

FINANCIAL INTERMEDIATION AND FLIGHTS TO SAFETY

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Abstract

I develop a continuous-time game between a population of investors and an intermediary whose type is private information and whose portfolio allocation is neither observable nor contractible. I define and characterize a sequential equilibrium of the game and solve for a Markovian equilibrium where investors' posterior beliefs are the key state variable. In my model, demand for riskless assets undergoes dramatic changes that resemble the episodes of flight to safety observed during financial crises. I show that a risk-neutral intermediary chooses a portfolio that minimizes risk when beliefs are near the threshold below which the intermediary is terminated.

1 INTRODUCTION

Many existing contributions have interpreted countercyclical demand for riskless assets as reflecting time-varying costs of external financing. However, evidence from the behavior of the banking system exposes the empirical gaps of this theory. Indeed, banks can currently rely on virtually unlimited funds from central banks, but they are nevertheless holding record-high levels of excess reserves.

In this paper, I show that episodes of sudden portfolio re-allocation towards riskless securities are a robust feature of a game of incomplete information between investors and an intermediary. Specifically, I model a continuous time interaction between a population of investors and a large intermediary. Investors can purchase riskless securities and let the intermediary manage the remaining part of their capital. In exchange of

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its services, the intermediary receives a proportional fee, which is determined endogenously. Besides the risk-free asset, the intermediary has also access to a risky investment opportunity whose expected return (above the risk-free rate or below the risk-free rate) is its private information and is associated with the intermediary's type (good or bad). Investors do not observe the type of the intermediary or its portfolio allocation, but can observe a signal that coincides with the returns generated by the intermediary.

The key state variable of the model is the intermediary's reputation, i.e. investors' posterior beliefs that the intermediary is of the good type. The law of motion of reputation is endogenous and depends on the entire history of the intermediary's performance. This provides incentives for the intermediary to choose a portfolio allocation between riskless and risky assets in order to manipulate the distribution of the signal and, consequently, the evolution of its reputation. Clearly, in equilibrium, investors will rationally anticipate the strategy of the intermediary, neutralizing signal manipulation and giving rise to instances of signal jamming of the type discussed in Stein (1989).

The model generates episodes of hoarding of riskless assets that bear a qualitative similarity to those observed during financial crises (see Acharya and Merrouche, 2012 Ashcraft et al., 2011 and Beber et al., 2009 for empirical investigations of liquidity hoarding during the latest crisis). Hoarding, in my model, takes place in the asset side of the intermediary's balance sheet, as soon as it heavily invests in riskless securities in order to control the evolution of its reputation.

Contrary to existing literature, liquidity hoarding, in my model, is not directly associated with changes in exogenous state variables. Indeed, demand for riskless assets, in a Markovian equilibrium, negatively depends on the intermediary's reputation, which evolves endogenously as investors learn by observing returns. Therefore, hoarding will typically happen after a sequence of particularly low returns.

The equilibrium of the model is characterized by investors learning the type of the intermediary and by intermediaries of different types choosing the same portfolio allocation. Investors use Bayes' rule to update their beliefs about the type of the intermediary after observing the entire history of returns. As long as investors, who are risk-neutral, expect the intermediary to deliver returns above the risk-free rate, the intermediary will survive and manage investors' capital. However, when beliefs fall below a threshold, the intermediary will be terminated, since investors will prefer to hold riskless securities. The bad intermediary, therefore, imitates the portfolio allocation of the good one. Indeed, if this was not the case, the two types would generate signals

with different volatility and, hence, they could be immediately separated, leading to the termination of the bad type.

I provide analytical results showing that, when reputation is low enough and the risk of early termination is high, the intermediary, rather than gambling for resurrection, will become risk averse and minimize the risk in its portfolio. This outcome is in stark contrast with most of the existing literature on risk-shifting and is linked to the learning dynamics and the infinite horizon of the model¹. Indeed, near the threshold below which the intermediary is terminated, the expected excess return on the intermediated asset is approximately zero, according to investors' beliefs. Therefore, a small amount of risk is sufficient for the good intermediary to exceed, on average, investors' expectations and, thus, to ensure that beliefs acquire a positive drift. Moreover, if a negative shock to returns, and hence to reputation, is realized, the good intermediary will be terminated and it will lose the option to signal its type through returns. Therefore, the optimal choice is to minimize the amount of portfolio risk.

In order to highlight the key mechanisms behind the model's results and test their robustness, I develop some variations and extensions of the basic set-up. I show that lack of commitment about the intermediary's trading strategy is crucial in delivering the results of the paper. Indeed, whenever the intermediary can ex-ante commit to a portfolio allocation, no hoarding happens, since the signal-manipulation incentives disappear. Moreover, I show that hoarding can be observed also in those situations in which the intermediary cannot control the law of motion of investors' capital. This remarks that the relevant incentives entirely come from the informational asymmetries in the model.

To account for the recurring nature of liquidity hoarding episodes, I also develop a model where the type of the intermediary evolves stochastically. Indeed, if the type is fixed, beliefs will converge with probability one to either zero or one, reflecting the fact that beliefs must be asymptotically correct. This would imply that, if we consider a partial equilibrium model with a single intermediary, liquidity hoarding episodes are transient and will disappear in the long run. By allowing the intermediary's type to be time-varying, episodes of liquidity hoarding may appear with a cyclical pattern. My analysis suggests that the model is robust to this extension.

¹Panageas and Westerfield (2009) have shown that the risk-taking incentives of convex payoff schemes may disappear if an asset manager has an infinite investment horizon.

The rest of the paper is organized as follows. In Section 2, I compare this paper to the existing literature. I then introduce the model in Section 3, where I also give the definition of sequential equilibrium and provide a characterization. In Section 4, I characterize and solve for a Markovian equilibrium where the state variables are the capital of investors and the reputation of the intermediary. Section 5 contains variations and extensions of the model. Finally, Section 6 concludes. All proofs are collected in Appendix A, while Appendix B contains additional plots.

2 RELATED LITERATURE

The relation between investors' confidence, reactions to news and liquidity hoarding has always been one of the reasons provided to explain historical patterns in demand for liquidity and riskless assets. Friedman and Schwartz (1971) wrote the following about the Great Depression:

“Excess reserves, which in January 1931 had for the first time since 1929, when data became available, reached the \$100 million level and had then declined as confidence was restored, again rose, reaching a level of \$125-\$130 million in June and July. Once bitten, twice shy, both depositors and bankers were bound to react more vigorously to any new eruption of bank failures or bank difficulties than they did in the final months of 1930.”

A sharp rise in excess reserves has also been observed during the recent financial crisis (see Figure 7 in Appendix B).

My model is able to rationalize these facts by appealing to one simple friction, i.e. the information asymmetry about the type of the intermediary and its portfolio allocation. This is in contrast with the banking and macro-finance literature, which usually appeals to moral hazard or unobservable cash flows as key frictions in financial markets (Bernanke and Gertler, 1989; Bernanke et al., 1999; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010).

A similar comparison can be made with the literature on intermediary asset pricing with frictions. The main result of this literature is the pro-cyclical variation of demand for risky assets and the key driving force is the presence of leverage constraints or market incompleteness (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012, 2013). In my model, the driving forces behind the same kind of result are simply the

inter-temporal incentives of intermediaries to manipulate the distribution of returns in bad times in order to avoid risks of reputation losses.

Flight to quality and liquidity hoarding have been subject to discussion under different perspectives in previous theoretical literature. In Bernanke et al. (1996) the key driver of these phenomena are time-varying agency costs. In Vayanos (2004) it is stochastic volatility coupled with risk of early liquidation of the intermediary. For Caballero and Krishnamurthy (2008) the reason behind liquidity hoarding lies in exogenous liquidity shocks and knightian uncertainty. Acharya et al. (2012), Acharya et al. (2007) and Acharya and Skeie (2011) focus instead on the role of external financing frictions and the variation in credit risk, investment opportunities and roll-over risk, respectively, to explain variation in demand for safe assets. Finally, in Gale and Yorulmazer (2013) the absence of contingent markets for the provision of liquidity creates incentives for investors to occasionally hoard liquidity for precautionary or speculative reasons.

This paper is also closely related to the literature on reputation in dynamic games. Fudenberg and Levine (1992) provide the classic benchmark in a discrete time model, where they derive bounds for the equilibrium payoffs of the rational large player, as its discount rate tends to zero. Faingold and Sannikov (2011) extend this model to continuous time and obtain a sharper characterization of the equilibrium payoffs.

These models share the common feature that reputation does not carry a premium *per se*, but it just affects the actions of players. Board and Meyer-ter Vehn (2013) model a reputation game between consumers and a firm whose product quality is private information. The firm enjoys reputational dividends, as reputation affects the market price of the product. Similarly, in my paper, reputation will directly affect the cash flow of the intermediary through the fee paid by investors.

My approach also bears connections with the recent literature of continuous time games. However, besides the fact that my set-up differs from the typical principal-agent problem, an additional point of departure is the fact that, in my model, the intermediary controls also the volatility of the signal, and not only its drift. Holmstrom and Milgrom (1987) is the first contribution to the literature of continuous time games where the signal follows a diffusion process whose probability distribution is controlled by an agent. Later, Sannikov (2007) and Sannikov (2008) extended this framework to more general settings and showed that the key state variable in this class of models is the continuation value of the agent. Indeed, this is the relevant variable to take into consideration in the provision of dynamic incentives. DeMarzo and Sannikov (2006) apply

these techniques to the optimal design of securities of firms and He (2009) extends the analysis to firms growing as a geometric Brownian motion whose drift is controlled by the agent.

The type of economic interaction explored in this paper is very similar to the typical setting in the delegated asset management literature. Hugonnier and Kaniel (2010) is probably the closest, in spirit, to my paper. Indeed, they characterize the optimal portfolio allocation of an asset manager who has to dynamically raise funds from external investors. However, their implicit assumption is that the asset manager is able to ex-ante commit to a dynamic trading strategy and that, in choosing it, the asset manager takes into account how the strategy will affect the dynamic incentives of investors. This leads to the possibility of sub-game imperfection. In my model, the equilibrium is sub-game perfect and is characterized by imperfect information and by the impossibility of the asset manager to commit to a state contingent trading strategy.

Vayanos (2004) explores the time variation of liquidity premia in a general equilibrium model with intermediation. Stochastic volatility is the main state variable of the model and intermediaries are subject to early termination according to an exogenously specified rule, that is based only on the short-term performance of the intermediary. In my model, time-varying demand for safe assets does not depend on stochastic volatility, but only on the endogenous evolution of reputation. Furthermore, in my set-up, the termination rule is endogenous and depends on the entire history of the intermediary's performance.

Other papers (Basak and Cuoco, 1998; Cuoco and Kaniel, 2011; Kaniel and Kondor, 2013) have analyzed the asset pricing implications of delegated asset management. Despite I limit the analysis to a partial equilibrium environment, I provide a micro-foundation to the asset manager's compensation when he faces a set of risk neutral and competitive investors. This makes the fees paid to the asset manager naturally dependent on the historical performance of the funds, while in previous contributions fees were either fixed or assumed to depend on past performance through an exogenously specified function. Fees, in my model, represent a premium that the intermediary receives for its reputation.

Indeed, contributions in the active portfolio management literature, e.g. Berk and Green (2004), showed that the presence of competitive capital markets is able to dramatically change the empirical asset pricing implications of models with intermediation. While, in Berk and Green (2004), equilibrium is achieved through changes in

the size of the fund and decreasing returns to scale, in my model adjustment happens through the price, i.e. the fee, paid by investors to access the fund's services.

3 MODEL

I consider a partial equilibrium model where a large intermediary and a population of investors are present. Investors have the choice of either investing in a riskless asset or let an intermediary manage their capital. The intermediary manages investors' fund in exchange of a proportional fee that, in equilibrium, constitutes a premium for its reputation. The intermediary can be one of two types and investors cannot observe the type of the intermediary.

3.1 INVESTMENT OPPORTUNITIES

There are three assets in the economy. One of them is a riskless asset whose price, S_t^0 follows the process

$$\frac{dS_t^0}{S_t^0} = r dt, \quad r > 0,$$

for a given S_0^0 .

The other two assets are risky. The price of the first risky asset, defined as the good asset, evolves according to

$$\frac{dS_t^1}{S_t^1} = \mu_1 dt + \sigma dW_t,$$

for a given S_0^1 , while the second asset, defined as the bad asset, follows

$$\frac{dS_t^2}{S_t^2} = \mu_2 dt + \sigma dW_t,$$

for a given S_0^2 . Note that both risky assets have the same volatility and are exposed to the same Weiner process.

The following assumption provides the reason why the two assets are defined as good and bad.

Assumption 1. *In an economy where no individual is risk-lover, the bad asset is dominated by the other two, i.e.*

$$\mu_1 > r > \mu_2.$$

Investors can only hold positive amounts of the riskless asset and let an intermediary manage the remaining part of their portfolio. The intermediary can be one of two types, good or bad. Both types can invest in the riskless security, but they differ in terms of the risky investment opportunity that is available to them. Indeed, the good intermediary can invest in the good asset but not in the bad one, while the bad intermediary can invest in the bad asset, but not in the good one.

3.2 PLAYERS

The economy is populated by a unit measure of small and atomistic investors so that none of them, taken individually, is able to influence the incentives of the intermediary.

Investors are risk neutral and discount future utility at rate r , thus evaluating a consumption process $(c_t)_{t \geq 0}$, conditional on the history of the game up to time t , according to

$$\mathbb{E} \left[\int_0^\infty e^{-rs} c_s ds \middle| \mathcal{F}_t^I \right].$$

where $(\mathcal{F}_t^I)_{t \geq 0}$ is the investors' information filtration, which will be fully characterized in Section 3.3.

Investors are endowed with a stock of capital K_0 at time zero. At each time t , they decide which fraction of their capital that will be invested in riskless assets. The remaining part will be managed by the intermediary from time t to time $t + dt$. Let $\theta_t \in [0, 1]$ be the fraction of capital that investors delegate to the intermediary and let dR_t be the return generated by the intermediated assets. Then, the law of motion of investors' capital is

$$\frac{dK_t}{K_t} = (1 - \theta_t)rdt + \theta_t(dR_t - f_t dt). \quad (1)$$

f_t is the proportional fee received by the intermediary for managing capital from t to $t + dt$. The fee is paid up-front at time t and investors will receive the full return, dR_t , generated by the intermediated assets. The fee will be endogenously determined by a break-even condition for investors. The existence of fees will provide incentives for the bad intermediary to keep operating despite the socially wasteful investment opportunity. The intermediary cannot save on its own account and immediately consumes all the fees it receives. Therefore fees have to be non-negative.

Once investors decide the amount of assets that the intermediary will manage, the latter will be free to choose a portfolio allocation between the riskless asset and the

risky investment opportunity available. In this sense, the portfolio allocation of the intermediary is not contractible.

Indeed, the intermediary will choose a process for the fraction of risky assets in its portfolio, which I call portfolio risk and denote with $(i_t)_{t \geq 0}$, after the investors' capital has been transferred to the intermediary and after fees have been collected. Investors cannot directly observe the choice of portfolio risk. Moreover, even if it was revealed, there would be no mechanism to impose a penalty on an intermediary that deviates from a previously agreed portfolio risk.

The intermediary is risk neutral and discounts future consumption at rate ρ . Given a process for the assets under management, $(\theta_t K_t)_{t \geq 0}$, the intermediary receives a lifetime utility, conditional on the history of the game up to time t , of

$$\mathbb{E} \left[\int_0^\infty e^{-\rho s} \theta_s K_s f_s ds \middle| \mathcal{F}_t \right]. \quad (2)$$

where $(\mathcal{F}_t)_{t \geq 0}$ represents the intermediary's information filtration (more on this in Section 3.3).

Therefore, given an investment strategy $(i_t)_{t \geq 0}$ for the intermediary, the return on the assets under management is described by

$$dR_t = r dt + i_t (h \mu_1 + (1 - h) \mu_2 - r) dt + i_t \sigma dW_t. \quad (3)$$

$h \in \{0, 1\}$ is random variable, drawn at time zero, that defines the type of the intermediary. $h = 1$ indicates that the intermediary is of the good type and, thus, has access to the good investment opportunity. $h = 0$ means that the intermediary is bad and can therefore invest in the bad asset.

Given the risk neutrality of the players, it is necessary that the portfolio risk, i_t , lies in a compact set in order to guarantee the existence of a solution to the portfolio allocation problem, at least in the limiting case of perfect information with a good intermediary. Hence, I assume the following.

Assumption 2. *The intermediary is subject to a minimum and a maximum portfolio risk constraints, i.e.*

$$i_t \in [\hat{i}, \bar{i}] \quad \forall t.$$

with $0 < \hat{i} < \bar{i}$.

A positive upper bound for i_t can be interpreted as a limit on risk-free borrowing. While a no-short selling constraint would impose $\hat{i} = 0$, this model actually requires the slightly stronger assumption that \hat{i} is strictly positive. The motivation is related to the learning part of the model that will be described in Section 3.4 and in Proposition 2 in particular. To explain briefly, the fact that the intermediary has always to hold some risk in its portfolio guarantees that returns are always stochastic and investors will always learn from them. This avoids theoretical complications in the dynamic game when considering deviation to a portfolio risk $i'_t > 0$ when the equilibrium risk is $i_t = 0$. In these cases, the equilibrium returns are deterministic and investors do not learn from them in equilibrium. If, however, the intermediary deviates, then it will provide investors with a return that, with probability 1, differs from the equilibrium (deterministic) one. Strong assumptions would then be needed to pin down the off-equilibrium beliefs of investors.

A further assumption will be needed to ensure the existence of a solution to the model in the perfect information case. As it will become clear in Section 4.1, when the intermediary is of the good type and there is perfect information, in equilibrium fees will be constant and capital will grow as a geometric Brownian motion with drift r . The following assumption is then necessary for the continuation value of the good intermediary to be bounded under perfect information.

Assumption 3. *The intermediary is more impatient than investors, i.e.*

$$\rho > r.$$

In the next subsection, I will precisely describe the information structure of the game and formalize the definition of the strategies of the players.

3.3 INFORMATION STRUCTURE

There are two sources of asymmetric information in this game. The first one is that the type of the intermediary (good or bad) is private information of the intermediary itself. The second one is that investors do not observe the portfolio allocation i_t of the intermediary, but only the total return dR_t . This implies that investors' strategies cannot directly depend on the type of the intermediary they are facing, or on the history of the Wiener process $(W_t)_{t \geq 0}$.

To set the notation, let $(\Omega, \mathcal{F}^*, P)$ be a probability space and let the Wiener process $(W_t)_{t \geq 0}$ and the type of the intermediary h be independent random variables on (Ω, \mathcal{F}^*) .

In order to formally define the strategies of the investors, let $(\mathcal{F}_t^I)_{t \geq 0}$ be the filtration generated by the process $(R_t)_{t \geq 0}$, possibly augmented by the collection of P -null sets. An action θ_t for the investors at time t is an \mathcal{F}_t^I -measurable function that maps histories of returns up to time t into the action space of delegated portfolio share, $[0, 1]$.

The intermediary is instead endowed with a larger filtration, namely a filtration $(\mathcal{F}_t)_{t \geq 0}$ that is generated by h and $(W_t)_{t \geq 0}$ and is possibly augmented by the collection of P -null sets. Therefore, an action i_t for the intermediary at time t is an \mathcal{F}_t -measurable function that maps its type and histories of the Wiener process up to time t into the action space $[\hat{i}, \bar{i}]$. Let i_t^G and i_t^B indicate the action i_t conditional on the type being good ($h = 1$) and bad ($h = 0$), respectively. Since I consider only pure strategy equilibria, the filtration $(\mathcal{F}_t)_{t \geq 0}$ coincides with the filtration generated by h and $(R_t)_{t \geq 0}$ ² and, hence, it is without loss of generality to assume that the actions of the two types, i_t^G and i_t^B , are public, i.e. they depend on the history of the signal $(R_s)_{0 \leq s \leq t}$.

Finally, the fee f_t is defined as an \mathcal{F}_t^I -measurable random variable that represents the premium investors are willing to pay to the intermediary to have access to its intermediation services.

Investors do not know the type of the intermediary they are facing. At time zero, they start with a prior $\phi_0 \in [0, 1]$ that the intermediary is of the good type. Conditional on the intermediary having a positive amount of assets under management, investors will receive a signal, coinciding with the return generated by the intermediary, dR_t . On the basis of the signal and of the equilibrium strategies of the two types, investors will update their beliefs. A belief process $(\phi_t)_{t \geq 0}$ is therefore a stochastic process, adapted to $(\mathcal{F}_t^I)_{t \geq 0}$, representing the probability that investors assign, at each time t , to the good intermediary. I refer to ϕ_t as the reputation of the intermediary at time t .

3.4 SEQUENTIAL EQUILIBRIUM DEFINITION AND CHARACTERIZATION

I will now define and characterize a sequential equilibrium of this game. The following definition implicitly assumes the existence of market clearing mechanism for the allocation of investors' funds. Moreover, it assumes that, whenever investors are indifferent

²See Appendix A for a proof.

between the riskless asset and the intermediary, they will let the intermediary manage their whole portfolio, provided that the intermediary generates a strictly positive expected excess return, according to their information³.

Definition 1 (Public Sequential Equilibrium). A public sequential equilibrium consists in a fee process $(f_t)_{t \geq 0}$, a process for the fraction of capital managed by the intermediary $(\theta_t)_{t \geq 0}$, a portfolio risk process $(i_t)_{t \geq 0}$ and a belief process $(\phi_t)_{t \geq 0}$, such that, for all times $t \geq 0$ and after every history of the game up to time t , the following conditions hold.

(i) Investors break even,

$$f_t = E[i_t(h\mu_1 + (1-h)\mu_2 - r) | \mathcal{F}_t^I].$$

(ii) $\theta_t = 1$ if and only if $E[i_t(h\mu_1 + (1-h)\mu_2 - r) | \mathcal{F}_t^I] > 0$, $\theta_t = 0$ otherwise.

(iii) Given the public strategy profile $(i_t)_{t \geq 0}$ and an initial $\phi_0 \in [0, 1]$, beliefs $(\phi_t)_{t \geq 0}$ are updated using Bayes' rule and are consistent with the public strategy profile,

$$\phi_t = E[h | \mathcal{F}_t^I].$$

(iv) $(i_t)_{t \geq 0}$ maximizes the intermediary's lifetime utility (2) given (i), (ii),(iii) and the law of motion of capital (1).

We have seen that we can restrict our attention to equilibria in public strategies. This means that the strategies of the two types may differ, but they will depend on the history of the process R_t . The following proposition provides a stronger characterization of the equilibrium strategies of the two types.

Proposition 1. *In equilibrium, i_t is \mathcal{F}_t^I measurable P -a.s. and, therefore, $i_t^G = i_t^B$ P -a.s..*

The intuition for this result is that, being $(i_t)_{t \geq 0}$ related to the volatility of the process $(R_t)_{t \geq 0}$, an observer is able to learn about it very quickly thanks to the high frequency movements of $(R_t)_{t \geq 0}$. Therefore, if the two types choose different portfolio risks, consistency of beliefs will impose to assign a correct and degenerate posterior probability

³In a partial equilibrium setting with risk-neutral investors like this one, it is not possible to precisely pin down quantities. One way to justify this assumption, is to think that the intermediary can always request an infinitesimally smaller fee and break the tie in its favor. This is the reason why, according to Definition 1, in equilibrium $\theta_t = 1$ if and only if $f_t > 0$.

distribution over types. Since the bad intermediary destroys value for investors, the bad intermediary will be terminated if it is separated. Given this belief formation rule, the bad type has incentives to pool with the good one.

Since zero measure deviations do not change the lifetime utility of the players, I will simply assume that the two types pool at every time t . We can therefore simply denote as $(i_t)_{t \geq 0}$ the common choice of portfolio risk process of the two types.

Proposition 1, however, leaves a large number of possible equilibria. Indeed, given investors' expectations about the process $(i_t)_{t \geq 0}$ chosen by the good type, both types will have an incentive to pool to such process in order not to be considered bad and terminated by investors.

For the purposes of this paper, I will simply assume that the bad intermediary imitates the choice of the good one. The good intermediary is free to choose the process $(i_t)_{t \geq 0}$ that maximizes its lifetime utility, without concerns of seeing its own reputation decreased or increased because of its choice of $(i_t)_{t \geq 0}$. In other words, the good intermediary chooses its portfolio risk while assuming that investors update their beliefs exclusively on the basis of the returns they observe. Investors understand this and understand that the bad intermediary will imitate the good one. Therefore, they will not penalize deviations towards strategies that are optimal for the good intermediary, given the way in which investors learn from returns. I leave the investigation of more general equilibrium refinement concepts for future work⁴.

Once the intermediary chooses its portfolio allocation, it will generate a return that will be delivered to investors without any agency friction. Since different types of intermediaries induce different probability distribution over returns, investors will exploit the history of returns to learn the type of the intermediary. Standard filtering results, extensively used in the literature on learning and continuous time strategic experimentation (Bolton and Harris, 1999; Hansen and Sargent, 2011; Pástor and Veronesi, 2009; Veronesi, 1999), lead to the following Proposition.

Proposition 2. *Given a prior at time 0 that the intermediary is good $\phi_0 \in [0, 1]$, and given an equilibrium strategy profile $(i_t)_{t \geq 0}$, a belief process $(\phi_t)_{t \geq 0}$ is consistent with the strategy profile if, when $\theta_t > 0$, it satisfies the stochastic differential equation*

$$d\phi_t = \varphi(\phi_t) (i_t \sigma)^{-1} (dR_t - \mu(i_t, \phi_t) dt), \quad (4)$$

⁴Extending the divinity refinement concept of Banks and Sobel (1987) to continuous time games seems to be a promising way to pin down the type of pooling equilibrium that I assume in this paper.

with

$$\varphi(\phi_t) = \phi_t(1 - \phi_t)\sigma^{-1}(\mu_1 - \mu_2)$$

and

$$\mu(i_t, \phi_t) = r + i_t [\phi_t\mu_1 + (1 - \phi_t)\mu_2 - r]$$

and where the expression for dR_t is given by (3), while, when $\theta = 0$, $d\phi_t = 0$.

The terms $\varphi(i_t, \phi_t)$, $(i_t\sigma)^{-1}$ and $\mu(i_t, \phi_t)$, as well as, the distribution of dR_t depend on i_t . However, they do so for different reasons. Indeed, the quantities $\varphi(i_t, \phi_t)$, $(i_t\sigma)^{-1}$ and $\mu(i_t, \phi_t)$ are set by investors as a function of the strategy played in equilibrium by the intermediary. It follows that those terms will be taken as given by the intermediary when it has to choose its trading strategy. On the contrary, the distribution of dR_t directly depends on the action of the intermediary. The intermediary, therefore, will consider how the distribution of dR_t changes when it considers deviations from the equilibrium strategies.

Of course, an equilibrium must be such that the intermediary has no incentives to deviate. But, by keeping the distinction clear, I intend to stress the main channel through which signal manipulation incentives arise.

Suppose that the intermediary is contemplating a deviation $(i'_t)_{t \geq 0}$ from the equilibrium $(i_t)_{t \geq 0}$. This will induce a different probability distribution for the signal dR_t so that, under the intermediary's information filtration, the law of motion of beliefs is given by

$$d\phi_t = \varphi(\phi_t) (i_t\sigma)^{-1} [r + i'_t(h\mu_1 + (1 - h)\mu_2 - r) - \mu(i_t, \phi_t)]dt + \varphi(\phi_t) (i_t\sigma)^{-1} (i'_t\sigma)dW_t.$$

In equilibrium, the intermediary must have no incentive to choose a trading strategy that is different from the equilibrium one. Therefore, under the information filtration of the intermediary, the equilibrium law of motion of beliefs is given by

$$d\phi_t = \varphi(\phi_t)(h - \phi_t)\frac{\mu_1 - \mu_2}{\sigma}dt + \varphi(\phi_t)dW_t.$$

The drift in the latest equation is positive if the intermediary is good, i.e. if $h = 1$, while it is negative when the intermediary is bad, i.e. when $h = 0$. This reflects the fact that, while under the information filtration of investors beliefs are martingales, under the information filtration of the intermediary they converge a.s. to either 0 or 1, depending on whether the intermediary is bad or good, respectively.

While the bad intermediary has to follow the trading strategy chosen by the good one, the latter can choose a trading strategy in order to control the evolution of capital and beliefs by manipulating the distribution of returns dR_t . Indeed, by controlling the evolution of K_t and ϕ_t , the intermediary is controlling the process for its cash flow $K_t f_t$. For example, the good intermediary may have an incentive to choose a low i_t in order to reduce the volatility of the signal and, consequently, of beliefs and capital, despite this will reduce their drift. However, in equilibrium, these incentives will be understood by investors who will then scale the signal by the appropriate liquidity ratio i_t . This could give rise to a phenomenon of signal-jamming in the demand for riskless assets. The intermediary may try to manipulate the distribution of the signal by demanding less risk, but such manipulation will be offset in equilibrium by the rational expectations of the investors.

The discussion so far has been quite general and players' actions at time t may depend on the entire history of returns up time t . In the next section, I discuss the properties of an equilibrium that is Markovian in capital and beliefs.

4 MARKOVIAN EQUILIBRIUM

Investors' capital and beliefs naturally embody the entire history of the game and constitute natural state variables of the model. In an equilibrium that is Markovian in these state variables, the intermediary has to consider two elements which affect the amount of assets under management and the fees in future periods: the growth rate of investors' capital (1) and the change in its reputation (4).

Thanks to the functional form of the intermediary's utility, it is possible to guess and verify⁵, that the continuation value of the intermediary will be linear in capital,

$$V(K, \phi) = Kg(\phi), \tag{5}$$

and that the players' strategies will be functions of the intermediary's reputation only and not of the level of capital, since capital is essentially a scaling variable. Therefore, let $i(\phi_t)$ denote the value of the intermediary's risk at time t in a Markovian equilibrium and let $\theta(\phi_t)$ be the value of θ_t in a Markovian equilibrium.

⁵See Proposition 3 below.

The break even condition (i) in Definition 1, will therefore become

$$f_t = f(\phi_t) \equiv i(\phi_t) [\phi_t \mu_1 + (1 - \phi_t) \mu_2 - r]. \quad (6)$$

Since $i(\phi) > 0$ for all ϕ by Assumption 2, then condition (ii) of Definition 1 can be expressed in terms of a threshold $\bar{\phi}$ for the intermediary's reputation below which the expected excess return of the intermediary's portfolio, given investors' information, is negative for any possible value of $i(\phi)$. Such threshold is given by

$$\bar{\phi} = \frac{r - \mu_2}{\mu_1 - \mu_2}. \quad (7)$$

For any $\phi > \bar{\phi}$ the intermediary will generate, according to investors' information, a strictly positive expected excess return. By condition (i) in Definition 1, this excess return equals $f(\phi)$ and represents reputation rent of the intermediary, collected in the form of fees, while investors will break even. If instead $\phi \leq \bar{\phi}$, investors expect the intermediary to generate negative excess returns. Since the intermediary cannot pay investors to allow it to experiment, investors will set $\theta(\phi) = 0$, beliefs will be no longer updated and the intermediary will be terminated forever.

In the rest of this section, I will derive boundary conditions for the limiting cases of $\phi = \bar{\phi}$ and $\phi = 1$. I will then characterize the equilibrium in the open set $(\bar{\phi}, 1)$ and provide numerical results. Since I am assuming that the bad intermediary pools on the trading strategy that is optimal for the good intermediary given (1) and (4), I can solve for an equilibrium by simply focusing on the trading strategy and continuation value of the good intermediary.

4.1 BOUNDARY CONDITIONS AND DEGENERATE BELIEFS CASE

It is straightforward to derive the boundary condition at $\phi = \bar{\phi}$. Since, at that point, the intermediary is terminated it must be the case that

$$g(\bar{\phi}) = 0 \quad (8)$$

and that $g(\phi) = 0$ for all $\phi \in [0, \bar{\phi}]$.

I now derive the equilibrium continuation value and trading strategy in the perfect information set-up. On the one hand, this provides a benchmark towards which we

can compare the imperfect information equilibrium outcomes. On the other hand it provides a second boundary condition that is needed to solve the model with imperfect information.

Let $\phi = 1$. By equation (4), this implies that there will be no learning. Given the guess (5), the Hamilton-Jacobi-Bellman (HJB) equation is

$$\rho g(1) = \max_{i \in [\hat{i}, \bar{i}]} \{f(1) + g(1)(r + i[(\mu_1 - r) - f(1)])\}.$$

It is clear that the solution requires $i(1) = \bar{i}$ and the continuation value exists under Assumption 3 with

$$g(1) = \frac{\bar{i}(\mu_1 - r)}{\rho - r}. \quad (9)$$

4.2 MARKOVIAN EQUILIBRIUM CHARACTERIZATION

I will now characterize a Markovian equilibrium of the game. I will show that, provided that a function $g(\phi)$ exists and that it satisfies certain conditions, then the continuation value is indeed given by $Kg(\phi)$. Finally, I will provide analytical results about the shape of $g(\phi)$ and the equilibrium portfolio risk $i(\phi)$ for ϕ in a neighborhood of $\bar{\phi}$. Contrary to what standard model with convex incentive schemes predict, the intermediary displays risk aversion near the termination threshold and, in equilibrium, it will keep as little risk as possible in its portfolio.

Since the intermediary maximizes (2) subject to (1) and (4), the HJB equation is

$$\begin{aligned}
\rho g(\phi) = \max_{i \in [\underline{i}, \bar{i}]} & \left\{ \underbrace{f(\phi)}_{\text{flow payoff}} + \right. \\
& + \underbrace{g'(\phi)\varphi(\phi) \frac{[i(\mu_1 - r) - f(\phi)]}{i(\phi)\sigma}}_{\text{first order effect of } d\phi} + \\
& + \underbrace{g(\phi)(r + i(\mu_1 - r) - f(\phi))}_{\text{first order effect of } dK} + \\
& + \underbrace{\frac{1}{2}g''(\phi)\varphi(\phi)^2 \frac{i^2}{i(\phi)^2}}_{\text{second order effect of } d\phi} + \\
& \left. + \underbrace{g'(\phi)\varphi(\phi)\sigma \frac{i^2}{i(\phi)}}_{\text{interaction effect of } d\phi \text{ and } dK} \right\}.
\end{aligned} \tag{10}$$

in the interval $(\bar{\phi}, 1)$. It is important to bear in mind the difference between $i(\phi)$ and i in equation (10). $i(\phi)$ represents the equilibrium trading strategy with respect to which beliefs are consistent. It is therefore taken as given by the intermediary. i is the choice variable of the intermediary, which is chosen in order to control the distribution of returns and, consequently, the distribution of capital growth and beliefs changes. Of course, in equilibrium, the optimal choice of i by the intermediary must coincide with $i(\phi)$.

An equilibrium $i(\phi)$ is therefore defined as a fixed point

$$i(\phi) \in \arg \max_{i \in [\underline{i}, \bar{i}]} \left\{ g'(\phi)\varphi(\phi) \frac{i(\mu_1 - r)}{i(\phi)\sigma} + g(\phi)i(\mu_1 - r) + \frac{1}{2}g''(\phi)\varphi(\phi)^2 \frac{i^2}{i(\phi)^2} + g'(\phi)\varphi(\phi)\sigma \frac{i^2}{i(\phi)} \right\} \tag{11}$$

with $f(\phi) = i(\phi)[\phi\mu_1 + (1 - \phi)\mu_2 - r]$.

Consequently, in equilibrium g must solve the following second order differential

equation

$$\begin{aligned}
\rho g(\phi) = & f(\phi) + \\
& + g'(\phi)\phi(1-\phi)^2 \left(\frac{\mu_1 - \mu_2}{\sigma} \right)^2 + \\
& + g(\phi)[r + i(\phi)(1-\phi)(\mu_1 - \mu_2)] + \\
& + \frac{1}{2}g''(\phi)\phi^2(1-\phi)^2 \left(\frac{\mu_1 - \mu_2}{\sigma} \right)^2 + \\
& + g'(\phi)\phi(1-\phi)(\mu_1 - \mu_2)i(\phi)
\end{aligned} \tag{12}$$

where $i(\phi)$ is defined as in (11) and $f(\phi) = i(\phi)[\phi\mu_1 + (1-\phi)\mu_2 - r]$.

For the purposes of this paper, I will work under the assumption that there exists a solution to (12). A careful exploration of questions related to existence and uniqueness of the solution to the HJB equation will be subject to future research.

Assumption 4. *There exists a bounded function $g: [\bar{\phi}, 1] \rightarrow \mathbb{R}$ that is twice continuously differentiable in $(\bar{\phi}, 1)$ and that satisfies (12) with boundary conditions $g(\bar{\phi}) = 0$ and $g(1) = \frac{\bar{i}(\mu_1 - r)}{\rho - r}$.*

So far, I have guessed that the continuation value of the good intermediary is linear in capital and heuristically proceeded to characterize an equilibrium by means of equations (11) and (12). The following proposition provides a verification theorem that confirms that my guess and characterization are valid if Assumption 4 holds.

Proposition 3. *Under Assumption 4, if $(\phi_t)_{t \geq 0}$ evolves according to (4) with $i_t = i(\phi_t)$, where $i(\phi)$ solves (11) for any $\phi \in (\bar{\phi}, 1)$, then $V(K_t, \phi_t) = K_t g(\phi_t)$ is the continuation value of the good intermediary at time t and $(i(\phi_t))_{t \geq 0}$ is the equilibrium trading strategy.*

A second order differential equation like (12) does not lend itself to straightforward analytical solutions. However, it is possible to characterize the shape of $g(\phi)$ when ϕ is sufficiently close to the termination threshold $\bar{\phi}$. Similarly, is it possible to solve for the equilibrium portfolio risk $i(\phi)$ in the proximity of $\bar{\phi}$.

While someone may expect the good intermediary to have incentives to risk-shift and gamble as much as possible in order to leave the neighborhood of $\bar{\phi}$, the following Proposition shows that exactly the opposite happens.

Proposition 4. *Under Assumption 4, there exists an $\epsilon > 0$ such that*

$$g''(\phi) < 0 \quad \forall \phi \in (\bar{\phi}, \bar{\phi} + \epsilon)$$

and

$$i(\phi) = \hat{i} \quad \forall \phi \in (\bar{\phi}, \bar{\phi} + \epsilon).$$

The proposition shows that the marginal value of capital $g(\phi)$ is concave near $\bar{\phi}$, so that the intermediary is averse to the risk of reputation losses in that region. Furthermore, rather than gambling for resurrection, the intermediary chooses the safest feasible portfolio near the termination threshold. This is in stark contrast with models of risk-shifting with convex payoffs.

The reason behind this result is that, when ϕ is close enough to $\bar{\phi}$, a very small portfolio risk is sufficient for the good intermediary to give a positive drift to investors' beliefs. Moreover, the intermediary has no incentive to increase the volatility of the belief process above the equilibrium one. Indeed, if a negative shock is realized and the intermediary is terminated, the good intermediary would lose the option to signal its type and gain reputation in the future. Therefore, near the threshold, for any equilibrium $i(\phi)$, the unconstrained maximizer of the right-hand side of (11) is strictly smaller than $i(\phi)$ itself and, hence, the only possible equilibrium is the one where $i(\phi) = \hat{i}$.

The result of Proposition 4 is consistent with evidence of liquidity hoarding and flight to quality during financial crises. Given a sequence of bad returns that drive down intermediaries' reputation, the model predicts a shift of their portfolio towards safe assets, which, in most cases, consist in currency or short-term government bonds.

In Section 5.1, I introduce a variation of the model where the intermediary can commit to a state contingent trading strategy. A comparison between Proposition 4 and Proposition 5 in Section 5.1 highlights the role of incomplete contracts in generating hoarding of safe assets. Indeed, if the intermediary could commit to a trading strategy, in equilibrium the intermediary would always invest in risky assets up to the limit \bar{i} . This is because the signal manipulation incentives disappear and only concerns about the growth rate of capital remain. On the contrary, when the intermediary cannot commit, it will have incentives to trade off expected returns for lower volatility in order to control the evolution of beliefs.

4.3 NUMERICAL RESULTS

I will now report numerical solutions to ODE (12).

Figure 1 shows the marginal value of capital $g(\cdot)$ of the good intermediary and the portfolio risk as a function of the beliefs of investors. The model in the Figure is pa-

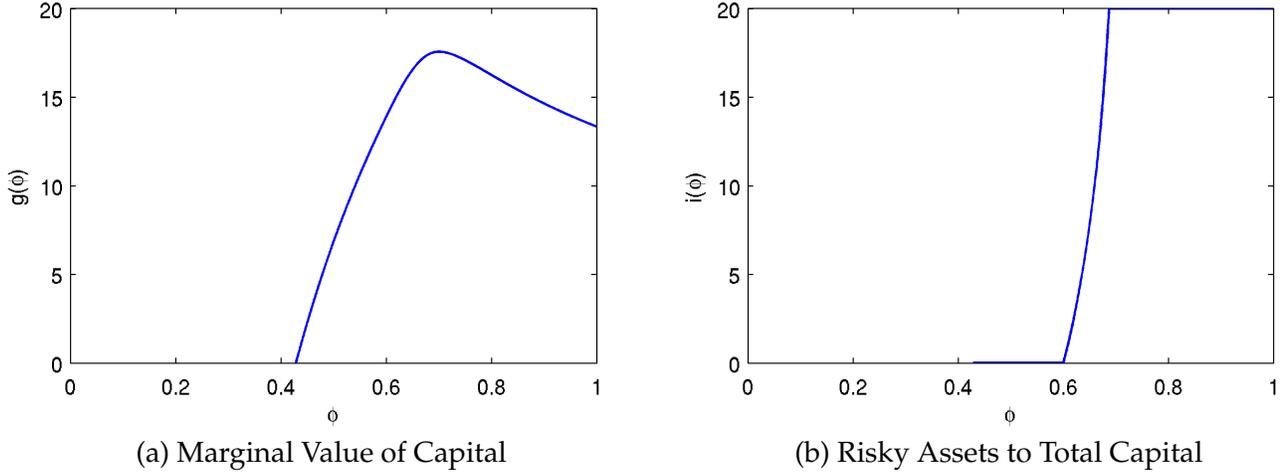


Figure 1: **Case 1.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are: $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

parameterized as follows: $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$. As a robustness check, in Appendix B, I report numerical results for eight alternative parameterizations of the model. The shape of the value function and the shape of the demand for risky assets are robust to changes in the parameters.

We can immediately notice a pattern that we can call of liquidity hoarding. For high levels of reputation, the intermediary will invest in the risky asset as much as possible. However, there is a level of reputation below which the demand for risky assets decreases very rapidly until it reaches its lower bound.

It is easy to see how this mechanism is likely to play a role during financial crises. Indeed, the dynamics of the reputation are tightly linked to the history of returns on the assets under management. Given a long enough sequence of bad return, the reputation of the intermediary may fall below a critical level and trigger a run towards liquid and safe assets.

For illustrative purposes, Figures 2 and 3 show the time series of the demand for risky assets in a simulated economy, when the returns are generated by the good and by the bad intermediary, respectively. This pattern, conditional on the bad type, is qualitatively similar to the evolution of excess reserves in Figure 7. There we see that, in a short interval of time, excess reserves saw a dramatic increase. This would be consistent with ϕ moving through the critical region from above, thus making i fall and

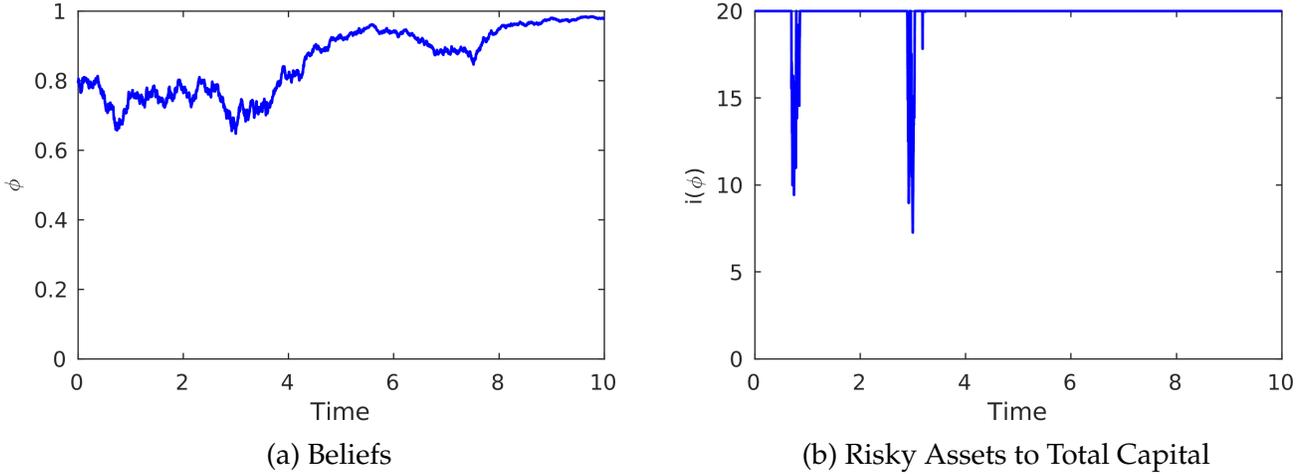


Figure 2: **Good Intermediary.** Time series of investors’s beliefs and demand for risky assets as share of total capital under management when returns are generated by the good type. Parameter values are : $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

demand for riskless assets increase.

Since beliefs are asymptotically correct⁶, the “typical” time series, when the type is fixed and ϕ_0 is high enough, will be characterized by small probabilities of liquidity hoarding if the intermediary is good, and high probability of liquidity hoarding if the type is bad, but with no recurring patterns. One way to obtain recurring episodes of hoarding is to allow the type to be time-varying. Preliminary results from this model are available in Section 5.4 and show the robustness of the model to this extension.

It is interesting to note that the function g is decreasing for high values of ϕ and this region coincides with the one where $i(\phi) = \bar{i}$. The reason is that, once i hits the upper limit, it can no longer be increased. Therefore, the expected return on the assets under management (under the probability measure induced by the good intermediary) stays constant but the fees raised by the intermediary keep increasing as ϕ increases. This has the effect of reducing the growth rate of capital⁷.

⁶In this model, learning stops when $\phi = \bar{\phi}$, so, conditional on the bad type, beliefs converges with probability one to $\bar{\phi}$.

⁷In Appendix 5 I show the results for a version of the model where the growth rate of capital is exogenous and where the marginal value of capital is always increasing in ϕ for all experimented parameterizations.

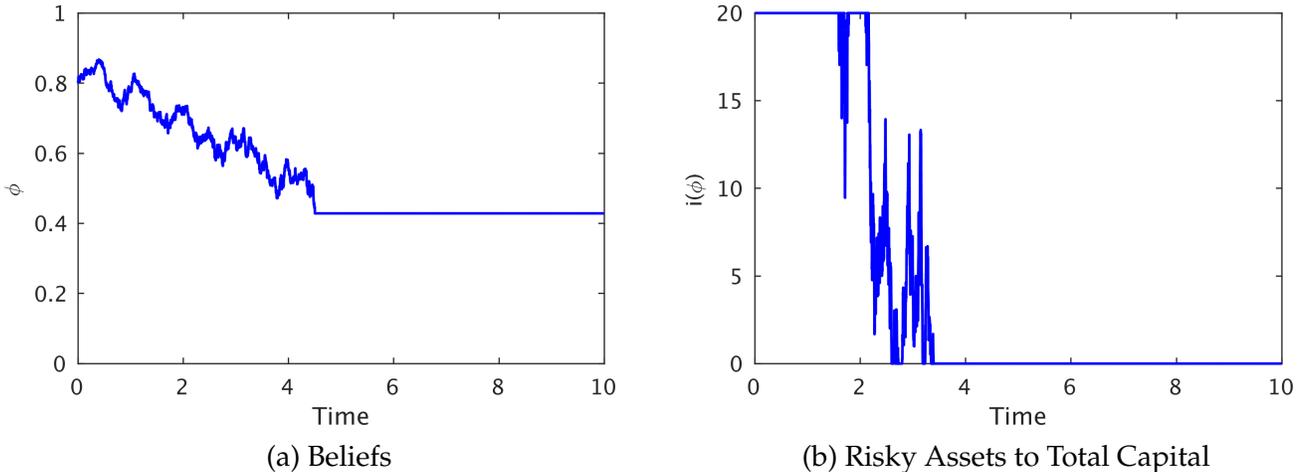


Figure 3: **Bad Intermediary.** Time series of investors’s beliefs and demand for risky assets as share of total capital under management when returns are generated by the bad type. Parameter values are : $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

5 VARIATIONS AND EXTENSIONS OF THE MODEL

In this section, I briefly introduce and discuss alternative specification of the model developed in Sections 3 and 4. The purpose is to convince the reader that the results of the model are due to the interaction between asymmetric information incomplete contracting and that the results of the model are robust to modifications of the model.

5.1 MODEL WITH COMMITMENT

Consider a model that is identical to the model of Section 3, but assume that the intermediary can commit to some portfolio risk i_t at each time t in a sequential game with investors. The assumption about the unobservability of the type is maintained but, at, each time t , the investor proposes and commit to a portfolio risk.

Investors are still learning the type of the intermediary by looking at the performance its portfolio. However, the signal manipulation incentive of the intermediary does not exist anymore. Indeed, as the intermediary commit to a particular trading strategy, investors will set their learning rule in order to update beliefs solely on the basis of the performance of the risky part of the intermediary’s portfolio. Therefore the continuation value is still linear in capital, $\tilde{V}(K, \phi) = K\tilde{g}(\phi)$, but the HJB equation takes

the form

$$\begin{aligned}
\rho \tilde{g}(\phi) = \max_{i \in [\bar{i}, \bar{i}]} & \left\{ i(\phi\mu_1 + (1-\phi)\mu_2 - r) + \right. \\
& + \tilde{g}'(\phi)\phi(1-\phi)^2 \left(\frac{\mu_1 - \mu_2}{\sigma} \right)^2 + \\
& + \tilde{g}(\phi)[r + i(1-\phi)(\mu_1 - \mu_2)] + \\
& + \frac{1}{2} \tilde{g}''(\phi)\phi^2(1-\phi)^2 \left(\frac{\mu_1 - \mu_2}{\sigma} \right)^2 + \\
& \left. + \tilde{g}'(\phi)\phi(1-\phi)(\mu_1 - \mu_2)i \right\}
\end{aligned} \tag{13}$$

Denote with $\tilde{i}(\phi)$ the equilibrium risk in the intermediary's portfolio.

A proposition analogous to Proposition 3 and lemmas analogous to 2 and 5 hold also for $\tilde{g}(\phi)$ if a bounded, continuously differentiable bounded function \tilde{g} exists that satisfies (13) in $(\bar{\phi}, 1)$ with boundary conditions $\tilde{g}(\bar{\phi}) = 0$ and $\tilde{g}(1) = \frac{\bar{i}(\mu_1 - r)}{\rho - r}$.

The following result, when compared with Proposition 4, highlights the importance of the lack of commitment in generating hoarding of riskless assets

Proposition 5. *Suppose that there exists a continuously differentiable bounded function \tilde{g} satisfying (13) in $(\bar{\phi}, 1)$ with boundary conditions $\tilde{g}(\bar{\phi}) = 0$ and $\tilde{g}(1) = \frac{\bar{i}(\mu_1 - r)}{\rho - r}$. If the intermediary can commit to a state contingent trading strategy $\tilde{i}(\phi)$, then $\tilde{i}(\phi) = \bar{i}$ for all $\phi \in (\bar{\phi}, 1]$.*

Numerical results are provided in Figure 4 and it is clear that, once the incentives to manipulate the signal are absent, the intermediary will make the socially optimal choice. Qualitatively, the results are robust to changes in the parameters.

5.2 EXOGENOUS CAPITAL GROWTH

Consider now a situation where the intermediary is free from concerns about the growth rate of investors' capital. This could be the case of a small fund that is active in a large market with very diversified investors. Specifically, assume that the law of motion of the capital under management is

$$\frac{dK_t}{K_t} = \mu dt + \nu dZ_t \tag{14}$$

where Z_t is a Weiner process independent of W_t .

The fee will be determined by market clearing in the same way as before, so that expected return for the investors is equal to the risk free rate.

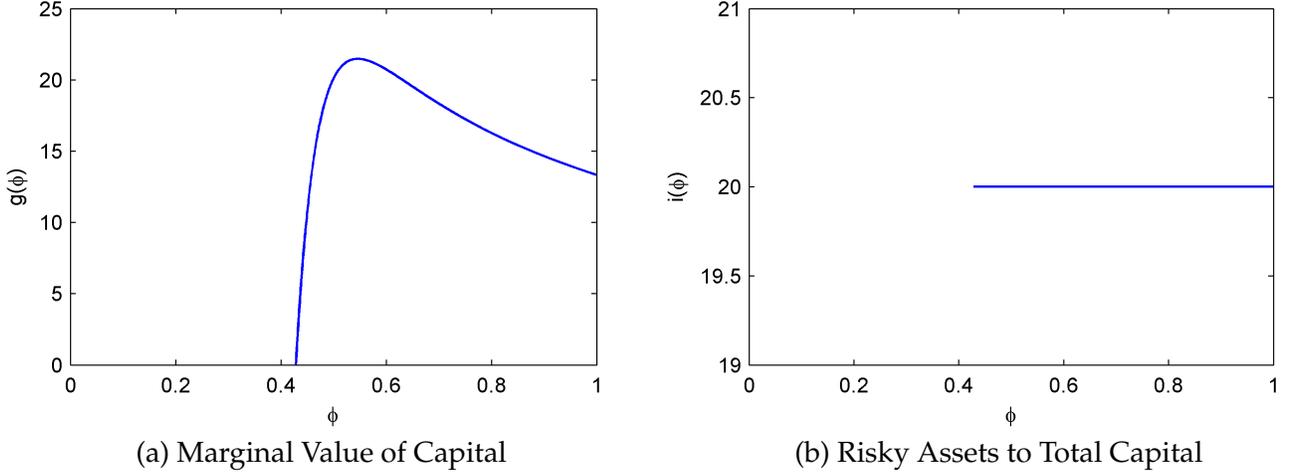


Figure 4: **Commitment. Case 1.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

The HJB equation for the good type is now

$$\begin{aligned} \hat{\rho}\hat{g}(\phi) = \max_{0 \leq i \leq \bar{i}} \left\{ \hat{f}(\phi) + \right. \\ \left. + \hat{g}'(\phi) \frac{\varphi(\phi)}{\hat{i}(\phi)\sigma} [i((1-a)\mu_1 + a\mu_2 - r) - \hat{f}(\phi)] + \right. \\ \left. + \frac{1}{2} \hat{g}''(\phi) \frac{\varphi(\phi)^2}{\hat{i}(\phi)^2} i^2 \right\} \end{aligned} \quad (15)$$

in the interval $(\bar{\phi}, 1]$ with terminal conditions $\hat{g}(\bar{\phi}) = 0$ and $\hat{g}(1) = \frac{\bar{i}(\mu_1 - r)}{\rho - r}$. In the previous equation, $\hat{\rho} = \rho - \mu$, $\hat{i}(\phi)$ is the equilibrium portfolio risk, defined in an analogous way to (11), and $\hat{f}(\phi) = \hat{i}(\phi)[\phi\mu_1 + (1 - \phi)\mu_2 - r]$ is the equilibrium fee.

Results for a parameterization of the model are reported in Figure 5 and are robust to changes in the parameters. The qualitative features of $\hat{i}(\phi)$ are the same as in Section 4. In fact, under existence conditions analogous to Assumption 4, Proposition 4 holds also for $\hat{g}(\phi)$ and $\hat{i}(\phi)$. However, from the numerical solutions, we can observe that the marginal value of capital, $\hat{g}(\phi)$, is always increasing in reputation. This provides suggestive evidence that the downward sloping part of the function $g(\phi)$ in Section 3 is due to the decreasing rate of capital growth when $i(\phi) = \bar{i}$.

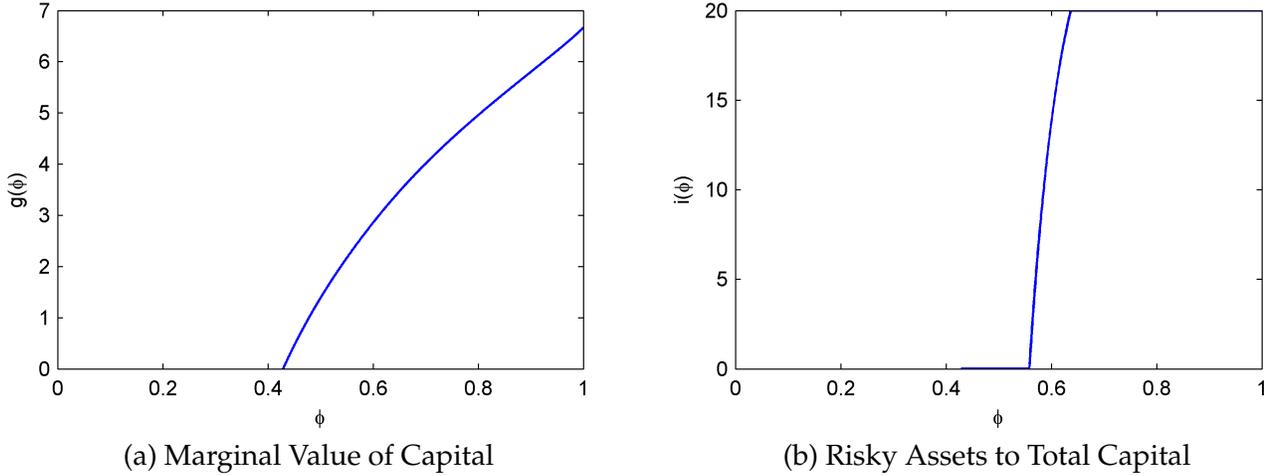


Figure 5: **Exogenous Capital Growth. Case 1.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

5.3 HETEROGENEOUS VOLATILITY

The results of the paper rely on the fact the two risky assets have the same volatility and that the two types choose the same portfolio risk. It is easy to see that the results continue to hold if the two assets have different volatility. Suppose that the good asset has volatility σ , while the bad asset has volatility $\frac{1}{k}\sigma$, for $k > 0$. This means that, in equilibrium $i_t^B = ki_t^G$ for every t , as a straightforward modification of Proposition 1. To simplify notation, let i_t be the equilibrium choice of the good type.

This implies that the expected return on the portfolio of the bad intermediary is $r + i_t k(\mu_2 - r)$, while the volatility is σi_t . But this is mathematically and economically equivalent to a model where both assets have the same volatility σ and the expected return of the bad asset is $\mu'_2 = k(\mu_2 - r) + r$, which is still lower than r , since $\mu_2 - r < 0$. Therefore, heterogeneous volatility does not pose any threat to the conclusions of the model.

5.4 TIME VARYING HIDDEN TYPES

In a model like the one presented in Sections 3 and 4, beliefs converge with probability one to either 1 or $\bar{\phi}$ depending on the type of the intermediary. This implies that

episodes of liquidity hoarding will disappear in the long run. A natural extension is to consider a case in which the type of the intermediary, rather than being a time-constant process, evolves over time. The definitions of Section 3 can be easily modified to explicitly deal with a process $(h_t)_{t \geq 0}$ for the type of the intermediary. However, Proposition 1 cannot be straightforwardly extended to this setting since, now, there is no guarantee that the continuation value of the bad intermediary is zero under perfect information. In this Section, I provide a brief discussion of this model, which will be subject to further research in the future.

Suppose that $(h_t)_{t \geq 0}$ is independent of $(W_t)_{t \geq 0}$ and that it follows a continuous-time Markov chain with generator

$$\Lambda = \begin{pmatrix} -\lambda_2 & \lambda_2 \\ \lambda_1 & -\lambda_1 \end{pmatrix}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$ and the state vector is $(1, 0)'$.

The law of motion of beliefs, conditional on $\theta_t > 0$ and on the two types pooling, is then modified as follows (see Theorem 9.1 in Liptser and Shiryaev, 2001):

$$d\phi_t = [-\lambda_2\phi_t + \lambda_1(1 - \phi_t)]dt + \varphi(i_t, \phi_t) (i_t\sigma)^{-1} (dR_t - \mu(i_t, \phi_t)dt);$$

while $d\phi_t = [-\lambda_2\phi_t + \lambda_1(1 - \phi_t)]dt$ when the intermediary does not generate signals.

At $\phi_t = 1$, reputation is deterministically decreased by an amount $-\lambda_2 dt$, while at $\bar{\phi}$ two cases have to be distinguished, depending on the stationary probability of the good type,

$$\pi = \frac{\lambda_1}{\lambda_1 + \lambda_2}.$$

When $\pi \leq \bar{\phi}$, the reputation of the intermediary deterministically decreases when no signal is given to investors. Therefore, when the value $\bar{\phi}$ is hit, the intermediary will be shut down. On the contrary, when $\pi > \bar{\phi}$ the process ϕ_t deterministically moves upward when $\bar{\phi}$ is hit and, therefore, the continuation value of the intermediary at that point will not, in general, be zero. These two situations give rise to different terminal conditions, different continuation values when $\phi \in [0, \bar{\phi}]$ as well as possibly different incentives for the bad intermediary to imitate the good one or to separate itself.

In this richer framework, it is necessary to explicitly model the continuation value of both types in the interval $[\bar{\phi}, 1]$, conditional on them pooling. For the good type we

have

$$\begin{aligned}
\rho g(\phi) = \max_{0 \leq i \leq \bar{i}} \left\{ & f(\phi) + \\
& + g'(\phi) \frac{\varphi(\phi)}{i(\phi)\sigma} [i(\mu_1 - r) - f(\phi)] + \\
& + g(\phi)(r + i(\mu_1 - r) - f(\phi)) + \\
& + \frac{1}{2} g''(\phi) \frac{\varphi(\phi)^2}{i(\phi)^2} i^2 + \\
& + g'(\phi) \frac{\varphi(\phi)}{i(\phi)} i^2 \sigma + \\
& + \underbrace{g'(\phi)[- \lambda_2 \phi + \lambda_1(1 - \phi)]}_{\text{deterministic trend of } d\phi} + \\
& + \underbrace{\lambda_2 [b(\phi) - g(\phi)]}_{\text{type's jump}} \left. \right\}.
\end{aligned} \tag{16}$$

and similarly, for the bad type,

$$\begin{aligned}
\rho b(\phi) = & f(\phi) + \\
& - b'(\phi) \phi^2 (1 - \phi) \left(\frac{\mu_1 - \mu_2}{\sigma} \right)^2 + \\
& + b(\phi)(r - i(\phi)\phi(\mu_1 - \mu_2)) + \\
& + \frac{1}{2} b''(\phi) \phi^2 (1 - \phi)^2 \left(\frac{\mu_1 - \mu_2}{\sigma} \right)^2 + \\
& + b'(\phi) \varphi(\phi) i(\phi) \sigma + \\
& + b'(\phi)[- \lambda_2 \phi + \lambda_1(1 - \phi)] + \\
& + \lambda_2 [g(\phi) - b(\phi)].
\end{aligned} \tag{17}$$

$i(\phi)$ is the equilibrium portfolio risk that such that, given $i(\phi)$ in equation (16), the good intermediary optimally chooses $i = i(\phi)$ and the bad intermediary imitates it.

Consider first the case of $\pi \leq \bar{\phi}$, so that $\bar{\phi}$ is an absorbing barrier. Then, Proposition 1 holds and the bad intermediary imitates the good one. Indeed, if the two types were separated and the intermediary was bad, then beliefs would be reset to 0 and would deterministically converge to $\phi \leq \bar{\phi}$. Under the investors' perspective, expected returns are below the risk-free rate and therefore the intermediary will be inactive forever. This,

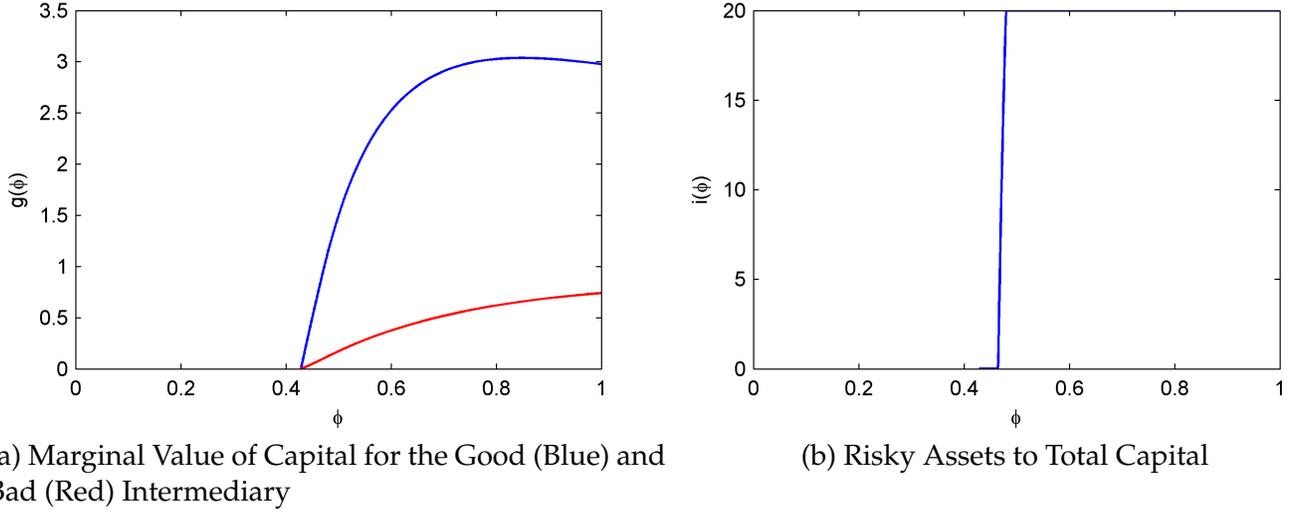


Figure 6: **Switching Types.** $\delta < \bar{\phi}$. **Case 6.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = \mathbf{0.04}$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

therefore, provides us with boundary conditions $g(\bar{\phi}) = 0$ and $b(\bar{\phi}) = 0$. Moreover, for all $\phi \in [0, \bar{\phi}]$ we will have $g(\phi) = 0$ and $b(\phi) = 0$.

At $\phi = 1$, no learning happens by observing returns, but ϕ will deterministically decrease. Since the choice of i now affects only the growth rate of capital, in equilibrium we will have $i(1) = \bar{i}$ and the relations between $g(1)$, $g'(1)$, $b(1)$ and $b'(1)$ will be given by

$$g(1) = \frac{\bar{i}(\mu_1 - r)}{\rho - r} - g'(1) \frac{\lambda_2}{\rho - r} + [b(1) - g(1)] \frac{\lambda_2}{\rho - r} \quad (18)$$

and

$$b(1) = \frac{\bar{i}(\mu_1 - r)}{\rho - r + \bar{i}(\mu_1 - \mu_2)} - b'(1) \frac{\lambda_2}{\rho - r + \bar{i}(\mu_1 - \mu_2)} + [g(1) - b(1)] \frac{\lambda_1}{\rho - r + \bar{i}(\mu_1 - \mu_2)} \quad (19)$$

A numerical result is shown in Figure 6. Note that the parameterization differs from Figure 1 in Section 4, but it can be compared with Figure 12 in Appendix B. The qualitative features of the model are unaffected. However, the flight to riskless assets seems to be more dramatic, since the region of the state space where i shifts from \bar{i} to \hat{i} is now narrower.

Consider now the case of $\pi > \bar{\phi}$, so that, at $\bar{\phi}$, investors beliefs are deterministically move upward. The boundary conditions at $\bar{\phi}$, provided that the two types are pooling, are

$$g(\bar{\phi}) = g'(\bar{\phi}) \frac{-\lambda_2 \bar{\phi} + \lambda_1 (1 - \bar{\phi})}{\rho - r} + [b(\bar{\phi}) - g(\bar{\phi})] \frac{\lambda_2}{\rho - r} \quad (20)$$

and

$$b(\bar{\phi}) = b'(\bar{\phi}) \frac{-\lambda_2 \bar{\phi} + \lambda_1 (1 - \bar{\phi})}{\rho - r} + [g(\bar{\phi}) - b(\bar{\phi})] \frac{\lambda_1}{\rho - r}. \quad (21)$$

At $\phi = 1$, the boundary conditions are the same as in (18) and (19), provided that the two types are pooling.

The formal investigation of the properties of this model is still ongoing, but I can provide a result that holds under reasonable conditions. First, we need an analogous of Assumption 4, i.e. we need to assume the existence of bounded, twice continuously differentiable functions g and b that solve (16) and (17) – for an $i(\phi)$ such that the maximizer in (16) coincides with $i(\phi)$ – and that satisfy the boundary conditions (18), (19), (20) and (21). Call this condition C1. Second, I will need to show that $g(\bar{\phi}) \geq b(\bar{\phi})$ and $g(\phi) \geq b(\phi)$ for ϕ is a right-neighborhood of $\bar{\phi}$. Let this be condition C2.

Claim 1. *If conditions C1 and C2 hold, then there exists an $\epsilon > 0$ such that $g''(\phi) < 0$ and $i(\phi) = \hat{i}$ for all $\phi \in (\bar{\phi}, \bar{\phi} + \epsilon)$.*

The proof is omitted, since it follows the same reasoning of the proof of Proposition 4, after noting two facts. The first one is that $g'(\bar{\phi}) > 0$ follows directly from C2 and (20). The second one is that, at least in a right-neighborhood of $\bar{\phi}$, the bad intermediary will pool, for, otherwise $g(\bar{\phi}) = g(0)$ and, since the intermediary receives no flow payoff when $\phi \in [0, \bar{\phi}]$, this would imply that $g(\bar{\phi}) = 0$.

The fact that Claim 1 holds when $\pi > \bar{\phi}$ underscores the intuition supporting that the value of the signaling option is a key driver of the model's results. Indeed, if reputation falls below the threshold, the good intermediary would be unable to signal its type until $\bar{\phi}$ is reached again. This option may be less valuable if $\pi < \bar{\phi}$. This is because the deterministic drift of beliefs may be negative enough that, even under the information filtration of the good type, beliefs may still be drifting downward.

6 CONCLUSIONS

I developed a model of financial intermediation with asymmetric information that delivers outcomes that qualitatively resemble episodes of liquidity hoarding. The incentives of the intermediary to manipulate the evolution of investors' beliefs, coupled with the impossibility to commit to a trading strategy, are the key drivers of the results. When reputation is close to the threshold below which the intermediary is terminated, a risk neutral intermediary displays risk aversion, despite being subject to a convex compensation scheme. This is because, in an infinite horizon game, termination would make the good intermediary lose the option to signal its type and gain reputation.

APPENDIX A PROOFS

PROOF OF LEMMA 1

Here, I provide a lemma supporting the claim that restricting our attention to public strategies is without loss of generality.

Let $(\mathcal{F}_t^P)_{t \geq 0}$ be the filtration generated by h and $(R_t)_{t \geq 0}$ and is possibly augmented by the collection of P -null sets. Recall that we are dealing with a probability space $(\Omega, \mathcal{F}^*, P)$ and that I have defined $(\mathcal{F}_t)_{t \geq 0}$ as the filtration generated by $(W_t)_{t \geq 0}$ and h .

Lemma 1. *Given the assumptions that $\mathcal{F}_0 = \mathcal{F}_0^P$ and that the filtrations $(\mathcal{F}_t)_{t \geq 0}$ and $(\mathcal{F}_t^P)_{t \geq 0}$ are augmented by the P -null sets, we have that $\mathcal{F}_t = \mathcal{F}_t^P$ for all $t \geq 0$. Therefore, there is no loss of generality in assuming that strategies are public.*

Proof. Let us proceed by contradiction and suppose that there exists a τ defined as

$$\tau = \inf\{t \geq 0 : \exists B \text{ s.t. } B \in \mathcal{F}_t \text{ and } B \notin \mathcal{F}_t^P\}$$

and note that, for every t , $\mathcal{F}_t^P \subseteq \mathcal{F}_t$.

Since both filtrations are generated by continuous processes, then they are left-continuous (see Karatzas and Shreve (1991), Chapter 2.7). A filtration $(\mathcal{G}_t)_{t \geq 0}$ is said to be left-continuous if $\mathcal{G}_t = \mathcal{G}_{t-}$, where $\mathcal{G}_{t-} \equiv \sigma(\cup_{s < t} \mathcal{G}_s)$. By convention, $\mathcal{G}_{0-} = \mathcal{G}_0$.

Suppose then that $\mathcal{F}_\tau^P \subset \mathcal{F}_\tau$. By definition of τ we must have, $\mathcal{F}_{\tau-}^P = \mathcal{F}_{\tau-}$, leading us to a contradiction, since, by left-continuity, $\mathcal{F}_{\tau-}^P = \mathcal{F}_{\tau-} = \mathcal{F}_\tau$.

Suppose instead that $\mathcal{F}_\tau^P = \mathcal{F}_\tau$. By Proposition 7.7 in Chapter 2.7 of Karatzas and Shreve (1991), the filtration $(\mathcal{F}_t)_{t \geq 0}$ is also right-continuous. A filtration $(\mathcal{G}_t)_{t \geq 0}$ is said to be right-continuous if $\mathcal{G}_t = \mathcal{G}_{t+}$, where $\mathcal{G}_{t+} \equiv \cap_{s > t} \mathcal{G}_s$.

This means that we must have $\mathcal{F}_\tau = \mathcal{F}_{\tau+}$. However, by definition of τ we must also have $\mathcal{F}_{\tau+}^P \subset \mathcal{F}_{\tau+}$. This would imply that $\mathcal{F}_{\tau+}^P \subset \mathcal{F}_\tau^P$, contradicting that $(\mathcal{F}_t^P)_{t \geq 0}$ is an increasing sequence of σ -algebras.

We therefore conclude that $\mathcal{F}_t^P = \mathcal{F}_t$ for all t . □

PROOF OF PROPOSITION 1

This proof follows the same lines of Theorem 7.17 and Lemma 5.2 in Liptser and Shiryaev (2001) with some minor modifications to allow the proof to work for any finite time t in an infinite horizon game.

Proof. Given a time interval $[0, t]$, consider a partition of n sub-intervals with end points $0 = t_{n,0} < t_{n,1} < \dots < t_{n,n} = t$ such that $\max_i |t_{n,i+1} - t_{n,i}| \rightarrow 0$ as $n \rightarrow \infty$.

Then consider

$$\begin{aligned} \sum_{i=0}^{n-1} (R_{t_{n,i+1}} - R_{t_{n,i}})^2 &= \sum_{i=0}^{n-1} \left(\int_{t_{n,i}}^{t_{n,i+1}} i_s \sigma_s dW_s \right)^2 + \\ &+ \sum_{i=0}^{n-1} \left\{ \int_{t_{n,i}}^{t_{n,i+1}} [r + i_s(h\mu_1 + (1-h)\mu_2 - r)] ds \right\}^2 + \\ &+ 2 \sum_{i=0}^{n-1} \left(\int_{t_{n,i}}^{t_{n,i+1}} i_s \sigma_s dW_s \right) \left\{ \int_{t_{n,i}}^{t_{n,i+1}} [r + i_s(h\mu_1 + (1-h)\mu_2 - r)] ds \right\} \end{aligned} \quad (22)$$

Note that $|r + i_s(h\mu_1 + (1-h)\mu_2 - r)| \leq r + \bar{i}D$, where $D \equiv |\mu_1 - r| + |\mu_2 - r|$, therefore

$$\sum_{i=0}^{n-1} \left\{ \int_{t_{n,i}}^{t_{n,i+1}} [r + i_s(h\mu_1 + (1-h)\mu_2 - r)] ds \right\}^2 \leq (r + \bar{i}D)^2 \cdot t \cdot \max_i |t_{n,i+1} - t_{n,i}| \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Also,

$$\begin{aligned} \left| \sum_{i=0}^{n-1} \left(\int_{t_{n,i}}^{t_{n,i+1}} i_s \sigma_s dW_s \right) \left\{ \int_{t_{n,i}}^{t_{n,i+1}} [r + i_s(h\mu_1 + (1-h)\mu_2 - r)] ds \right\} \right| &\leq \\ &\leq (r + \bar{i}D) \cdot t \cdot \max_i \left| \int_{t_{n,i}}^{t_{n,i+1}} i_s \sigma_s dW_s \right| \rightarrow 0 \quad \text{as } n \rightarrow \infty. \end{aligned}$$

since $|t_{n,i+1} - t_{n,i}| \rightarrow 0$ for all i and $\{i_s \sigma\}_{u \geq 0}$ is a bounded process.

As for the remaining term, note that, by Ito's lemma

$$d \left(\int_{t_{n,i}}^s i_u \sigma dW_u \right)^2 = 2 \left(\int_{t_{n,i}}^s i_u \sigma dW_u \right) i_s \sigma dW_s + (i_s \sigma)^2 ds$$

and hence

$$\left(\int_{t_{n,i}}^{t_{n,i+1}} i_s \sigma dW_s \right)^2 = 2 \int_{t_{n,i}}^{t_{n,i+1}} \left(\int_{t_{n,i}}^{s_{n,i}} i_u \sigma dW_u \right) i_{s_{n,i}} \sigma dW_{s_{n,i}} + \int_{t_{n,i}}^{t_{n,i+1}} (i_s \sigma)^2 ds,$$

with $s_{n,1} \in [t_{n,i}, t_{n,i+1}]$.

We can then substitute the latter expression in the first term in the right-hand side

of (22) and obtain

$$\sum_{i=0}^{n-1} \left(\int_{t_{n,i}}^{t_{n,i+1}} i_s \sigma_s dW_s \right)^2 = \int_0^t (i_s \sigma)^2 ds + 2 \int_0^t \left(\int_{t_{n,i}}^s i_u \sigma dW_u \right) i_s \sigma dW_s. \quad (23)$$

The second term in the right-hand side of (23), however, converges in probability to zero, since

$$\left| \int_{t_{n,i}}^{t_{n,i+1}} \left(\int_{t_{n,i}}^{s_{n,i}} i_u \sigma dW_u \right) i_s \sigma dW_s \right| \leq \left[\max_i \sup_{s_{n,1} \in [t_{n,i}, t_{n+1,i}]} \left(\int_{t_{n,i}}^{s_{n,i}} i_u \sigma dW_u \right)^2 \right] \cdot (\bar{i}\sigma)^2 \cdot t \rightarrow 0$$

as $n \rightarrow \infty$

since $|t_{n,i+1} - t_{n,i}| \rightarrow 0$ for all i and $\{i_u \sigma\}_{u \geq 0}$ is a bounded process.

The left-hand side of (22) is clearly \mathcal{F}_t^I -measurable and converges to an \mathcal{F}_t^I -measurable random variable as $n \rightarrow \infty$, while the right-hand side converges in probability to $\int_0^t (i_s \sigma)^2 ds$. It therefore follows that $\int_0^t (i_s \sigma)^2 ds$ is itself \mathcal{F}_t^I -measurable.

We can then straightforwardly apply Lemma 5.2 in Liptser and Shiryaev (2001) and use the fact that both i_t and σ are strictly positive, to conclude that there exist an \mathcal{F}_t^I -measurable process $(\tilde{i}_s)_{0 \leq s \leq t}$, that is progressively measurable with respect to $(\mathcal{F}_s^I)_{0 \leq s \leq t}$ and such that $\tilde{i}_t = i_t$ P -a.s..

Therefore, let $(i_t^G)_{t \geq 0}$ be the equilibrium public strategy of the good type, then consistency of beliefs imposes

$$\phi_t = 0 \quad \text{if} \quad \exists s \geq 0, \Delta > 0 \text{ s.t. } \int_s^{s+\Delta} (\tilde{i}_u \sigma)^2 du \neq \int_s^{s+\Delta} (i_u^G \sigma)^2 du.$$

Therefore, it is optimal for the bad intermediary to pool P -a.s. □

PROOF OF PROPOSITION 2

In equilibrium, the law of motion of the signal is given by

$$dR_t = [r + i_t(h\mu_1 + (1-h)\mu_2 - r)]dt + i_t \sigma dW_t$$

where $(i_t)_{t \geq 0}$ is an $(\mathcal{F}_t^I)_{t \geq 0}$ progressively measurable process, while h is hidden to investors. The fact that only the drift changes depending on the type of the intermediary calls for an application of Girsanov's theorem to find an appropriate change of measure and derive the likelihood ratio of the two types. However, care should be taken since Girsanov's theorem cannot be extended straightforwardly to the infinite horizon case. Fortunately, Huang and Pages (1992) and Revuz and Yor (2013) provide a framework to extend the result to the infinite horizon case.

Proof. Let P^G be a probability measure on $(\Omega, \mathcal{F}_\infty^I)$ induced on the good type. Define P^B in an analogous way for the bad type.

We are interested in finding a random variable ξ_t representing the ratio between the likelihood that the path $(R_s)_{0 \leq s \leq t}$ is generated by the good type and the likelihood that the same path is generated by the bad type. Note that $\tilde{W}_t \equiv \int_0^t [dR_s - rds - i_s(\mu_2 - r)](i_s\sigma)^{-1} ds$ is a Wiener process conditional on the bad type.

Let

$$\eta = (\mu_1 - \mu_2)\sigma^{-1}$$

and define the local martingale

$$\xi_t = \exp \left\{ \eta \tilde{W}_t - \frac{1}{2} \eta^2 t \right\}.$$

It can be seen that $E_{P^B}[\xi_t] = 1$ for all t . From Proposition 1 in Huang and Pages (1992) it follows that there exists a measure Q on $(\Omega, \mathcal{F}_\infty^I)$ such that the restriction of Q on \mathcal{F}_t^I is equivalent to the restriction of P^B on \mathcal{F}_t and, moreover, if restricted to $(\Omega, \mathcal{F}_t^I)$, ξ_t is the likelihood ratio dQ/dP^B . Furthermore,

$$\tilde{W}_t^Q = \tilde{W}_t - \eta t$$

is a standard Wiener process for $(\Omega, \mathcal{F}_\infty^I, Q)$. We can think of Q as a probability measure whose restriction on $(\Omega, \mathcal{F}_t^I)$ coincides with the restriction on $(\Omega, \mathcal{F}_t^I)$ of P^G , for all t .

By consistency of beliefs in equilibrium

$$\phi_t = \frac{p\xi_t}{p\xi_t + (1-p)} \tag{24}$$

If we then apply Ito's lemma we obtain

$$d\phi_t = -\frac{(1-p)p^2}{(p\xi_t + (1-p))^3}(\eta\xi_t)^2 dt + \frac{(1-p)p}{(p\xi_t + (1-p))^2}\eta\xi_t d\tilde{W}_t.$$

Finally recall that $d\tilde{W}_t = [dR_s - rds - i_s(\mu_2 - r)ds](i_s\sigma)^{-1}$. It is then sufficient to substitute for $d\tilde{W}_t$ in the previous expression and use (24) to conclude that

$$d\phi_t = (1 - \phi_t)\phi_t \frac{\mu_1 - \mu_2}{\sigma} (i_t\sigma)^{-1} (dR_t - \mu(i_t, \phi_t)dt)$$

where $\mu(i_t, \phi_t) = r + i_t [\phi_t\mu_1 + (1 - \phi_t)\mu_2 - r]$. □

PROOF OF PROPOSITION 3

The proof of this Proposition follows the same lines of standard verification theorems as those in Chapter 10 of Oksendal (2013). However, given the game-theoretic set-up of the model, it is essential to keep in mind that, given an equilibrium portfolio risk $i(\phi)$, the good intermediary is free to choose its portfolio risk. The two will coincide in equilibrium by the consistency condition given by (11). Expectations are understood to be expectations conditional on \mathcal{F}_0 .

Proof. To begin with, note that $V(0, \phi) = 0$ and the function $V(K, \phi) = Kg(\phi)$ satisfies this condition. Similarly, $V(K, \phi) = 0$ if $\phi \leq \bar{\phi}$ and $Kg(\phi)$ satisfies also this condition. Let us therefore focus on the case $K_0 > 0$ and $\phi_0 > \bar{\phi}$.

Define $\tau = \inf\{t : \phi_t \leq \bar{\phi}\}$. I want to show that

$$K_0g(\phi_0) \geq \mathbb{E} \left[\int_0^\tau e^{-\rho t} f(\phi_t) K_t dt \right]$$

for any feasible process of the control $(i_t)_{t \geq 0}$ when K_t follows (1), ϕ_t follows (4) and $f(\phi)$ and $i(\phi)$ are defined by (6) and (11).

Let

$$W_t = e^{-\rho \min\{t, \tau\}} K_{\min\{t, \tau\}} g(\phi_{\min\{t, \tau\}}) + \int_0^{\min\{t, \tau\}} e^{-\rho s} f(\phi_s) K_s ds \quad .$$

Define

$$\begin{aligned} \mathcal{D}[Kg(\phi), i] = & \left\{ f(\phi) + g'(\phi)\varphi(\phi) \frac{[i(\mu_1 - r) - f(\phi)]}{i(\phi)\sigma} + g(\phi)(-\rho + r + i(\mu_1 - r) - f(\phi)) + \right. \\ & \left. + \frac{1}{2}g''(\phi)\varphi(\phi)^2 \frac{i^2}{i(\phi)^2} + g'(\phi)\varphi(\phi)\sigma \frac{i^2}{i(\phi)} \right\} K \end{aligned}$$

and

$$\mathcal{V}[Kg(\phi), i] = \left\{ g'(\phi)\varphi(\phi) \frac{i}{i(\phi)} + g(\phi)\sigma i \right\} K.$$

By Ito's lemma

$$W_t = W_0 + \int_0^{\min\{t, \tau\}} e^{-\rho s} \mathcal{D}[K_s g(\phi_s), i_s] ds + \int_0^{\min\{t, \tau\}} e^{-\rho s} \mathcal{V}[K_s g(\phi_s), i_s] W_s. \quad (25)$$

By (10) and (11), $\mathcal{D}[K_s g(\phi_s), i_s]$ attains its maximum when $i_s = i(\phi_s)$ and $\mathcal{D}[K_s g(\phi_s), i_s] \leq 0$ for any i_s . Therefore

$$\int_0^{\min\{t, \tau\}} e^{-\rho s} \mathcal{V}[K_s g(\phi_s), i_s] W_s \geq W_t - W_0. \quad (26)$$

with equality if the control $(i_s)_{s \geq 0}$ coincides with the optimal one.

The left-hand side of (26) is a local martingale that is bounded from below by $-W_0 = -V_0 > -\infty$. Hence, it is a supermartingale. Taking expectations in (25) we obtain

$$\mathbb{E}[W_t] \leq W_0 + \mathbb{E} \left[\int_0^{\min\{t, \tau\}} e^{-\rho s} \mathcal{D}[K_s g(\phi_s), i_s] ds \right] \leq W_0 = K_0 g(\phi_0)$$

with equalities if the control $(i_s)_{s \geq 0}$ is the optimal one.

Hence,

$$K_0 g(\phi_0) \geq \mathbb{E} \left[\int_0^{\min\{t, \tau\}} e^{-\rho s} f(\phi_s) K_s ds \right] + \mathbb{E} \left[e^{-\rho \min\{t, \tau\}} K_{\min\{t, \tau\}} g(\phi_{\min\{t, \tau\}}) \right].$$

Using the fact that we must have

$$\lim_{t \rightarrow \infty} \mathbb{E} \left[e^{-\rho \min\{t, \tau\}} K_{\min\{t, \tau\}} g(\phi_{\min\{t, \tau\}}) \right] = 0.$$

then, by the monotone convergence theorem, as $t \rightarrow \infty$, we conclude

$$K_0 g(\phi_0) \geq \mathbb{E} \left[\int_0^T e^{-\rho s} f(\phi_s) K_s ds \right]$$

for any feasible portfolio risk process $(i_s)_{s \geq 0}$, with equality if $i_s = i(\phi_s)$ P -a.s..

It follows that $K_0 g(\phi_0)$ is the continuation value and $(i(\phi_t))_{t \geq 0}$ is indeed the strategy played by the intermediary in equilibrium. □

PROOF OF PROPOSITION 4

Let us re-write (12) with a more parsimonious notation as

$$\frac{1}{2} g''(\phi) \phi^2 (1-\phi)^2 \kappa^2 = -i(\phi)(-\delta_2 + \phi \delta_1) - g'(\phi) \phi (1-\phi)^2 \kappa^2 + g(\phi)(\delta - i(\phi)(1-\phi)\delta_1) - g'(\phi) \phi (1-\phi) \delta_1 i(\phi) \quad (27)$$

where the parameters $\kappa, \delta_1, \delta_2$ and δ are all strictly positive.

I break down the proof in a series of Lemmas. Assumption 4 is taken for granted in all of them. All limits in the form $\phi \rightarrow \bar{\phi}$ are understood to be for sequences converging to $\bar{\phi}$ from above.

Lemma 2. $\liminf_{\phi \rightarrow \bar{\phi}} g'(\phi) \geq 0$.

Proof. Suppose, towards a contradiction, that $\liminf_{\phi \rightarrow \bar{\phi}} g'(\phi) < 0$. This means that, in any interval $(\bar{\phi}, \bar{\phi} + \varepsilon)$, there exists a sequence $\phi_n \rightarrow \bar{\phi}$ such that $\lim_{n \rightarrow \infty} g'(\phi_n) < 0$. Let $\varepsilon_n = \phi_n - \bar{\phi}$. By a first order Taylor expansion

$$g(\bar{\phi}) = g(\phi_n) - g'(\phi_n) \varepsilon_n + o(\varepsilon_n) \implies g(\phi_n) = \left(g'(\phi_n) + \frac{o(\varepsilon_n)}{\varepsilon_n} \right) \varepsilon_n$$

Then, there exists an $\bar{n} > 0$ large enough such that $\left(g'(\phi_{\bar{n}}) + \frac{o(\varepsilon_{\bar{n}})}{\varepsilon_{\bar{n}}} \right) \varepsilon_{\bar{n}} < 0$, which contradicts the fact that $g(\phi_{\bar{n}})$ must be non-negative. □

Lemma 3. *There exists an $\varepsilon > 0$ such that, for all $\phi \in (\bar{\phi}, \bar{\phi} + \varepsilon)$, $g''(\phi) \leq 0$.*

Proof. By lemma 2 and by equation (27) we must conclude that $\limsup_{\phi \rightarrow \bar{\phi}} g''(\phi) \leq 0$, that is, there exists an $\varepsilon > 0$ such that, for all $\phi \in (\bar{\phi}, \bar{\phi} + \varepsilon)$ we have that $g''(\phi) \leq 0$. □

Lemma 4. $\liminf_{\phi \rightarrow \bar{\phi}} g'(\phi) = \limsup_{\phi \rightarrow \bar{\phi}} g'(\phi)$ and possibly both are equal to $+\infty$. Moreover, $\liminf_{\phi \rightarrow \bar{\phi}} g''(\phi) = \limsup_{\phi \rightarrow \bar{\phi}} g''(\phi)$ and possibly both are equal to $-\infty$.

Proof. Recall that, by Lemmas 2 and 3, there exists a neighborhood of $\bar{\phi}$ where $\liminf_{\phi \rightarrow \bar{\phi}} g'(\phi) \geq 0$ and $\limsup_{\phi \rightarrow \bar{\phi}} g''(\phi) \leq 0$ for all ϕ in that neighborhood.

Suppose, by way of contradiction, that $\liminf_{\phi \rightarrow \bar{\phi}} g'(\phi) = H \geq 0$ and that $\limsup_{\phi \rightarrow \bar{\phi}} g''(\phi) > H$. This means that there exists an $H' > H$ such that $g'(\phi) = H'$ for infinitely many ϕ in any neighborhood of $\bar{\phi}$, contradicting the fact that $g''(\phi) \leq 0$ for all ϕ in a neighborhood of $\bar{\phi}$.

This, together with equation (27), implies the claim. □

With some abuse of notation, let $g'(\bar{\phi}) \equiv \lim_{\phi \rightarrow \bar{\phi}} g'(\phi)$ and $g''(\bar{\phi}) \equiv \lim_{\phi \rightarrow \bar{\phi}} g''(\phi)$. It is understood that both can be infinity.

Lemma 5. $g'(\bar{\phi}) > 0$ and $g''(\bar{\phi}) < 0$.

Proof. Suppose, by way of contradiction, that $g'(\bar{\phi}) = 0$. By (27), this also implies $g''(\bar{\phi}) = 0$. Let $\hat{\phi} \equiv \sup\{\phi : \phi \geq \bar{\phi}, g''(\phi) = 0\}$. We must have $g''(\phi) > 0$ for all $\phi \in (\hat{\phi}, \hat{\phi} + \varepsilon)$ for some small enough ε , for otherwise we would have $g(\phi) < 0$, since

$$g(\phi) = \int_{\hat{\phi}}^{\phi} \int_{\hat{\phi}}^{\phi} g''(x) dx dx$$

Consider $\varepsilon' \in (0, \varepsilon)$ and let $\phi' = \hat{\phi} + \varepsilon'$. By a first order Taylor expansion

$$g(\phi') = g(\hat{\phi}) + g'(\hat{\phi})\varepsilon' + o(\varepsilon) = o(\varepsilon) \tag{28}$$

$$g'(\phi') = g'(\hat{\phi}) + g''(\hat{\phi})\varepsilon' + o(\varepsilon) = o(\varepsilon) \tag{29}$$

Substituting (28) and (29) into (27) we obtain

$$\frac{1}{2}g''(\phi')(\phi')^2(1 - \phi')^2\kappa^2 = \left(-\delta_1 i(\phi') + \frac{o(\varepsilon')}{\varepsilon'}\right)\varepsilon'$$

Since $\delta_1 > 0$ and $i(\phi')$ is strictly positive and bounded away from zero, it follows that, for ε' small enough, $g''(\phi') < 0$, contradicting that $g''(\phi) > 0$ for all $\hat{\phi} < \phi < \hat{\phi} + \varepsilon$.

Therefore it follows that $g'(\bar{\phi}) > 0$ and $g''(\bar{\phi}) < 0$. □

Lemma 6. $i(\phi) = \hat{i}$ in a neighborhood of $\bar{\phi}$.

Proof. An immediate corollary of the Lemmas of this part of the appendix is that

$$\frac{1}{2}g''(\phi)\frac{\varphi(\phi)}{i(\phi)^2} + g'(\phi)\frac{\varphi(\phi)\sigma}{i(\phi)} \leq 0$$

in a neighborhood of $\bar{\phi}$.

It follows that, in this neighborhood, the unconstrained maximizer in (10) is positive and equal to

$$M(\phi) = -\frac{g'(\phi)\varphi(\phi)\frac{\mu_1-r}{\sigma} + g(\phi)i(\phi)(\mu_1-r)}{g''(\phi)\frac{\varphi(\phi)^2}{i(\phi)} + 2g'(\phi)\varphi(\phi)\sigma}.$$

I want to show that $M(\phi) < i(\phi)$ if ϕ is close enough to $\bar{\phi}$ and, hence, that the only possible equilibrium is the one where $i(\phi) = \hat{i}$.

Towards a contradiction, suppose that, in equilibrium, $M(\phi) \geq i(\phi)$. It then follows that

$$\frac{1}{2}g''(\phi)\varphi(\phi) \geq -g'(\phi)\varphi(\phi)\sigma i(\phi) - \frac{1}{2}g'(\phi)\varphi(\phi)\frac{\mu_1-r}{\sigma} - \frac{1}{2}g(\phi)i(\phi)(\mu_1-r)$$

Substituting the latter expression in (12) we obtain

$$\rho g(\phi) \geq f(\phi) + g'(\phi)\frac{\varphi(\phi)}{\sigma} \left[\frac{1}{2}(\mu_1-r) - \frac{f(\phi)}{i(\phi)} \right] + g(\phi) \left[r + \frac{1}{2}i(\phi)(\mu_1-r) - f(\phi) \right].$$

Since $f(\phi) \rightarrow 0$ and $g(\phi) \rightarrow 0$ as $\phi \rightarrow \bar{\phi}$, then we would conclude that $\lim_{\phi \rightarrow \bar{\phi}} g'(\phi) \leq 0$. But this contradicts Lemma 5. Therefore, for ϕ close enough to $\bar{\phi}$, we must have $M(\phi) < i(\phi)$. \square

PROOF OF PROPOSITION 5

The right-hand side of equation (13) is linear in i . To prove the claim, it is therefore sufficient to show that

$$C(\phi) \equiv (\phi\mu_1 + (1-\phi)\mu_2 - r) + \tilde{g}(\phi)(1-\phi)(\mu_1 - \mu_2) + \tilde{g}'(\phi)\phi(1-\phi)(\mu_1 - \mu_2) > 0 \quad \forall \phi \in (\bar{\phi}, 1].$$

Proof. Recall that an analogous of Proposition 4 holds for $\tilde{g}(\phi)$, so that, $\tilde{g}'(\bar{\phi}) > 0$. Since $\tilde{g}'(\bar{\phi}) > 0$, if ϕ is sufficiently close to $\bar{\phi}$, then $C(\phi) > 0$. By assumption, $C(\phi)$ is contin-

uously differentiable in $(\bar{\phi}, 1)$. Therefore, consider $\phi' \equiv \inf\{\phi : \phi > \bar{\phi} \text{ and } C(\phi) \leq 0\}$. By way of contradiction, suppose such ϕ' exists. Then we must have $C(\phi') = 0$ and $C'(\phi') \leq 0$.

From $C(\phi') = 0$, I can derive that

$$-\tilde{g}(\phi') - \tilde{g}'(\phi')\phi' = \frac{(\phi'\mu_1 + (1 - \phi')\mu_2 - r)}{\phi'(1 - \phi')(\mu_1 - \mu_2)} > 0 \quad (30)$$

while from (13), we obtain

$$2\tilde{g}'(\phi')(1 - \phi') = 2\frac{(\rho - r)\tilde{g}(\phi')}{\phi'(1 - \phi')\kappa^2} - \tilde{g}''(\phi')\phi'(1 - \phi') \quad (31)$$

where $\kappa = \frac{\mu_1 - \mu_2}{\sigma}$.

Compute

$$\frac{C'(\phi')}{\mu_1 - \mu_2} = 1 + \tilde{g}'(\phi')(1 - \phi') - \tilde{g}(\phi') + \tilde{g}''(\phi')\phi'(1 - \phi') + \tilde{g}'(\phi')(1 - \phi') - \tilde{g}'(\phi')\phi'$$

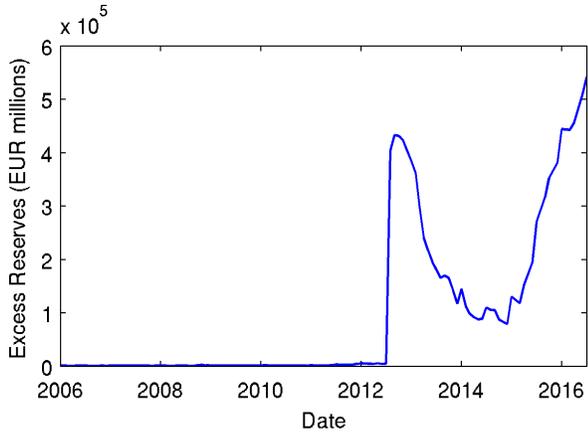
and use (30) and (31) to obtain

$$\frac{C'(\phi')}{\mu_1 - \mu_2} = 1 - \tilde{g}(\phi') - \tilde{g}'(\phi')\phi' + 2\frac{(\rho - r)\tilde{g}(\phi')}{\phi'(1 - \phi')\kappa^2} > 0$$

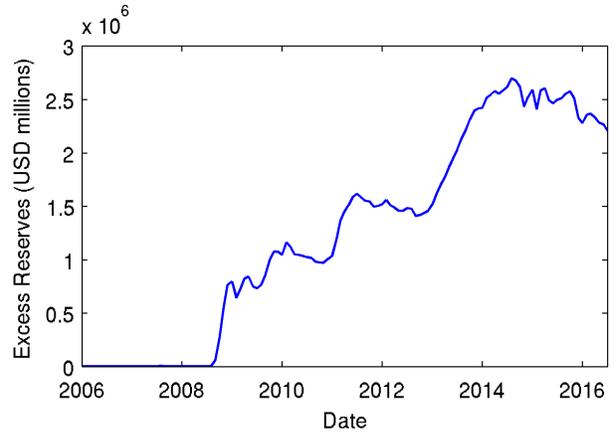
which contradicts that $C'(\phi') \leq 0$.

□

APPENDIX B ADDITIONAL PLOTS

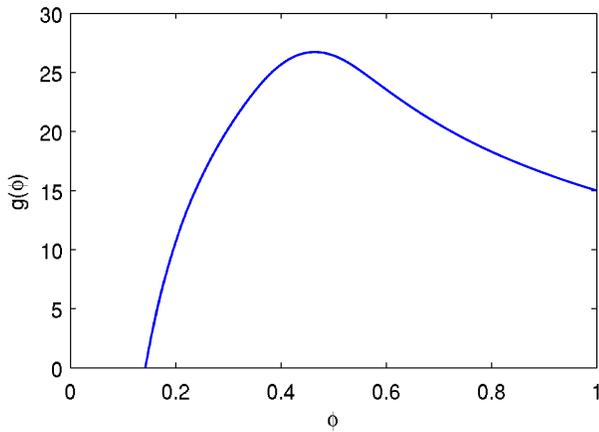


(a) Euro Area

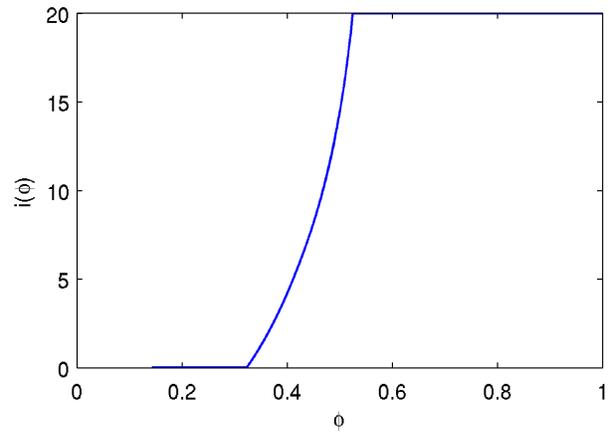


(b) United States

Figure 7: Excess reserves of institutions subject to minimum reserve requirements in the Euro Area and the United States

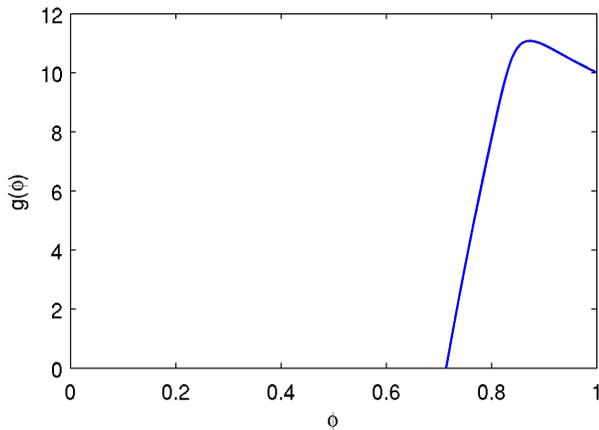


(a) Marginal Value of Capital

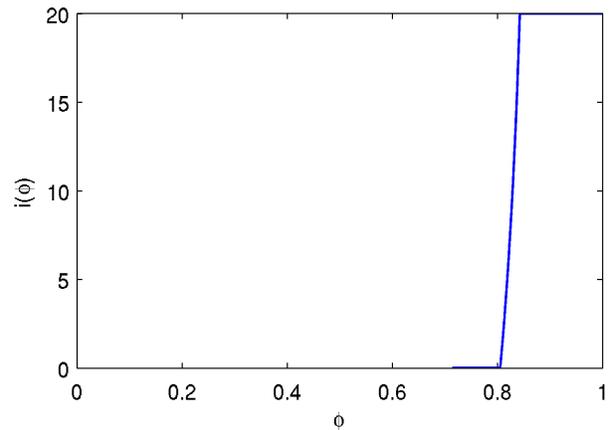


(b) Risky Assets to Total Capital

Figure 8: **Case 2.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = \mathbf{0.02}$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

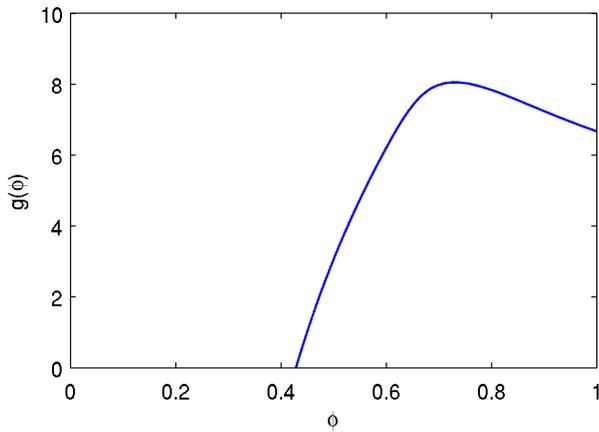


(a) Marginal Value of Capital

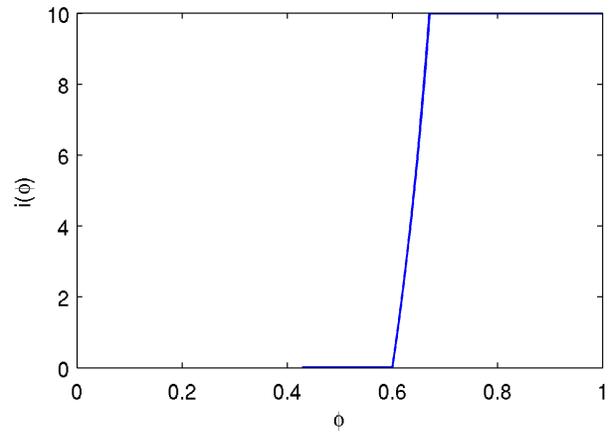


(b) Risky Assets to Total Capital

Figure 9: **Case 3.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = \mathbf{0.04}$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

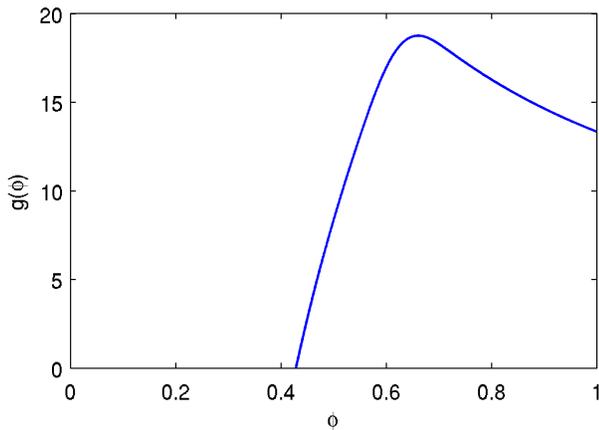


(a) Marginal Value of Capital

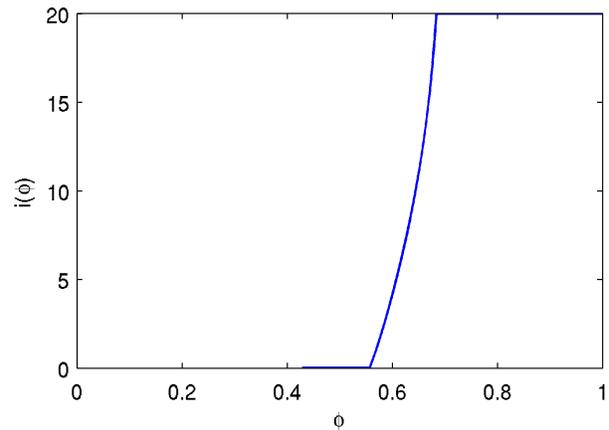


(b) Risky Assets to Total Capital

Figure 10: **Case 4.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 10$ and $\hat{i} = 10^{-9}$.

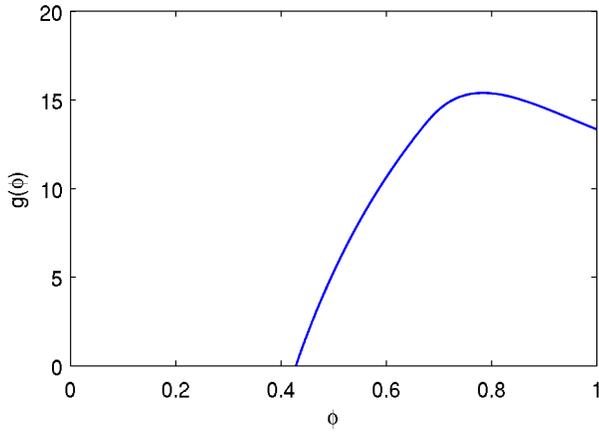


(a) Marginal Value of Capital

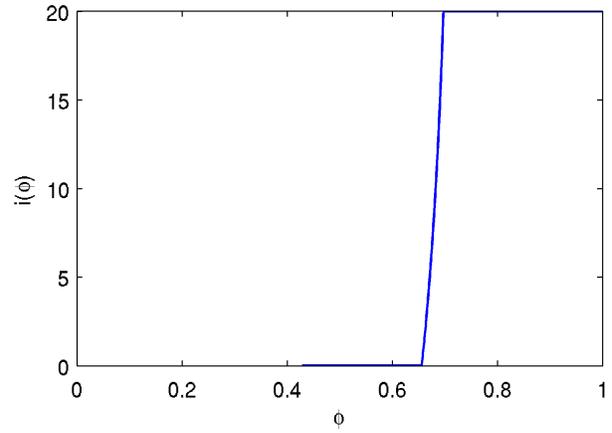


(b) Risky Assets to Total Capital

Figure 11: **Case 5.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.10$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

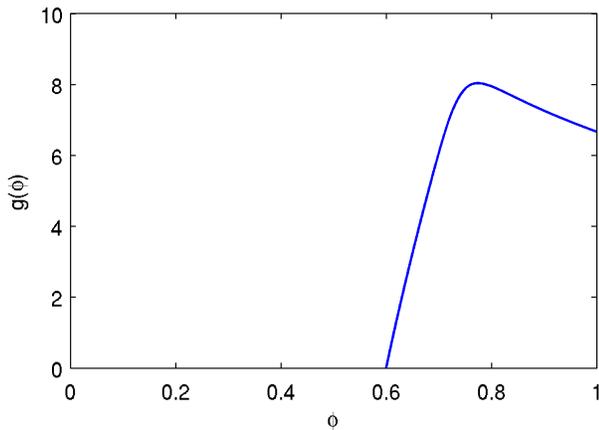


(a) Marginal Value of Capital

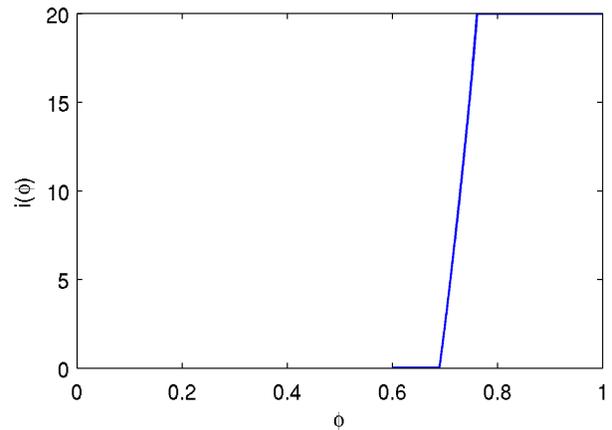


(b) Risky Assets to Total Capital

Figure 12: **Case 6.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = \mathbf{0.04}$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

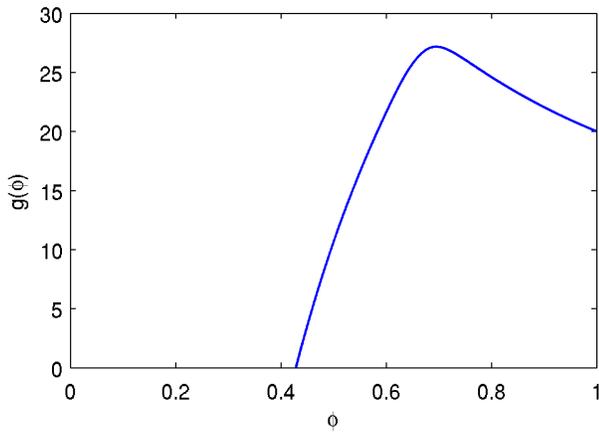


(a) Marginal Value of Capital

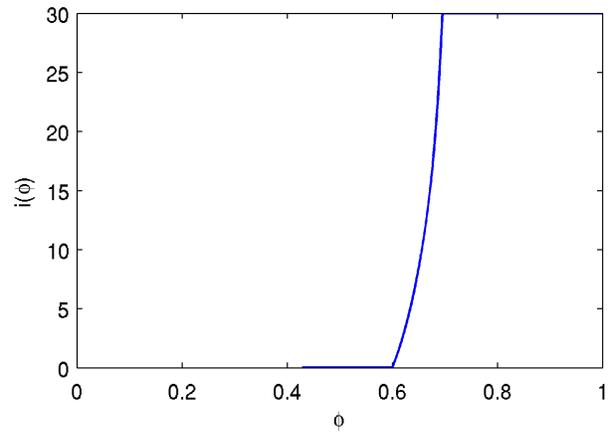


(b) Risky Assets to Total Capital

Figure 13: **Case 7.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = \mathbf{0.04}$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

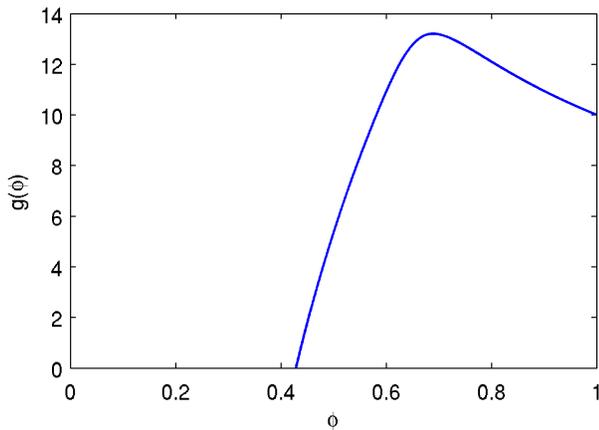


(a) Marginal Value of Capital

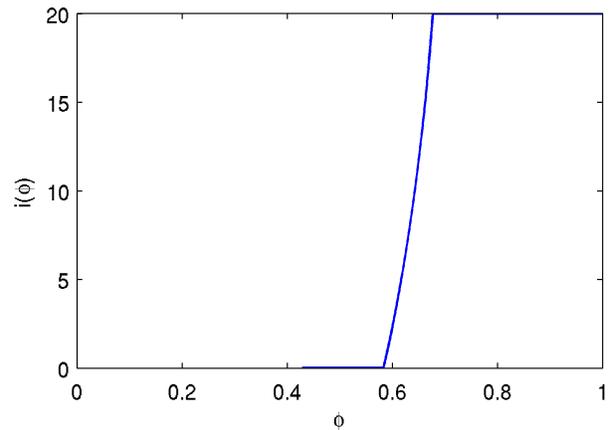


(b) Risky Assets to Total Capital

Figure 14: **Case 8.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = 0.06$, $\bar{i} = \mathbf{30}$ and $\hat{i} = 10^{-9}$.



(a) Marginal Value of Capital



(b) Risky Assets to Total Capital

Figure 15: **Case 9.** Marginal value of capital for the good intermediary and demand for risky assets as share of total capital under management. Parameter values are (parameters that differ from Case 1 are in bold): $r = 0.03$, $\mu_1 = 0.05$, $\mu_2 = 0.015$, $\sigma = 0.07$, $\rho = \mathbf{0.07}$, $\bar{i} = 20$ and $\hat{i} = 10^{-9}$.

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