding the optimal nonlinear price under average-price bias is straightforward. The firm maximizes expected profit, subject only to (3) and a separation requirement. We proceed by considering the relaxed program (ignoring separation) and verify that the solution does not induce pooling on positive outputs.

In the relaxed program, the firm chooses the allocation q pointwise in θ to maximize

$$E_\theta [P(q(\theta)) - C(q(\theta))] = E_\theta [u_q(q(\theta), \theta)q(\theta) - C(q(\theta))].$$

This is exactly the same program as determining the optimal allocation under

¹⁵We use "curvature" to evoke the idea that nonlinear utility is the source of consumer surplus. This differs from the mathematical definition of curvature used in differential geometry.

perfect third-degree price discrimination. That is, it is as if the firm can price conditionally on observed θ , but the firm is restricted to offering a linear price. Given our assumption that demand elasticities $\varepsilon(q, \theta)$ are nonincreasing in q , the solution to this program is determined by pointwise maximization. Given our assumption that marginal revenue is increasing in θ , the resulting allocation is fully separating among purchasing consumers.¹⁶

PROPOSITION 1: *The optimal allocation under average-price bias satisfies*

$$(4) \quad u_q(\bar{q}(\theta), \theta) - C'(\bar{q}(\theta)) = -u_{qq}(\bar{q}(\theta), \theta)\bar{q}(\theta)$$

for $\bar{q}(\theta) > 0$ and $\bar{q}(\theta) = 0$ otherwise; the optimal price schedule is

$$(5) \quad \bar{P}(q) = u_q(q, \bar{\vartheta}(q))q,$$

where $\bar{\vartheta}(q)$ is the inverse of $\bar{q}(\theta)$.

Several comparisons with the standard model are now available. First, the optimal allocation is independent of the type distribution, F . As suggested above, average-price bias eliminates the costs of incentive compatibility; information rents (indicated by the inverse hazard rate of the type distribution) are no longer earned by consumers.

Second, although there are no information rents, there are curvature rents. Indeed, one can rewrite the firm's expected profits as social surplus minus curvature rents,

$$E_{\theta} [u(q(\theta), \theta) - C(q(\theta)) - \Delta(q(\theta), \theta)q(\theta)],$$

and thus interpret the first-order condition as a marginal tradeoff between efficient consumption and reducing such rents.

Third, for all types that consume, consumption is inefficient – this is true even for the highest type. This everywhere-downward distortion reflects the tradeoff between efficiency and curvature rents, in sharp contrast with the standard nonlinear pricing model. Moreover, absent consumer fixed costs, the marginal purchasing consumer is the same as in the first best. In other words, on the extensive margin there is efficiency at the bottom.

Fourth, the lowest consumer type with average-price bias has (weakly) higher surplus than the same type in the unbiased setting. In the average-price bias setting, the lowest-type consumer earns $\bar{U}(\theta_0) = \Delta(\bar{q}(\theta_0), \theta_0)\bar{q}(\theta_0)$. This is positive if $\bar{q}(\theta_0) > 0$ which arises if $u_q(0, \theta_0) > C'(0)$. The monopolist leaves rents to a type- θ_0 consumer because it cannot extract the agent's surplus without raising the agent's perceived average price, leading to reductions in purchases.

¹⁶Differentiating (4),

$$\bar{q}'(\theta) = \frac{R_{q\theta}(\bar{q}(\theta), \theta)}{C''(\bar{q}(\theta)) - R_{qq}(\bar{q}(\theta), \theta)} > 0.$$

Fifth, quantity premia are typically optimal. Recall that if u is quadratic and unit cost is constant, then the optimal price schedule in the unbiased model exhibits quantity discounts. In contrast, consider the optimal price schedule when consumers have average-price bias. Letting $\bar{\vartheta}(q)$ represent the inverse allocation, substituting (4) into (5) reveals

$$(6) \quad \bar{P}(q) = C'(q)q - u_{qq}(q, \bar{\vartheta}(q))q^2.$$

If utility is quadratic and $C'''(q) \geq 0$, then $\bar{P}''(q) > 0$ and there are quantity premia rather than discounts. Intuitively, an increase in the true marginal price raises the perceived marginal price less for high-consumption agents than low-consumption agents, and so (ignoring terms above second-order) the firm will want to exploit this misperception by increasing marginal prices with consumption. Such quantity premia (or increasing block tariffs) are a common feature in electricity pricing, although prices are typically set without a singular profit motive.¹⁷ Quantity premia are also observed in a variety of profit-maximizing settings. Cellular phone plans historically featured higher marginal prices for minutes over a fixed monthly allocation, Michael Grubb (2015). Kenneth Manning, David Sprott and Anthony Miyazaki (1998) find quantity surcharges in grocery stores for 27 percent of the product brands offering packages of multiple sizes. And Yuri Levin, Mikhail Nediak and Andrei Bazhanov (2014) document airlines selling multiple seats at average prices which exceed the single-priced ticket.

We have exceptionally clear empirical predictions across the two settings. In the standard setting with rational consumers, nonlinear prices depend upon the underlying distribution of types and typically (e.g., with preferences well approximated by a quadratic function) exhibit quantity discounts with full efficiency for the highest-value consumer and no consumer surplus for the lowest types. In contrast, when consumers exhibit average-price bias, the optimal nonlinear price schedule does not depend upon the details of the type distribution, the schedule typically exhibits increasing marginal prices, consumption is distorted downward even for the highest-type consumer, and every consumer who purchases earns a curvature rent.

III. Welfare comparisons

We are interested in how profits, consumer surplus and welfare differ across the standard model and the average-price bias setting. If, for example, such a bias reduces a firm's profits, we might expect firms to mitigate its presence. If such a bias reduces social welfare, changes in policy may be advisable. Given our previous observation that optimal pricing under average-price bias is equivalent to perfect third-degree price discrimination, welfare comparisons between settings with unbiased and biased consumers are equivalent to comparisons between

¹⁷See Ito (2014) and Shaffer (2018) for examples.

second-degree and perfect third-degree price discrimination environments.

We have previously noted that the lowest-type consumer may earn a strictly positive rent with average-price bias if $\bar{q}(\theta_0) > 0$, implying that such consumers do better compared to the standard model. This preference may not hold for all consumers, however, as the slope of rent with respect to type may be flatter in the bias setting. There is also ambiguity in terms of profits. In the bias game, the monopolist eliminates all information rents, but at the cost of leaving curvature rents. It is not *a priori* obvious which effect will dominate.¹⁸ In order to explore these different effects, we investigate a model with quadratic preferences and uniformly-distributed heterogeneity. We then illustrate how variations from constant unit costs lead to very different welfare conclusions.

A. Results for Uniform-Quadratic Preferences

Consider the following *uniform-quadratic* model: (i) unit costs are constant, (ii) consumers have linear demand curves with identical slopes but heterogeneity in the intercepts, and (iii) consumer heterogeneity is distributed uniformly. Formally, assume $C(q) = cq$,

$$u(q, \theta) = \theta q - \frac{\gamma}{2} q^2,$$

$\gamma > 0$, and θ is uniformly distributed on $[\theta_0, \theta_1]$. Consequently, the θ -type consumer's demand curve is characterized by

$$p = \theta - \gamma q.$$

We also assume $\theta_1 > c > \theta_0$ so that it is inefficient to serve all types.

In the standard model without bias, using (1),

$$(7) \quad q^*(\theta) = \arg \max_{q \in \mathcal{Q}} u(q, \theta) - cq - (\theta_1 - \theta)q = \frac{1}{\gamma} \max \{0, 2\theta - \theta_1 - c\},$$

and the corresponding optimal nonlinear price is

$$(8) \quad P^*(q) = \frac{q}{2} \left(\theta_1 + c - \frac{\gamma}{2} q \right).$$

Consumer surplus for type θ is

$$U^*(\theta) = \max_{q \in \mathcal{Q}} u(q, \theta) - P^*(q) = \frac{1}{4\gamma} (\max \{0, 2\theta - \theta_1 - c\})^2,$$

¹⁸Sobel (1984) assumes $u(q, \theta) = \theta v(q)$, $C(q) = cq$ and the elasticity of demand is nondecreasing in q , which together imply that the firm prefers unbiased consumers. In a two-type consumer model, Liebman and Zeckhauser (2004) consider a restriction on price schedules which causes the firm to prefer bias. We demonstrate below that assuming demand elasticity magnitudes weakly decrease in q and allowing for unrestricted price schedules, the firm's preference for bias depends upon marginal utility and costs.

with ex ante consumer surplus

$$CS^* = E[U^*(\theta)] = \frac{(\theta_1 - c)^3}{24\gamma(\theta_1 - \theta_0)}.$$

Similarly, we compute the firm's profit from a type- θ consumer and its ex ante expected profit using (7) and (8):

$$\begin{aligned}\pi^*(\theta) &= P^*(q^*(\theta)) - cq^*(\theta), \\ \Pi^* &= E[\pi^*(\theta)] = \frac{(\theta_1 - c)^3}{12\gamma(\theta_1 - \theta_0)}.\end{aligned}$$

Note that $\Pi^* = 2CS^*$.

Now consider the average-price bias regime. The consumption allocation is given by (4), which specializes to

$$(9) \quad \bar{q}(\theta) = \arg \max_{q \in \mathcal{Q}} (\theta - \gamma q - c)q = \max \left\{ 0, \frac{\theta - c}{2\gamma} \right\},$$

and

$$(10) \quad \bar{P}(q) = cq + \gamma q^2.$$

The corresponding consumer surpluses and profits are

$$\begin{aligned}\bar{U}(\theta) &= \frac{1}{8\gamma} (\max\{0, \theta - c\})^2, \quad \overline{CS} = E[\bar{U}(\theta)] = \frac{(\theta_1 - c)^3}{24\gamma(\theta_1 - \theta_0)}, \\ \bar{\pi}(\theta) &= \frac{1}{4\gamma} (\max\{0, \theta - c\})^2, \quad \bar{\Pi} = E[\bar{\pi}(\theta)] = \frac{(\theta_1 - c)^3}{12\gamma(\theta_1 - \theta_0)}.\end{aligned}$$

As in the case of unbiased consumers, expected profits are twice the expected consumer surplus, $\bar{\Pi} = 2\overline{CS}$. As is evident, these ex ante values are also equal across the regimes. The expected benefit of eliminating information rents equals the expected cost of curvature rents.

PROPOSITION 2: *In the uniform-quadratic model,*

$$\bar{\Pi} = \Pi^*, \quad \overline{CS} = CS^*.$$

Although profits and consumer surpluses are equal across the regimes, we emphasize that the optimal allocations in the two regimes are very different. Furthermore, the marginal non-participating consumer type under average-price bias, $\theta = c$, coincides with the first-best marginal consumer, which is lower than the marginal consumer in the standard setting, $\theta = \frac{1}{2}(c + \theta_1)$. Figure 1 illustrates these differences.

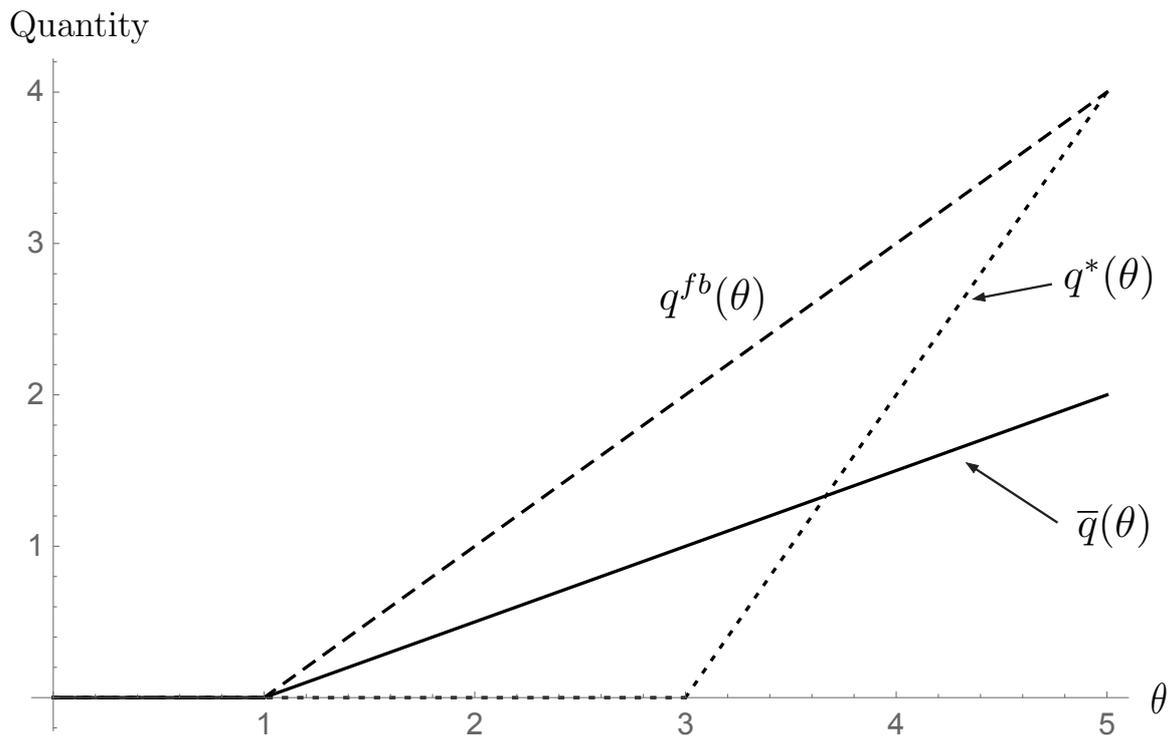


FIGURE 1. FIRST-BEST, STANDARD AND BIASED ALLOCATIONS

Note: Allocations generated using $\theta_0 = 3$, $\theta_1 = 5$, $c = 3.25$, $\gamma = 1$. First-best allocation, $q^{fb}(\theta) = \max\{0, \theta - 3.25\}$; second-degree price discrimination allocation, $q^*(\theta) = \max\{0, 2\theta - 8.25\}$; “average-price bias” allocation, $\bar{q}(\theta) = \frac{1}{2} \max\{0, \theta - 3.25\}$.

B. Welfare for nonlinear costs

The result in Proposition 2 is knife-edge and relies, in part, on constant unit costs. By changing the cost function to be either strictly convex or to contain a fixed cost, we will arrive at different comparisons across the regimes. If marginal costs are strictly increasing, for example, the firm can more easily pass on marginal costs to biased consumers (who spread the increases across all units) than to unbiased consumers (who correctly perceive the marginal price increase). As such, the introduction of strictly increasing marginal costs leads firms to prefer biased consumers over rational ones. The introduction of a fixed cost has the opposite effect. Because a fixed cost cannot be passed along to a biased consumer without leading to the perception that the marginal price is higher, the introduction of a fixed cost will lead firms to earn lower profits with biased consumers than rational ones.

To provide the precise details for the above argument, we consider a more general cost function $C(q) = c_0 + cq + \frac{c_2}{2}q^2$ and explore the relative impacts of $c_0 > 0$

and $c_2 > 0$ on surplus and profit.

Strictly convex costs: Suppose that $c_0 = 0$ but $c_2 > 0$; i.e., the cost function is strictly convex with $C(0) = 0$. The previous analysis easily extends to this setting, albeit with slightly more complex expressions. We summarize these in the following proposition.

PROPOSITION 3: *In the quadratic-uniform setting with $C(q) = cq + \frac{c_2}{2}q^2$, for all $c_2 \in (0, \gamma)$,*

$$\begin{aligned}\frac{d}{dc_2} (\bar{\Pi} - \Pi^*) &> 0, \\ \frac{d}{dc_2} (\overline{CS} - CS^*) &< 0, \\ \frac{d}{dc_2} (\bar{\Pi} + \overline{CS} - \Pi^* - CS^*) &> 0.\end{aligned}$$

An increase in c_2 lowers profit and consumer surplus in both regimes, but such an increase adversely impacts profits less (and consumer surplus more) under average-price bias than in the unbiased setting. An increase in c_2 by dc_2 raises marginal cost by $q dc_2$. In either setting, the firm increases the marginal price for higher outputs in response. In the standard setting,

$$P^*(q) = \frac{q}{2} \left(\theta_1 + c + \frac{c_2 - \gamma}{2} q \right),$$

and thus

$$\frac{d}{dc_2} P^{*'}(q) = \frac{q}{2};$$

half of the increase in marginal cost is passed along to unbiased consumers in higher marginal prices. In the case of average-price bias, however, increases in the price margin are less salient and a greater amount of the marginal cost increase is passed along:

$$\overline{P}(q) = cq + (\gamma + c_2)q^2,$$

and thus

$$\frac{d}{dc_2} \overline{P}'(q) = 2q.$$

Although marginal pass-through is higher, the consumer perceives only half as much pass-through of the marginal cost increase:

$$\frac{d}{dc_2} \left(\frac{\overline{P}(q)}{q} \right) = q.$$

It follows that a slight convexity in costs will tilt a firm toward a preference for consumers with average-price bias. The reverse is the case for changes in fixed

per-consumer costs.

Fixed costs: Suppose that marginal costs are constant, but there is a positive per-consumer fixed cost: $c_0 > 0$. We assume that c_0 is sufficiently small that it remains optimal to sell to a measure of types under either regime. With the introduction of a fixed cost, the optimal choice of the marginal purchasing type in each regime requires an additional condition. In the standard model, the marginal consumer is the type for which virtual surplus is zero; in the bias setting, the marginal consumer is the type for which the firm's profit is zero. Care must be taken in when deriving comparisons because both $q^*(\theta)$ and $\bar{q}(\theta)$ will exhibit positive jumps at these respective marginal types. Nonetheless, computing a general comparative static result for c_0 is straightforward if c_0 is not too large relative to the marginal surplus generated by sales to the highest type.¹⁹

PROPOSITION 4: *In the quadratic-uniform setting with $C(q) = c_0 + cq$, for c_0 sufficiently small*

$$\begin{aligned}\frac{d}{dc_0} (\bar{\Pi} - \Pi^*) &< 0, \\ \frac{d}{dc_0} (\overline{CS} - CS^*) &> 0, \\ \frac{d}{dc_0} (\bar{\Pi} + \overline{CS} - \Pi^* - CS^*) &< 0.\end{aligned}$$

The introduction of a small fixed cost reduces profitability and consumer surplus, but such a cost increase adversely impacts profits more (and consumer surplus less) under average-price bias than in the textbook setting. Fixed costs can be easily passed onto rational consumers with increases in the fixed component of a price schedule, but introducing a fixed increase in \bar{P} will be misunderstood as an increase in marginal price for consumers with average-price bias. Consequently, less of the fixed fee will be passed along to biased consumers which is evident in the price functions:

$$P^*(q) = \begin{cases} \frac{1}{2} (c_0 + q(\theta_1 + c) - \frac{\gamma}{2}q^2) & \text{if } q \geq \sqrt{\frac{2c_0}{\gamma}} \\ \infty & \text{otherwise,} \end{cases}$$

$$\bar{P}(q) = \begin{cases} cq + \gamma q^2 & \text{if } q \geq \sqrt{\frac{c_0}{\gamma}} \\ \infty & \text{otherwise.} \end{cases}$$

Half the fixed cost is passed along to rational consumers, yet it is absorbed by the firm serving average-price biased consumers.

¹⁹It is sufficient that $(2 + \sqrt{2})\sqrt{c_0\gamma} < \theta_1 - c$.

IV. Extensions

A few extensions suggest themselves. First, it is plausible that the population of consumers contains some who are unbiased and others that suffer from average-price bias. This is certainly consistent with the experimental work in de Bartolome (1996) and in recent work by Shaffer (2018) which identified distinct behavioral types. How should the firm alter its nonlinear price schedule if it must offer the same schedule to all consumers? Second, given a model of adaptive consumer learning with misspecification, how should the firm manipulate the consumer *on the path to a steady state*? In a fully dynamic model, the firm may find it optimal to depart from the steady-state contract in Proposition 1 to increase profits by manipulating the consumer's learning dynamics.

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MATHEMATICAL APPENDIX

Proof of Proposition 3:

The standard model: Maximizing virtual surplus pointwise yields

$$q^*(\theta) = \max \left\{ 0, \frac{2\theta - c - \theta_1}{\gamma + c_2} \right\}.$$

For participating types, the inverse allocation is

$$\vartheta^*(q) = \frac{c + q(\gamma + c_2) + \theta_1}{2}.$$

The marginal consumer's type is determined by $q^*(\theta^*) = 0$, if such a type exists, or $\theta^* = \theta_0$, otherwise.

$$\theta^* = \max \left\{ \theta_0, \frac{c + \theta_1}{2} \right\}.$$

The optimal price schedule is obtained by integrating the consumer's marginal utility function:

$$P^*(q) = \int_0^q u_q(x, \vartheta^*(x)) dx = \frac{q}{2} \left(\theta_1 + c + \frac{c_2 - \gamma}{2} q \right).$$

Direct computations yield

$$U^*(\theta) = u(q^*(\theta), \theta) - P^*(q^*(\theta)) = \frac{(c + \theta_1 - 2\theta)^2}{4(\gamma + c_2)}.$$

$$\pi^*(\theta) = u(q^*(\theta), \theta) - C(q^*(\theta)) - U^*(\theta) = \frac{(c + 2\theta - 3\theta_1)(c + \theta_1 - 2\theta)}{4(\gamma + c_2)}.$$

Integrating over the interval $[\theta^*, \theta_1]$ yields

$$CS^* = \int_{\theta^*}^{\theta_1} U^*(\theta) d\theta = \frac{(\theta_1 - c)^3}{24(\gamma + c_2)(\theta_1 - \theta_0)},$$

$$\Pi^* = \int_{\theta^*}^{\theta_1} \pi^*(\theta) d\theta = \frac{(\theta_1 - c)^3}{12(\gamma + c_2)(\theta_1 - \theta_0)}.$$

Average-price bias: For all purchasing consumers, the optimal allocation satisfies (4)

$$\bar{q}(\theta) = \max \left\{ 0, \frac{\theta - c}{2\gamma + c_2} \right\},$$

with associated inverse allocation for $q > 0$

$$\bar{\vartheta}(q) = c + (c_2 + 2\gamma)q.$$

Because $c > \theta_0$, the marginally-participating type is $\bar{\theta} = c$. Following (6), the optimal price schedule is

$$\bar{P}(q) = cq + (\gamma + c_2)q^2.$$

Consumer surplus for type θ is

$$\bar{U}(\theta) = u(\bar{q}(\theta), \theta) - P(\bar{q}(\theta)) = \frac{\gamma(\theta - c)^2}{2(2\gamma + c_2)^2},$$

and profit is

$$\bar{\pi}(\theta) = P(\bar{q}(\theta)) - C(\bar{q}(\theta)) = \frac{(\theta - c)^2}{2(2\gamma + c_2)}.$$

Integrating over the interval $[\bar{\theta}, \theta_1]$ yields

$$\bar{CS} = \int_{\bar{\theta}}^{\theta_1} \bar{U}(\theta) d\theta = \frac{\gamma(\theta_1 - c)^3}{6(\theta_1 - \theta_0)(2\gamma + c_2)},$$

$$\bar{\Pi} = \int_{\bar{\theta}}^{\theta_1} \bar{\pi}(\theta) d\theta = \frac{(\theta_1 - c)^3}{6(\theta_1 - \theta_0)(2\gamma + c_2)}.$$

Combining the surplus expressions for the two regimes, we obtain

$$\bar{\Pi} - \Pi^* = \frac{c_2(\theta_1 - c)^3}{12(\theta_1 - \theta_0)(2\gamma + c_2)(\gamma + c_2)},$$

$$\bar{CS} - CS^* = -\frac{c_2^2(\theta_1 - c)^3}{24(\theta_1 - \theta_0)(2\gamma + c_2)^2(\gamma + c_2)},$$

$$(\bar{\Pi} + \bar{CS}) - (\Pi^* + CS^*) = \frac{c_2(4\gamma + c_2)(\theta_1 - c)^3}{24(\theta_1 - \theta_0)(2\gamma + c_2)^2(\gamma + c_2)}.$$

Differentiating with respect to c_2 ,

$$\frac{d}{dc_2} (\bar{\Pi} - \Pi^*) = \frac{(2\gamma^2 - c_2^2)(\theta_1 - c)^3}{12(\theta_1 - \theta_0)(2\gamma + c_2)^2(\gamma + c_2)^2},$$

$$\frac{d}{dc_2} (\bar{CS} - CS^*) = \frac{c_2(c_2^2 - 2c_2\gamma - 4\gamma^2)(\theta_1 - c)^3}{24(\theta_1 - \theta_0)(2\gamma + c_2)^3(\gamma + c_2)^2},$$

$$\frac{d}{dc_2} (\bar{\Pi} + \overline{CS} - \Pi^* - CS^*) = \frac{(8\gamma^3 - 6c_2^2\gamma - c_2^3)(\theta_1 - c)^3}{24(\theta_1 - \theta_0)(2\gamma + c_2)^3(\gamma + c_2)^2}.$$

Noting that $\theta_1 > c$, we have the following relationships:

$$\frac{d}{dc_2} (\bar{\Pi} - \Pi^*) > 0 \iff c_2 < \sqrt{2}\gamma,$$

$$\frac{d}{dc_2} (\overline{CS} - CS^*) < 0 \iff c_2^2 - 2c_2\gamma - 4\gamma^2 < 0,$$

$$\frac{d}{dc_2} (\bar{\Pi} + \overline{CS} - \Pi^* - CS^*) > 0 \iff 8\gamma^3 > c_2^2(c_2 + 6\gamma).$$

By assumption, $c_2 < \gamma$, and all three inequalities are satisfied. \square

Proof of Proposition 4:

The standard model: As in the linear-cost case, participating consumers choose

$$q^*(\theta) = \frac{2\theta - c - \theta_1}{\gamma},$$

with inverse allocation

$$\vartheta^*(q) = \frac{c + q\gamma + \theta_1}{2}.$$

The marginal consumer type is determined by setting virtual surplus to zero:

$$u(q^*(\theta^*), \theta^*) - cq^*(\theta^*) - (\theta_1 - \theta)q^*(\theta^*) = c_0.$$

Solving for θ^* , we obtain

$$\theta^* = \frac{c + \theta_1 + \sqrt{2\gamma c_0}}{2};$$

$\theta^* < \theta_1$ by assumption. The marginal type chooses $q^*(\theta^*) = \sqrt{\frac{2c_0}{\gamma}}$. The optimal allocation is

$$q^*(\theta) = \begin{cases} \frac{2\theta - c - \theta_1}{\gamma} & \text{if } \theta \geq \theta^* \\ 0 & \text{otherwise.} \end{cases}$$

We compute the price schedule by integrating the consumer's marginal utility:

$$P^*(q) = u(q^*(\theta^*), \theta^*) + \int_{q^*(\theta^*)}^q u_q(x, \vartheta^*(x)) dx = \frac{1}{2}c_0 + \frac{q}{2}(\theta_1 + c) - \frac{\gamma}{4}q^2.$$

Type- θ consumer surplus and profit are therefore

$$U^*(\theta) = u(q^*(\theta), \theta) - P^*(q^*(\theta)) = \frac{(2\theta - \theta_1 - c)^2}{4\gamma} - \frac{c_0}{2},$$

$$\pi^*(\theta) = u(q^*(\theta), \theta) - C(q^*(\theta)) - U^*(\theta) = \frac{1}{4\gamma} ((3\theta_1 - 2\theta - c)(2\theta - \theta_1 - c)) - \frac{c_0}{2}.$$

Integrating over the interval $[\theta^*, \theta_1]$ yields

$$CS^* = \int_{\theta^*}^{\theta_1} U^*(\theta) d\theta = \frac{4\sqrt{2}(c_0\gamma)^{\frac{3}{2}} + (\theta_1 - c)^3 - 6c_0\gamma(\theta_1 - c)}{24\gamma(\theta_1 - \theta_0)},$$

$$\Pi^* = \int_{\theta^*}^{\theta_1} \pi^*(\theta) d\theta = \frac{4\sqrt{2}(c_0\gamma)^{\frac{3}{2}} + (\theta_1 - c)^3 - 6c_0\gamma(\theta_1 - c)}{12\gamma(\theta_1 - \theta_0)}.$$

Average-price bias: Because c_0 doesn't impact the intensive margin, the optimal allocation for participating consumers under average-price bias satisfies

$$\bar{q}(\theta) = \frac{\theta - c}{2\gamma},$$

with associated inverse allocation

$$\bar{\vartheta}(q) = c + 2\gamma q.$$

The profit and surplus earned for participating types are

$$\bar{\pi}(\theta) = \frac{(\theta - c)^2}{4\gamma} - c_0,$$

$$\bar{U}(\theta) = \frac{(\theta - c)^2}{8\gamma}.$$

The marginal consumer is the type for which $\bar{\pi}(\bar{\theta}) = 0$,

$$\bar{\theta} = c + 2\sqrt{c_0\gamma};$$

$\bar{\theta} < \theta_1$ by assumption. The marginal type chooses $\bar{q}(\bar{\theta}) = \sqrt{\frac{c_0}{\gamma}}$. Integrating over the interval $[\bar{\theta}, \theta_1]$ yields

$$\overline{CS} = \int_{\bar{\theta}}^{\theta_1} \bar{U}(\theta) d\theta = \frac{(\theta_1 - c)^3 - 8(c_0\gamma)^{\frac{3}{2}}}{24\gamma(\theta_1 - \theta_0)},$$

$$\overline{\Pi} = \int_{\bar{\theta}}^{\theta_1} \bar{\pi}(\theta) d\theta = \frac{16(c_0\gamma)^{\frac{3}{2}} + (\theta_1 - c)^3 - 12c_0\gamma(\theta_1 - c)}{12\gamma(\theta_1 - \theta_0)}.$$

Combining the surplus expressions for the two regimes yields

$$\begin{aligned}\bar{\Pi} - \Pi^* &= \frac{c_0((8 - 2\sqrt{2})\sqrt{c_0\gamma} - 3(\theta_1 - c))}{6(\theta_1 - \theta_0)}, \\ \overline{CS} - CS^* &= \frac{c_0(3(\theta_1 - c) - (4 + 2\sqrt{2})\sqrt{c_0\gamma})}{12(\theta_1 - \theta_0)}, \\ (\bar{\Pi} + \overline{CS}) - (\Pi^* + CS^*) &= \frac{c_0((4 - 2\sqrt{2})\sqrt{c_0\gamma} - (\theta_1 - c))}{4(\theta_1 - \theta_0)}.\end{aligned}$$

Differentiating with respect to c_0 ,

$$\begin{aligned}\frac{d}{dc_0} (\bar{\Pi} - \Pi^*) &= \frac{(4 - \sqrt{2})\sqrt{c_0\gamma} - (\theta_1 - c)}{2(\theta_1 - \theta_0)}, \\ \frac{d}{dc_0} (\overline{CS} - CS^*) &= \frac{(\theta_1 - c) - (2 + \sqrt{2})\sqrt{c_0\gamma}}{4(\theta_1 - \theta_0)}, \\ \frac{d}{dc_0} (\bar{\Pi} + \overline{CS} - \Pi^* - CS^*) &= \frac{3(2 - \sqrt{2})\sqrt{c_0\gamma} - (\theta_1 - c)}{4(\theta_1 - \theta_0)}.\end{aligned}$$

Noting that $\theta_1 > c$,

$$\begin{aligned}\frac{d}{dc_0} (\bar{\Pi} - \Pi^*) < 0 &\iff (4 - \sqrt{2})\sqrt{c_0\gamma} < \theta_1 - c, \\ \frac{d}{dc_0} (\overline{CS} - CS^*) > 0 &\iff (2 + \sqrt{2})\sqrt{c_0\gamma} < \theta_1 - c, \\ \frac{d}{dc_0} (\bar{\Pi} + \overline{CS} - \Pi^* - CS^*) < 0 &\iff 3(2 - \sqrt{2})\sqrt{c_0\gamma} < \theta_1 - c,\end{aligned}$$

which are satisfied for c_0 sufficiently small. □