

Is *Sarvajñatā* Synthetic or Gestalt?

Kundakunda, Cantor, and the ‘Inaccessibility’ of the Absolute

Although operating within very different religious and conceptual frameworks, the Jain mystic Kundakunda and the mathematician Georg Cantor contended with similar *a priori* problems concerning the relationship between the limited, relative purview of human knowledge and the singular, independent nature of the absolute. I argue that they solved these conceptual problems in their respective religious frameworks with strikingly analogous forms of reasoning. More precisely, both thinkers demanded an independently existent, transcendental absolute to render consistent their own respective systems of thought, which depended upon resolving the formal quandary of ‘inaccessibility’: i.e., the inability for any sequential, determinate objectifications to ever sum up – viz., a mereological “synthesis” - to the simultaneous comprehension of a genuine absolute - viz., a holistic “gestalt.” Though one thinker adhered to a quasi-Vedāntic form of Jainism, and the other was a devout Roman Catholic, both discovered that the *sequential* features of human cognition precluded access to the *kind* of simultaneous knowledge that the absolute must comprehend. In other words, both thinkers grasped *a priori* why we can never reach a genuine ‘absolute’ perspective from ‘the-bottom-up’, as a mereological sum of the *kinds* of objective knowledge-facts that figure into our limited, relatively conditioned epistemic states. I end the paper with two points: First, Kundakunda’s view of omniscience does not necessarily contradict traditional Jainism, given a few charitable hermeneutic maneuvers. Secondly, I address the topic of the paper from the perspective of process philosophy, as there are deep phenomenological relationships between ignorance, process, and the metaphysical possibility of omniscience. In the spirit of Charles Hartshorne and A. N. Whitehead, we end on the speculative note that Cantor’s

results do not, in fact, substantiate a theistic vision of a completed absolute; rather, it indirectly supports the idea that absolute reality is forever incomplete *qua* the creative process of becoming.

The Jain Conception of Omniscience:

Although the exact nature of omniscience has been the subject of debate in the Jain tradition, no sect denies that the *Tīrthankaras* were *Kevala Jñāna*, or *sarvajñatā*, possessed of absolute knowledge. The idea of the Jina as omniscient traces back to the earliest Jain *Sūtras*, perhaps most notably the *Ācārāṅga*, *Uttarādhyayana*, *Kaṣāya Pāhuḍa*, and the *Prajñapānā*. But omniscience in most of these cases is just mentioned as a given, without a systematic elaboration of its phenomenal content. Soteriologically speaking, the spiritual progress of the Jain ascetic is measured by the degree to which the soul unfetters itself from the ‘baser’ vibrations of heavy karmic particles. At each stage of spiritual progress, the scope of the practitioner’s knowledge correspondingly broadens to include greater and greater domains of awareness, with the terminal state being *sarvajñatā*.

In Umāsvāmi’s *Tattvārtha Sūtra* we get what is arguably the classic formulation of omniscience, which is contrasted with both acquired, or ‘mediate’ forms of knowledge, and other types of “innate” knowledge, such as mind-reading and clairvoyance. (*Tattvārtha*, 1.10) Since acquired knowledge is empirical or articulate (i.e. linguistically-based) and gained through the senses, its scope is limited to physical objects, the result of a combination of sensory and cognitive interaction. (ibid, 1.14) The clairvoyant and mind-reading forms of innate knowledge, on the other hand, represent stages at which partial elimination of karmic particles has been achieved, and thus the relations between the substance of the soul and ‘finer’ material clusters of other minds become identifiable in consciousness.

According to the progressive scheme outlined above, empirical knowledge extends over the domain of perceptible substances and their attendant modes. As karmic material burns off and the soul gains knowledge of ‘lighter’ or ‘subtler’ dimensions of reality, it is granted access to the innate forms of knowledge; “*rūpiṣu avadheḥ*. “[The purview of clairvoyance is] in regards to all matter [not in regard to its modes]”, and “*tadananatabāhge manaḥparyāyasya*. The scope of ‘mind-reading’ [is] only in regard to an infinitesimal portion of that [the purview of clairvoyance].” (1.28-29) Accordingly, “*mohakṣayāj jñānadarśanāvaraṇāntarāyākṣayāc ca kevalam*. And omniscience [emerges] from the destruction of delusion because of the attendant elimination of the karma covering [*varaṇā*] knowledge and perception.” (10.1) The complete elimination of heavy, gross karmic particles that obscure innate perception is liberation, and omniscience represents the pure consciousness that subtly interpermeates all other material substances: “*sarvadravya-paryāyeṣu kevalasya*. [The purview] of omniscience [*kevala*] is in regards to all substances in all their modes.” (1.30)¹

The pressing question is: what exactly does this state of omniscience consist of in the Jain dialectic? The *Kalpa Sūtra*, for instance, plainly describes Mahāvīra’s *sarvajñatā* as a trans-spatiotemporal comprehension of the state of all realities:

When the Venerable Ascetic Mahāvīra had become a Jina and Arhat (Arihant), he was a *Kevali*, omniscient and comprehending all objects; he knew and saw all conditions of the world, of gods, men, and demons: whence they come, whither they go, whether they are born as men or animals or become gods or hell-beings, the ideas, the thoughts of their minds, the food, doings, desires, the open and secret deeds of all the living beings in the whole world; he the Arhat, for whom there is no secret, knew and saw all conditions of all living beings in the world. (Jaini, pp. 99–100)

For the mainstream Jain interpretation, then, omniscience means literally knowing everything in the *prima facie* sense: The Jina has access to every fact there is, or could be, to know, in all times

¹ One could argue that the innate knowledge of clairvoyance and mind-reading are still tied to ignorance, because as long as even one mode or substance remains obscure from the soul, base karmic material still binds consciousness. As Tatia comments, “strictly speaking, only omniscience is perfectly innate because it alone arises out of the total elimination of knowledge-covering karma and therefore knows everything, past, present, and future.” (Tatia, p.13)

and places. The first Digambara commentary on the *Tattvārtha Sūtra* (1.30) Pūjyapāda's *Sarvārthasiddhi* (464 - 524 CE), strongly reinforces this impression:²

The substances of the souls are infinite infinities,³ and the substances of material objects are also infinite infinities. Atoms are the divided portions of bodies; the mediums of motion, rest and space are threefold; and time is innumerable. And the modes of each of these three mediums - existing through past, present and future - are infinite infinities: There is absolutely no substance or mode that surpasses the scope of the knowledge of *Kevala* consciousness. (*Sarvārthasiddhi* 1.29)

Given that this is among the earliest authoritative commentaries on the *Tattvārtha*, we can safely say that the standard interpretation amongst mainstream Jain thinkers from the beginning has been more or less the kind of all-knowingness put forth here by Pūjyapāda.⁴

Although nobody but the Jina has complete knowledge, Jain logicians formulated epistemic structures to approximate a complete understanding of his multifaceted view of nature. As Siddhasēna writes in the *Nyāyavatāra*, “An object has a multifaceted nature [and thus] is [only] within the range of omniscience. [But] an object is considered suitable for a *naya* (perspective) when distinguished from a single side.”⁵ (*Nyāyavatāra*, v.29) The *Tattvārtha* (1.35) states that there are seven kinds of standpoints, which are generally held among both the Digamabara and Śvētāmbara sects: *naigama* (the common view), *saṃgraha* (the general view), *vyavahāra* (the practical view), *ṛjusūtra* (the linear view), *śabda* (the verbal view), *samabhirūḍha* (the etymological view), and *evambhūta* (the actuality view). A complete account of the substances and modes of an object is rendered when we employ this seven-fold scheme of the *nayavāda* to

² *jīvadravyāṇi tāvadanantānantāni tato 'py anantānantāni pudgaladravyāṇi ca anuskandhabhedabhinnāni dharmādharmākāśāni trīṇi, kālaścāsakhyeyasteṣāṃ paryāyāśca trikālabhuvāḥ pratyekamanantānantāsteṣu, dravyam paryāyajātaṃ vā na kiṃ citkevalajñānasya viṣayabhāvamatikrāntamasti.*

³ See footnote on numerical significance of ‘infinite’ [*ananta*] in Jain thought below.

⁴ Part of the motivation, no doubt, for such a totalizing vision of omniscience is a practical one; the Jina's *āgama* must withstand challenges from the Buddhists and other rival doctrines. If the Jina is omniscient in this absolute sense, “his teaching had full authority, against which the claims of other sects and schools were flawed and incomplete.” (Dundas, p.89) We will see in the conclusion that the concept of omniscience has important implications for the picture of Jainism as a tradition that promotes religious tolerance.

⁵ *anekāntātmakam vastu gocaraḥ sarvaṃvidam | ekadeśaviśiṣṭo arthā nayasya viṣayo mataḥ*

any propositional statement. For our purposes, the details of each of these views are not important.⁶ What *is* significant about this scheme is that any propositional statement about reality can be dissected and analyzed from each of these perspectives, and each is *equally valid*, albeit incomplete independent of the simultaneous knowledge of the others (i.e. not perceived from the comprehensive view of omniscience).

In addition to *nayavāda*, there is the complimentary doctrine of seven-fold predication called *syādvāda* which proclaims that the factuality of any proposition given from a limited standpoint depends upon proper ‘parameters’ or conditions (indicated by the optative 3rd person form of the verb ‘as’ - namely, ‘*syāt*’, or ‘may be’). Matilal (p. 55) symbolizes these predicates by the set (+, -, +-,0). The ‘inexpressible’ (0) predication represents the simultaneous instantiation of both the positive and the negative predicates, while the ‘+’ signifies their *gradual* application. Since there is no way to state in ordinary language this simultaneous instantiation of the positive and negative assertion, it is called ‘inexpressible.’ If we multiply the possibilities of the combinations of these three, we get the seven-fold schema of *syādvāda*.⁷ More specifically, for any given proposition, we get these seven possible predications subject to certain assertability conditions:

1. From a certain point of view, or in a certain sense, [*syāt*] *x* exists. (+)
2. *syāt...x* does not exist. (-)
3. *syāt...x* both exists and does not exist. (-+)
4. *syāt...x* is inexpressible. (0)
5. *syāt...x* exists and is inexpressible. (+0)
6. *syāt...x* does not exist and is inexpressible. (-0)
7. *syāt...x* exists, does not exist, and is also inexpressible. (-+0)

⁶ See Tatia (p.26) for an elaboration of these categories in terms of the propositional case ‘there is a forest’.

⁷ We must take into account the communicative property of conjunction, in which order is immaterial, such that ‘+-’ is the same as ‘-+’, etc.

The ideas outlined in the *syādvāda* and the *nayavāda* doctrines represent the methodological counterparts of *anekāntavāda* for epistemological or philosophical purposes. As Matilal writes, “according to the *anekānta* doctrine, a thing or entity is supposed to possess infinite or innumerable aspects or characters. But the sevenfold formula...will be applicable to each attribution of a property, i.e. to each individual predication.” (p.56) Thus every single predicate attributed to any given property has a limited number of possible qualities from an ignorant perspective. From the conventional viewpoint, the way we approximate a systematic and complete understanding of an entity is through subjecting it to an analysis based on these seven-fold criteria of *syādvāda* and *nayavāda*. Accordingly, we could plausibly affirm that the omniscient state that extends to ‘the domain of all substances and modes’ involves access to the set of predicates associated with each member of the set of all substances and modes as perceived from all perspectives. In the next section we show that this understanding of omniscience poses *a priori* problems because of its ‘inaccessibility’, and present Kundakunda’s gnostic approach that overcomes these difficulties.

The Problem of Inaccessibility and Kundakunda’s ‘Gestalt’ Theory of Omniscience:

Suppose we accept that ‘the domain of omniscience extends to all substances in all their modes’. Then ‘omniscience’ plausibly signifies a synthetic construction of the total set of relative knowledge-facts about all substances and modes from all *nayas* as conditioned by all *syāt* predicates. In simple terms, one might then suppose: isn’t the sum of all relative knowledge equal to absolute knowledge? The standard Jain response is “no,” at least according to Samantabhadra, who makes a distinction between the *simultaneous* nature of omniscience and the *sequential* nature of limited perspectives: “As *pramāṇas*, true knowledge is that which reveals everything *simultaneously* (*yugapat*), [in contrast to] that knowledge which exists in *sequence* (*krama*) subject

to the doctrines of *syādvāda* and *naya*.⁸ (*Āptamīmāṃsā*, v.101) Samantabhadra argues that omniscience is ‘*yugapat*,’ or simultaneous, which contrasts to the limited relative perspectives of the successive *nayas*, characterized by *krama*, or sequence. As noted, the *yugapat* vs. *krama* distinction represents the crucial point from which the entire ensuing discussion proceeds, because the difference between absolute knowledge existing all-at-once, as opposed to some mereological sum of discrete, relative facts (even *an infinite amount of facts*, as we will discuss below) corresponds to the difference between an inaccessible set of absolute truths (‘*gestalt*’) and a sum total of any set of relative, conceptually accessible truths (‘*synthetic*’), respectively.

Seen through the metaphysical lens of the *Tattvārtha*, the *yugapat* and *krama* distinction entails that the former is inherent to the nature of the unfettered soul. So Samantabhadra recognizes that since omniscience reflects the natural state of the karmically unsullied Self, this perspective must pre-exist in a totalizing sense, or it does not exist at all. In other words, *the idea of any sum of relative truths ‘adding up’ to the state of omniscience is wrong* - and soteriologically so, because omniscience is the end result of complete *deconditioning* through sole identification with the Self, rather than any infinite sum of discrete *kramatic facts* about the *external* states of things. The Jina recognized that facts only pertain to objects of the external world as perceived from the *kramatic* limitations of the relative perspective; *they appear as they do* only because we are in karmic synchronization with them, which obfuscates the pure, all-pervasive consciousness of the Jina.

The important point here is that no sum of relative truths about substances and modes can add up to omniscience *even though orthodox Jainism formulates the notion of omniscience in precisely those terms*. Thus we have a seeming tension between the idea of the soul as inherent all-knowingness, a simultaneously pre-existent state of absolute knowledge, and the traditional

⁸ *tattvajñānaṃ pramāṇaṃ te yugapatsarvabhāsanam | kramabhāvi ca yacjñānaṃ syādvādanayasamiskṛtam.*

definition of omniscience as knowledge of all possible discrete facts about all substances in all their modes. It is this tension that I believe Kundakunda both recognizes and successfully defuses in his ‘gestalt’, or ‘gnostic,’ understanding of omniscience. For him, omniscience does not equal the sum of all distinct relative truths – it must be a kind of knowledge fundamentally transcendent, but somehow encompassing, of all relative truth. Kundakunda took *sarvajñāna* to mean that there is no terminal sum or totality of facts about the world within the framework of *nayavāda* and *syādvāda* that could ever constitute absolute knowledge.

More specifically, given the *a priori* desirability of the holistic conception of omniscience as an intrinsic property of the soul - and (perhaps reluctantly acknowledged) the general implausibility of a human being with conscious access to a set of infinitely arbitrary facts about the external world - Kundakunda expressed a “gnostic” take on omniscience; the liberated soul represents the Absolute, the One Reality completely exempt from partaking in the karmic cycle of ignorance. In arguably his most important work, the *Samayasāra*, Kundakunda puts forth his ‘two-truths’ doctrine of *vyavahāranaya*, the ‘mundane perspective’, and the *niścayanaya*, or ‘ultimate perspective’: “It is impossible for the mundane perspective to show the real, but the absolute perspective reveals the real. The soul [possessed of the absolute perspective] has true perception.”⁹ Thus no empirical realities can relate to the transcendental nature of the liberated soul.¹⁰ In this case, decidedly *unlike* Vedantic mysticism, liberation *qua* self-knowledge obtains, not from a non-dual conception of *Brahman*, *Ātman*, and the external world, but rather because of the absolute *independence* of the pristine nature of the Self from the karmic flow of matter that defines the *saṃsāric*, empirical world: “In so far as the soul does not realize that the two categories of *Ātma*

⁹ *vyavahāro 'bhūtārtho bhūtārthodeśitastu śuddhanaya bhūtārthamāśrita khalu samyagdr̥ṣṭirbhavati jīva*

¹⁰ *arasamarūpamagandhamavyakta cetanāguṇamaśabdām jānīhi aliṅagrahaṇa jīvamanirdiṣṭasasthānam.* (*Samayasāra*, v. 49) Know that the soul is without taste, form, smell, touch and sound. It cannot be grasped by inferential knowledge and is impossible to elucidate; it is characterized by pure consciousness.

and *āsrava* [the transcendental Self and the changing empirical conditions of karma] are completely distinct, to that extent it is ignorant and identifies with anger, etc...”¹¹ Thus, for Kundakunda, this ‘Vedāntic’ perception of the Self as the transcendental Reality, unsullied by any conceptual or empirical category, results from a decidedly *dualistic* metaphysics. The matter of the world that is generated by the self-perpetuating procession of karmic manifestations is ontologically distinct from the transcendental Self. This realization is liberation: “Whenever there is knowledge of this categorical distinction between *ātma* and *āsrava* by the Jiva, then there is no more karmic bondage.”¹²

This strict metaphysical dualism is characterized by a kind of transcendental Self that reflects an absolute stasis, devoid of any novelty associated with the epistemic sequencing of karmic conditions:

The Jiva does not effect the qualities of *karma*, likewise, [*karma* does not effect] the qualities of the Jiva. Know rather that the changes of the two are because of mutual instrumental-causation. According to this position, the Self is the [substantial cause] of its own becoming, not the cause of the becoming of any *karmic* matter. Thus from the perspective of ultimate knowledge [*niścayanaya*] it is understood that the Self only produces its own Self repeatedly; know that the Self is only [of the nature] of the Self. But from the perspective of mundane knowledge [*vyavahāra*] the Self produces sundry [effects in] karmic material, and likewise, these sundry effects are known [by the Self.]¹³ (v. 81-83)

Kundakunda refers here to two kinds of causation: *nimitta kāraṇa*, instrumental causes, and *upādāna kāraṇa*, substantive causes. The substratum of the Self, being pure consciousness, cannot represent the substantive cause of karmic modifications – they can only be instrumentally related. Thus what characterizes the absolute knowledge of the Self is that it represents its own prehension; the Self has no constraints because *its only object of knowledge is itself*. In Kantian terms, an

¹¹ *yāvanna vetti viśeṣāntara tvātmāstravayordvayorapi ajñānī tāvatsa krodhādiṣu varttate jīva* (ibid, v.69)

¹² *yadānena jīvenātmana āstravāṇā ca tathaiva jñāta bhavati viśeṣāntara tu tada na bandhastasya* (ibid, v. 71)

¹³ *nāpi karoti karmaguṇān jīva karma tathaiva jīvaguṇān anyonyanimittena tu pariṇāma jānīhi dvayorapi eteṇa kāraṇeṇa tu kartā ātmā svakena bhāvena pudgalakarmakṛtānā na tu kartā sarvabhāvānām niścayanayasyaivamātmamānameva hi karoti vedayate punasta caiva jānīhi ātmā tvātmānām vyavahārasya tvātmā pudgalakarma karoti anekavidham taccaiva punarvadayate pudgalakarmānekavidham* (ibid, v.81-83)

unlimited mind is characterized by the fact that the changing content of its intuitions are not constrained as a receptive capacity of an *unknown* ‘external world’; the bridge between the activity of the understanding and the passivity of sensibility has dissolved in an inconceivable self-reference of objectification.

The idea that the matter we experience in *saṃsāric* life depends upon perpetual karmic modification is critical for our argument, because it relates Kundakunda’s metaphysical dualism directly to the *yugapat* vs. *krama* distinction in the epistemic realm of omniscience; the kind of processual disclosure that *defines* limited knowledge from the *syad* and *nayavāda* doctrines is relinquished in favor of a simultaneously existent stasis of conceptual development that represents the innate omniscience of the Self. There is no perceived *kramic* change anymore, because all activity now only occurs between the Self and itself! Indeed, in the *Pravacanasāra*, Kundakunda affirms along with Samantabhadra the importance of the simultaneous nature of omniscience: “If those past and future objects of knowledge were not directly illuminated knowledge, who would say, ‘that [knowledge] is truth’? It is proclaimed that those who know objects of sense-perception by means of stages and discrimination, they are not able to know the existence [of objects] beyond the senses.”¹⁴ (*Pravacanasāra* v. 1.39-40) Kundakunda emphasizes that omniscience is not congruous in any way to the *kramatic* sequence of *vyavaharic* knowledge, which we interpret as empirical change. The nature of omniscience necessarily discloses a holistic structure wherein reality is no longer known via the usual avenues of *karmic* perception attuned to the lower frequencies of *kramic* succession - rather knowledge becomes identified with Being:¹⁵ “I am truly

¹⁴ *yadi pratyakṣo’jātaḥ paryāyaḥ pralayitaśca jñānasya na bhavati vā tat jñānaṃ divyamiti hi ke prarūpayanti. arthamakṣanipatitamīhāpūrvaiḥ ye vijānanti teṣāṃ parokṣabhūtaṃ jñātumaśakyamiti prajñāptam (Pravacanasāra v. 1.39-40)*

¹⁵ *aḥameka khalu śuddhanirmamaśca jñānadarśanasamagra tasmin sthitastaccitta sarvānetān kṣaya nayāmi*

one, pure, free of worldly connections, possessing complete knowledge and perception. Abiding in the awareness of this [Self], I will lead all these [*āsravas*] to destruction.” (*Samayasāra*, v. 73)

As discussed, the *vyavaharānaya* is the relative perspective of the traditional *nayas*, those that characterize the world of conditioned experienced as emerging, perishing and enduring. Epistemically, this represents the fallen state of ignorant, non-omniscient souls still karmically bound to the rounds of *saṃsāra*. The other perspective, *niścaya*, is the true and absolute perspective of the omniscient Jina who has achieved liberation. Although Long notes that - “his two perspectives are not, like the seven *nayas* of mainstream Jainism, simply alternative, partly valid ways of viewing the soul. The *niścayanaya*, rather, is a *true* perspective, and the *vyavahāranaya*, as an impediment to liberation, is ultimately a *false* one.” (Long, p.128) – he ultimately claims that Kundakunda’s distinction is “largely a verbal matter.” (p.255)

According to our position, however, this is not the case. What we are claiming here is that the traditional notion of *kevalajñāna* is theoretically *accessible* from the traditional seven *nayas*, albeit in terms of an infinite sum of *nayas* (this would represent the infinite set of all facts concerning ‘knowledge of all substances in all their modes’). We mean accessible here in a technical sense, which we will discuss in the next section. But for now, it means something like ‘unequal only in terms of amount, not kind.’ That is to say, accessibility for us means that the *kind* of knowledge that figures into the sequential epistemic states of ignorant beings can *theoretically* equal the epistemic state of the omniscient Jina when taken as an infinite collection; namely, synthetic knowledge, symbolized by the *kevalajñāna* (*qua* the potentially infinite sum of all possible *nayas*), as opposed to the ‘gestalt’ knowledge of omniscience, represented by *niścayanaya*. In other words, I have presented the case that Kundakunda believes that *kevalajñāna* is *not* accessible ‘from-the-ground-up’ - that is, as *any mereological sum of the kind of kramatic*

epistemic states that figure into the limited perspective of vyavahāranaya. The omniscience that Kundakunda proposes is not constituted by the *kind* of knowledge that comprises that of the seven *nayas*, and, as we will see, that includes any infinite sum of discrete facts about substances and their modes as perceived from all the *nayas* taken collectively - which is precisely Long's understanding of the matter. If knowledge genitively relates to the knower in any way, we are still perceiving from the *vyavaharic* perspective, projecting a vision of omniscience in those terms - that is, as an inconceivably vast set of discrete facts about the world that represents the static, monolithic object of absolute knowledge. But in the state of omniscience, one's *being* is identical to one's *knowledge*. When knowledge is viewed as *possessed* - as a set of conceptually discrete 'knowledge-facts' about the external world - it is necessarily objectified in a *kramatic* format, and thus cannot have any type of relation to the pure knowledge of the Self.

For the purposes of our discussion, Kundakunda articulated, firstly, the intuition that just *one* kind of knowledge, viz. self-knowledge, represents the meaning of the Jain conception of omniscience. It does not depend upon any *conventional* idea of 'knowing each and every discrete fact about all of nature', since the degree to which one knows about the true multifaceted constitution of a thing only depends upon the extent to which they have realized the true unfettered nature of the soul. Secondly, the state of *niścayanaya* presents a *holistic* and *transcendental* notion of truth as more than simply the sum of any set of relative facts about the external world. The conventional *nayas* of *vyavahāranaya* can never add up to the ultimate truth, so omniscience cannot merely represent a synthesis of discrete facts about substances and their modes; it must fundamentally encompass these truths but also inconceivably go beyond them. In short, omniscience must be a 'gestalt' state – a whole that is more than the sum of its parts, eternally self-

complete in an *absolute* and *transcendental* sense, rather than any infinite collection of discrete facts or relative truths about an objectified reality.

Georg Cantor and the ‘Absolute Infinite’:

In order to explain why Cantor’s vision of the absolute jibes with Kundakunda’s, we need to briefly explain the conceptual origins of set theory. The lynchpin of George Cantor’s mathematics of *transfinite cardinals* (or non-finite numbers), and its relevance to our discussion of omniscience, is his rejection of the classic Aristotelian distinction between ‘actual’ and ‘potential’ infinity. ‘Potential infinity’ was the only sensible notion of infinity for Aristotle: "For generally the infinite has this mode of existence: one thing is always being taken after another, and each thing that is taken is always finite, but always different." (Aristotle, *Physics*, III.vi.) Before Cantor, most mathematicians and philosophers agreed with this Aristotelian picture of the infinite as a hypothetical limit point, but did not treat it as an actual infinity (this ‘limit point’ is the type of infinity assumed in calculus and algebra, for instance). Potential infinity represents a deficient kind of infinity; it amounts to the abstract ability to continually add a new discrete element to a given *finite* set of elements (the only ones that exist, in this picture). If we continue this process forever, Aristotle held that it would asymptotically *approach* an ‘actual infinity’, i.e. a *closed* and *complete* set of infinitely many elements whose mathematical structure pre-exists in some formal way, but could never actually reach it: “Nothing is complete (*teleion*) which has no end (*telos*); and the end is a limit.” (ibid) Aristotle believed that if we cannot *add* an element to a set and simultaneously change its quantity, it is a paradox by conventional standards, and so cannot be admitted into an account of the actual world.¹⁶ We can never *reach* an actual infinity – it is ‘inaccessible’.

¹⁶ For Aristotle, this concept meant the “annihilation of number”. Technically speaking, given any two finite numbers a and b , both greater than zero, their sum $a+b>a$ and $b+a>b$. However, if either a or b is infinite, no matter what finite

Cantor adamantly opposed Aristotle's distinction, which relegated actual infinity to the realm of absurdity because it did not adhere to the same rules as the arithmetic of integers. Cantor was not troubled by this 'paradox' of arithmetic; why should the transfinite numbers obey the same kinds of mathematical rules that limited integers do? He believed that the formal laws governing transfinite numbers were internally consistent, given their distinctive axiomatic structure. Although he offered several philosophical arguments in favor of the reality of transfinite numbers, (Dauben, pp. 125-7) the most germane involves a somewhat Platonic reversal of Aristotle's view; instead of starting with the presumption of 'potential infinity' and concluding that its asymptotic *limit* represents the impossible completion of 'actual infinity,' the latter is what *really* exists, and the finite integers of potential infinite are actually limited *abstractions* derived from this actual infinity:

One can irrefutably prove the occurrence of the actual infinite and its indispensability both in analysis and in number theory and algebra. There is no doubt that we cannot do without variable quantities in the sense of the potential infinite; and from this the necessity of the actual infinite can also be proven, as follows: In order for there to be a variable quantity in some mathematical inquiry, the 'domain' of its variability must strictly speaking be known beforehand through a definition. However, this domain cannot itself be something variable, since otherwise each fixed support for the inquiry would collapse. Thus, this 'domain' is a definite, actually infinite set of values. Thus, each potential infinite, if it is rigorously applicable mathematically, presupposes an actual infinite. (Cantor, 1887-88, p. 410-411)

This argument – that each variable presupposes a comprehensive domain of unity – should appear familiar, as it maps onto Kundakunda's insistence that a pre-existent, independent absolute is necessary to explain the Jina's knowledge of all relative entities. For Cantor, it is impossible to establish a variable range of possible magnitudes without already abstracting those magnitudes from a completed conceptual domain. Just as every potential infinity presupposes its own actual infinity, so too does every variable magnitude “presuppose its own domain of variability.”¹⁷

value we choose for either number, a or $b + \infty = \infty$. For Aristotle, the “annihilating” behavior of an actual infinite meant that it violated the consistency of arithmetic, and thus proved it was absurd.

¹⁷ Cantor used this argument to make an analogy between the totality of the transfinite numbers and the irrational numbers: “The transfinite numbers themselves are in a certain sense *new irrationals*, and in fact I think the best way

It should certainly be noted that Cantor was motivated in large part by an earnest religious sensibility. Although Jewish by heritage, he was raised as a devout Roman Catholic and had always perceived his mathematics as indissolubly linked to his theological and metaphysical attitudes. Cantor believed that by “setting mathematics free” to investigate the transfinite numbers, humans gained access to the divine realm of the actual infinite, one which had theretofore been barred from the masses. He thus often thought of set theory as divine revelation: “I entertain no doubts as to the truth of the transfinite, which I have recognized with God’s help...” (Bendiek 1965, 68) And while Cantor’s conception of the absolute undoubtedly had theological origins, it plays an instrumental role within Cantor’s system of transfinite numbers.

It is necessary at this point to outline the basic methods for creating the hierarchy of well-ordered sets in the set theoretical universe. A set S is a certain collection of things, which may be either finite or infinite. The cardinality of a set S is the number of things in that set, or, in other words, the size of the set. A subset of a set S is some collection of the things in S - which may include all of those things, or just some of them. There are only two arithmetical operations that allow sets to get bigger: the *union* operation, denoted by \cup ($A \cup B = \{x : x \in A \text{ or } x \in B\}$), which is analogous to addition, joining the elements of two distinct sets to form a new larger set; and the *power set* operation, denoted by $\mathcal{P}(S)$, which is analogous to multiplication and constructs a new set out of all of the subsets of a given set (this is generalized as the power relation 2^n where n represents the number of elements in the set). For our discussion, the latter is far more theoretically important. Consider, for example, a set with just two members: $\{a, b\}$. The cardinality of this set

to define the *finite* irrational numbers is entirely similar...One can absolutely assert: the transfinite numbers *stand or fall* with the finite irrational numbers; they are alike in their most intrinsic nature; for the former like these latter [numbers] are definite, delineated...” (ibid) So, according to Cantor, it would be impossible to define the irrational numbers without presupposing an infinite set of rational numbers. If a mathematician is willing to accept the definite reality of irrational quantities, they should be willing to accept the definiteness of transfinite numbers. (Jané, p.385)

is 2. The subsets of this set would be: $\{a\}, \{b\}, \{a,b\}$ (since a set is always considered a subset of itself), and the empty set $\{\}$ (which is axiomatically a subset of any set). Thus, if we perform the power set operation on this set, we would get a larger set with these four sets as members - $\{\{a\}, \{b\}, \{a,b\}, \{\}\}$ - and its cardinality would be 4. So from the operations of union and power we obtain a hierarchy of cardinal sets wherein each can be stacked on the other in terms of cardinal sequence. Cantor's Theorem¹⁸ states that *the cardinality of the power set of S will always be strictly greater than the cardinality of S , even for infinite sets.*

The notion of the “absolute infinite”, and its function within Cantor's system, emerges from the ordinal hierarchy of transfinite numbers. In the same way that we can list sets of natural numbers in a hierarchy of values, with each set in the hierarchy corresponding to a distinct ‘place’ *à la* the order of the natural numbers (in other words, we can number sets in terms of their consecutive size), Cantor believed that if he could prove that each *transfinite* number corresponds to an ordinal value (a generalization of the natural numbers used for counting infinite sets) in a hierarchy of distinct, well-ordered sets, he will have proved the reality of the transfinite domain.¹⁹

¹⁸ The formal proof of Cantor's Theorem works for both finite and infinite sets by *reductio*:

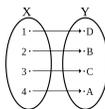
*Theorem: Let f be a map from set A to its power set $\mathcal{P}(A)$. Then $f: A \rightarrow \mathcal{P}(A)$ is not surjective.
As a consequence, $\text{card}(A) < \text{card}(\mathcal{P}(A))$ holds for any set A .
Proof: Consider the set $B = \{x \in A \mid x \notin f(x)\}$.
Suppose to the contrary that f is surjective. Then there exists $\xi \in A$ such that $f(\xi) = B$.
But by construction, $\xi \in B \Leftrightarrow \xi \notin f(\xi) = B$. This is a contradiction.
Thus, f cannot be surjective. On the other hand, $g: A \rightarrow \mathcal{P}(A)$ defined by $x \mapsto \{x\}$ is an injective map
Consequently, we must have $\text{card}(A) < \text{card}(\mathcal{P}(A))$ (Enderton, p. 132 – 33)*

¹⁹ For Cantor, and most mathematicians, to be real meant to be distinct, to have a definite value independent of other mathematical entities. If he could prove that we can ‘count’ transfinite sets with ordinal numbers, like we ‘count’ the *natural numbers*, he will have established their distinctness *from each other*, and thus will have shown that actual infinity manifests in different sizes. Infinity does not consist merely in the potential absolute, with a leap of faith to some God as a *true* absolute. Rather, there is another universe of infinity in between the potential infinity and the absolute: “The assumption that apart from the absolute - unreachable by any determination - and the finite, no modifications can exist which, though not finite, are nonetheless determinable by numbers and consequently are what I call the proper infinite; this assumption I do not find justified by anything... What I maintain and believe I have proved... is that after the finite there is a *transfinitum* (which could also be called *suprafinitum*), i.e., an unlimited gradation of definite modes which in their nature are not finite but infinite, yet which, just as the finite, can be determined by definite, well-defined and mutually distinguishable numbers.” (Cantor, 1883b, p. 176)

In order to compare the size of infinite sets, Cantor formalized the idea that two sets have an equivalent cardinality if there is a bijection (one-to-one correspondence) between their members. More specifically, bijection is a function between the elements of two sets, where each element of one set is paired with exactly one element of the other set, and each element of the other set is paired with exactly one element of the first set. There are no unpaired elements.²⁰

We begin with the first transfinite cardinal, \aleph_0 (aleph-zero or aleph-naught), which represents the cardinality of the smallest transfinite number, the set of all the natural numbers \mathbb{N} . In order to generate a new transfinite set from \aleph_0 , Cantor used the power set operation, 2^{\aleph_0} . Cantor's first article on set theory in 1874 - inconspicuously titled "On a Property of the Collection of All Real Algebraic Numbers" so as not to arouse ire from mainstream mathematicians who still rejected the actual infinite - proved that no bijective function obtains between the discrete, countably infinite set of the natural numbers and the infinity of the continuum, or the set of all real numbers on the number line. According to Cantor's Theorem, the power set of a countably infinite set must then be uncountably infinite, or the power set of natural numbers (\mathbb{N}) can be put in a one-to-one correspondence with the set of real numbers (\mathbb{R}).²¹ In other words, he had established that

²⁰ In mathematical terms, a bijective function $f: X \rightarrow Y$ is a one-to-one (injective) and onto (surjective) mapping of a set X to a set Y . The picture below represents a bijective function, $f: X \rightarrow Y$, where set X is $\{1, 2, 3, 4\}$ and set Y is $\{A, B, C, D\}$ (for example, $f(1) = D$):



²¹ In 1891 Cantor refined and simplified his argument regarding the impossibility of bijection between the natural and real numbers, producing his most famous proof of the idea that there are infinite sets with a greater cardinality than \aleph_0 . It's called the diagonal argument, and it goes like this: Imagine lining up all of the members in a well-ordered set of the infinite collection T of binary digits (a base 2 system of numeration, by which we can theoretically represent any natural number.) We can label these s_1, s_2, s_3 , and organize them into an ascending order. Now let's consider a new sequence s , constructed by choosing the 1st digit as complimentary to the 1st digit of s_1 (swapping 0s for 1s and vice versa), the 2nd digit as complementary to the 2nd digit of s_2 , the 3rd digit as complementary to the 3rd digit of s_3 , and generally for every n , the n^{th} digit as complementary to the n^{th} digit of s_n . By construction, s differs from each s_n , since each n^{th} digit represents the binary compliment of its respective place in the number s . Hence, s cannot occur in the list and we can conclude that the initial set T is *uncountable* (viz. it cannot

different sizes of infinity were *distinct*, and that the cardinality of the continuum was strictly greater than the cardinality of the set of natural numbers. He had obtained what he believed to be the next ordinal in his hierarchy.²²

The sensitive reader might ask at this point: “Ok, so, Cantor proved that we generate the cardinality of the uncountable set of \mathbb{R} from the power set operation on the countably infinite set of \mathbb{N} . But Cantor does not tell us how to generate the *countably* infinite cardinality of \mathbb{N} to begin with! If we perform a power set operation on any finite set of numbers (i.e. a subset of the natural numbers whose cardinality is strictly less than \aleph_0), we will never be able to reach the cardinality of \aleph_0 .” Yes, indeed! This is basically the problem of the potential infinite restated, which we can now present as the basic set theoretical definition of inaccessibility: Certain sets are so large that no additive operations of cardinal arithmetic (specifically, the power set operation) on their constituent members could ever generate them.²³

For Cantor, “the infinity of this sequence [1,2,3...v] affords the proof that the totality of all finite numbers, considered as a thing in itself, is an actually infinite set, a transfinite.” (Cantor

be put into one-to-one correspondence with the natural numbers). The uncountability of the real numbers was already established by Cantor's first uncountability proof in 1874, but it also follows from the above result. (To prove this, an injection will be constructed from the set T of infinite binary strings to the set \mathbb{R} of real numbers. Since T is uncountable, the image of this function, which is a subset of \mathbb{R} , is uncountable. Therefore, \mathbb{R} is uncountable. Also, by using a method of construction devised by Cantor, a bijection can be constructed between T and \mathbb{R} . Therefore, T and \mathbb{R} have the same cardinality, which is called the “cardinality of the continuum” and is typically denoted by \mathfrak{c} .)

²² The question of whether the power set of the natural numbers *must equal* the cardinality of the continuum is known as the continuum hypothesis: it asserts that no set exists whose cardinality is strictly between that of the integers and the real numbers. Formally, it means that $2^{\aleph_0} = \aleph_1$. (The continuum hypothesis can also be generalized; for every ordinal α , $2^{\aleph_\alpha} = \aleph_{\alpha+1}$) Establishing the truth of the continuum hypothesis was crucial for Cantor, since his theory of ordinals depended on revealing the distinct ordinal structure of the transfinite domain - indeed, he went mad trying to solve the problem. (Aczel, p. 157) Cantor has since been vindicated, as it was only with the results of both Kurt Gödel (1940) and Paul Cohen (1963), after Cantor's death, that it was known that the continuum hypothesis is *independent* of the axioms of set theory; it cannot be proved or disproved via the formal determinations of our mathematical systems (although one wonders why God didn't just let Cantor know this – he could have been spared his sanity!)

²³ More precisely: A cardinal number κ is said to be *inaccessible* iff it meets the following three conditions: (a) κ is greater than \aleph_0 (b) For any cardinal λ less than κ , we have $2^\lambda < \kappa$. (c) It is not possible to represent κ as the supremum of fewer than κ smaller ordinals. That is, if S is a set of ordinals less than κ , and if $\text{card } S < \kappa$, then the ordinal $\sup S$ is less than κ . (Enderton, p. 254) Note that \aleph_0 fulfills conditions (b) and (c), but obviously not (a).

1887-88, p. 419) That is to say, Cantor considers this set, and all others, as a thing in itself, a definite unity over and above all its elements. In other words, \aleph_0 must be a *gestalt* - a whole that is more than the sum of its parts: “This set [of all finite numbers] is a thing in itself and constitutes, completely apart from the natural sequence of numbers belonging to it, a firm in all its parts, determinate quantum, an *aphorismenon*.” (ibid, p. 401) This is why the set of all natural numbers, as the first infinite cardinal \aleph_0 , is sometimes considered inaccessible (see footnote 31), because we cannot ‘reach it’ through any arithmetical operation on its members; we must take its gestalt unity for granted in order for the theory of transfinite numbers to get off the ground in the first place. (Cameron, p.130)

The same reader may now inquire: “Alright, I see that we can never reach the gestalt unity of \aleph_0 through merging or adding to any of its subsets (less than itself) – that is, any operation using finite numbers will always inevitably yield another finite number. But does the same hold true for the *ordinal* numbers? If we can count transfinite numbers by their ordinal value like integers, are there any *transfinite* cardinals that are so exceptionally huge they cannot be reached through any power set operations on lower tier *transfinite* subsets?” Well, this is an interesting question. The existence of “large” inaccessible Cardinals is pretty much taken for granted among most set theorists, and there are many definite mathematical results that distinguish unique properties among these ultra-gigantic classes of objects. After all, if we are willing to accept the existence of \aleph_0 , why should we deny this gestalt structure to the set of all first-tier ordinal numbers? Granting existence to large cardinals, the set theoretical universe of transfinite numbers has an endless, gestalt-sandwich structure.

Cantor therefore felt that no transfinite number was *truly* infinite, since each could be put into a graded *sequence*. In Ignacio Jané’s words, “the set of all natural numbers is not absolutely

infinite. This means that while it is certainly infinite, it is nonetheless quantitatively bounded, as there are other sets of larger magnitude (power, cardinality), e.g., the set of all real numbers. Thus, as opposed to the absolute, both the finite and the proper [transfinite] infinite share the quantitative *boundedness* or limitation.” (p.379) For Cantor, the sequential aspect of ordinal numbers entails that the formal unity of a transfinite object determines its limited nature; the sequential progression means that each ordinal number is invariably limited by the existence of a greater unity, since one can always construct ever-greater sets beyond any given transfinite number.

We noted above that Cantor believed that a range of variable magnitudes presupposes a fixed domain of unified elements from which such magnitudes are abstracted. If the potential infinity of the countable integers presupposes the completed actual-infinity of \aleph_0 , then the succession of transfinite ordinal numbers must presuppose some inconceivable cardinality over and above the system of transfinite numbers, one that gives definite *meaning* to the succession. This is one way to interpret the function of the absolute infinite in Cantor’s system:

There is no doubt in my mind that in this way we will get farther and farther ahead, never reaching an *insurmountable* limit, but also attaining not even an approximate grasp of the absolute. The absolute can only be acknowledged, but never known, not even approximately. For just as within the first number-class (I) for every finite number, no matter how great, we are always confronted by the same power of the finite numbers greater than it, in the same way every transfinite number, no matter how large, of any of the higher number classes (II) or (III) etc. is followed by a totality of numbers and number-classes which has not suffered the least in power in comparison to the whole of the absolutely infinite totality of numbers starting at 1. This is a situation similar to what Albrecht von Haller says of eternity: 'I subtract it (the enormous number) and you (eternity) still lie complete in front of me'²⁴... The transfinite with its wealth of arrangements and forms necessarily *points at* an absolute, at the "true infinite", whose magnitude is unable

²⁴ We feel obliged to mention that these paradoxes of infinity had already pervaded Indian thought by the time Kundakunda formulated his gnostic vision of omniscience, thus setting the conceptual stage for grappling with these issues in regards to the Self and the nature of the absolute. The *Bṛhadāraṇyaka Upaniṣad* aphoristically articulates Albrecht von Haller’s observation of the additive paradox of Brahman (thousands of years, of course, before Cantor was dealing with the quizzical properties of transfinite numbers): *Auṃ pūrṇamadah pūrṇamidaṃ pūrṇāt pūrṇamudacyate pūrṇasya pūrṇamādāya pūrṇamevāvaśiṣyate*. That is absolute, this is absolute. From the absolute, the absolute proceeds; taking the absolute from the absolute, the absolute remains.” (*Bṛhadāraṇyaka Upaniṣad* 5.1.1) Thus the kinds of conceptual issues that Kundakunda identifies in the *kramatic* knowledge of the traditional *nayavāda* (i.e. the necessity of a gestalt conception of omniscience) stem from mystical roots such as these, but they also express the same religious intuitions (or so we are claiming) about the nature of the absolute that inspired Cantor’s formalization of the notions of inaccessibility and Ω .

to increase or decrease at all, and thereby must be considered quantitatively as an absolute maximum. The latter exceeds, so to speak, all human comprehension and eludes mathematical determination. (Cantor 1887-88, pp. 205, 405)

Cantor maintained that God, the absolute infinite, denoted by Ω , is a cardinal that is greater than any conceivable quantity – it exists beyond the system of transfinite numbers, and yet its infinite nature is somehow mirrored in each respective transfinite totality, in the sense that each one is an inaccessible ‘gestalt’ of actual infinity. The criteria for this jump to Ω are similar to those we mentioned above for inaccessible cardinals: “I use the word 'absolute' for that which cannot be further increased or perfected, in analogy with the 'absolute' in metaphysics. My proper infinite, or, if you prefer, transfinite numbers ω , $\omega+1$, $\omega+2$, etc...²⁵ are not 'absolute', because although not finite, they are however able to increase. But the absolute is unable to further increase and therefore it is *inaccessible* to us. (Cantor, 1991, p. 139 [emphasis mine])

Although the ‘gestalt’ property of Ω entails that it cannot be accessed from the bottom-up, so to speak, there are rather crucial differences between the inaccessibility of a transfinite set like \aleph_0 and the absolute infinite of Ω . While \aleph_0 , for example, indicates a gestalt-jump ‘outside’ of the system of natural numbers, Ω represents a jump outside of the *entire representational system*. More specifically: “it is obvious that by [the transfinite] we must not understand the absolute maximum (*sive Deus*), which can be determined only by itself, but not by us.” (Jané, p. 384) Like Kundakunda’s Self-begetting-itself, the absolute exists only for itself – it has no relative determinations whatsoever. Unlike the transfinite numbers, there is no order or sequence with respect to the self-determination of the unbounded absolute; it represents a form of reality that wholly transcends all empirical and formal objectifications:

²⁵ ω represents the first ordinal number (like the integer 1) for counting the hierarchy of transfinite numbers.

The actual infinite was distinguished by three relations: first, as it is realized in the Supreme Perfection, in the completely independent, extrawordly Existence, in *Deo*, where I call it Absolute Infinite or simply Absolute; second to the extent that it is represented in the dependent, creatural world; third as it can be conceived in *abstracto* in thought as a mathematical magnitude, number or ordertype. In the latter two relations, where it obviously reveals itself as limited and capable for further proliferation and hence familiar to the finite, I call it *Transfinitum* and strongly contrast it with the Absolute. (Cantor 1887-88, p. 378)

The inaccessible cardinal and the absolute are both gestalts - wholes that are more than the sum of their parts. But, according to Cantor, if transfinite gestalts within the set-theoretical universe have a genuine reality, it can only be in virtue of the implied wholeness of the transcendent absolute that grants each distinct cardinality its mathematical structure and ordinal value in the endless hierarchy of sets. (In the same way that, according to Cantor, we could not make sense of the order of the integers without the unified and complete conceptual domain of \aleph_0).

But, of course, we cannot possibly *conceive* of any other *kinds* of infinite entities other than as potential objects in the sequential hierarchy of ordinal numbers! In this sense, the set theoretical universe represents our own conceptual horizon, thus Cantor's dictum: "the essence of mathematics is freedom." (Dauben 1977, p. 132.) In other words, we cannot comprehend the ordinals *simultaneously*; they must exist in some *sequence* for us limited beings. The absolute is always on the far side of what we can construct or objectify for ourselves; it inherently escapes our representational capacity, even though it is presupposed in the gestalt nature of actual infinity, a quality possessed by each transfinite set.²⁶ In this respect, Cantor fused his spiritual and logical

²⁶ While on the subject of the limits of formal representation, there is a deep analogy here with Douglas Hofstadter's construal of the limitative theorems of formal systems. For instance, in one of Hofstadter's most provocative passages, he writes that "[a]ll the limitative Theorems of metamathematics and the theory of computation suggest that once the ability to represent your own structure has reached a certain critical point, that is the kiss of death: it guarantees that you can never represent yourself totally. Gödel's Incompleteness Theorem, Church's Undecidability Theorem, Turing's Halting Problem, Tarski's Truth Theorem -- all have the flavour of some ancient fairy tale which warns you that 'To seek self-knowledge is to embark on a journey which . . . will always be incomplete, cannot be charted on a map, will never halt, cannot be described.'" (Hofstadter, p.697) This is certainly a statement of a fundamental 'unrepresentability' in the *vyavaharic* domain. Here, the kinds of theorems that can be proved in formal systems (i.e. ones structured by symbolic manipulations within a limited axiomatic framework) resemble the discrete fact-objects of the *kramatic nayavāda*. Gödel's and Cantor's work both intimate that as we formulate symbolic or conceptual systems to enumerate *all* determinate, isolated truths, such systems are inherently rendered unable to do so. For example, only a formal system designed to be powerful enough to express all arithmetical truths and retain consistency

sensibilities in the idea of the absolute infinite as an independent, otherworldly being that contrasted strongly with any relative or sequential scheme of conventional thought. The absolute cannot be objectified as any transfinite quantity, because to be *determinate at all* means to be *delimited* in the ordinal hierarchy, to occupy some distinct *sequential* space relative to all other transfinite cardinals.

Bringing Kundakunda and Cantor Together:

In this section, we apply Cantor's formal insights to the Jain system to show that Kundakunda's vision of omniscience is preferable *a priori* over the traditional *nayavāda*. Let's begin by defining *omniscience* as 'knowledge of every member of the set that represents the infinite sum of all relative truths.' Let's call the set of all the truths O . Now consider the power set of O - the set of all the subsets of the set of all the truths. It would appear that corresponding to each subset of all the truths will be at least one truth. For example, if O^* is a subset of O , then it will either be true that the proposition that $2+2=4$ is a member of O^* or that the proposition that $2+2=4$ is not a member of O^* . But if there is a truth corresponding to each member of the power set of O , there must be as many truths as there are members of the power set of O . But we know from Cantor's Theorem that there are more members of the power set of O than there are members of O - from which it follows that there are more truths than there are members of O . *But this contradicts our initial proposition that O is the set of all the truths.* Ergo, there cannot be any set of all truths. Cantor's Theorem is crucial here, because it shows that as soon we try and *represent* any set of

becomes capable of the self-reference of Godel's incompleteness theorem - a result that renders it incapable of expressing all arithmetical truths and unable to prove its own consistency. In other words, as a system develops the capacity to express itself it ensures its own failure, which precisely mirrors the self-undermining 'infinity' of a transfinite gestalt within the ordinal sequence of the set-theoretical universe.

truths, infinite or not, we will always be able to construct a set with a greater number of truths.

The strength of this argument in our case, however, depends upon whether we can represent the account of reality in the *Tattvārtha* as a set of discrete facts about the modes of substances. Well, as it happens, the radical substantivalism of the Jaina metaphysics is ironically well-disposed to such a complete description, given its tendency towards explicating the minutia of the atomic quantities of substances – precisely the testament to the omniscience of the Jina himself! The fifth chapter outlines the properties of the six substances of the universe, namely, the four non-sentient entities and the soul. As we noted, a substance is defined as that which passes through modes and qualities, or is passed through by modes and qualities. Substances are ultimately composed of an ‘innumerable’ number of units whose modal qualities determine the constitution of the entire universe at any given moment. The ‘innumerable’ units are formally defined as “the total number of mustard seeds that have passed through and are in the four stores” which are extremely large areas whose dimensions are defined in terms of 100,000x1000 *yojanas*. (Tatia, p.269) For the classes of substance identified with time and matter, the number of units can be “infinite.”²⁷

Now: If the three motionless, extended substances are composed of an “innumerable number of units” (5.7); and if there are also “innumerable soul units in a soul” (5.8); and “the number of units in material clusters of matter may be numerable, innumerable, or infinite” (5.10); and even if, remarkably, “time consists of an infinite number of time units” (5.39); we can still construct a set of truths within the Jain system that represents the sum of the states of all modes

²⁷ It should be noted that ‘infinite’ for the Jains technically does not even reach the cardinality of \aleph_0 : the ‘infinite’ is ‘reachable’ by arithmetic operations on the ‘innumerable’ cardinal, which in turn is defined by the sum of mustard-seeds in the four stores – very very large areas, but still not transfinite. It is interesting to think about why this might be; the set of facts that constitutes omniscience for the Jains does not even have to reach the first \aleph_0 to be complete. Under our interpretive scheme, this limit (perhaps unconsciously) accords with the traditional view that the relative *nayas* give us *partial* access to the the *whole* truth. If “the whole truth” was \aleph_0 then perhaps it would make the relative *nayas* useless, as infinity in this sense would entail a gestalt perspective completely outside of the system of relative *nayas*, which the traditional reading would want to avoid (i.e. the gnostic view that Kundakunda embraces).

and all substances. This set - the *simultaneous* knowledge of all perspectives at once – could then plausibly be identified with the content of the state of omniscience. As noted, the crucial epistemic distinction between the unenlightened and the Jina, respectively, is the limited, *kramatic* movement through the set of *nayas* in contrast to the absolute knowledge of the Jina who has *simultaneous* access to the whole. Indeed, the fact that Mahāvīra laid out such an authoritatively precise account of nature *exemplifies* for the Jains the very fact that he *was* actually omniscient. This is what Samantabhadra indirectly acknowledged when he noted that “the various items of knowledge arrived at through the alternative *nayas* do not and cannot take place simultaneously but in succession.” The formulation of the *nayas* was intended to grant humans correct understanding of a *single* perspective, while the Jina has access to *all* perspectives instantaneously.

In sum, therefore, both Cantor and Kundakunda address similar *a priori* difficulties with equating omniscience with knowledge of any sum of relative truths - as a unified or completed *object*, akin to a set with some finite or transfinite cardinality. No matter how great or infinite a set we construct, something will forever escape explicit representation. Cantor recognized this when he posited the cardinality of Ω : The absolute infinite simultaneously encompasses all transfinite numbers within its being, but must also infinitely transcend them. To be expressible as any infinite sum is to be limited; if the actual infinite is *determinate* through a distinct conceptual slot in a sequence of cardinal powers, it cannot be absolute. We now appreciate that Kundakunda, like Cantor, understood the true metaphysical and epistemological significance of the *yugapat* vs *krama* distinction that Samantabhadra first articulated. Just as Kundakunda recognized that the *kind* of *kramatic* knowledge that figures into the traditional *nayavāda* can never access the omniscience of the Self, so too did Cantor realize the ordinal sequence of cardinals implied a transcendent totality. If we take Cantor and Kundakunda seriously, any knowledge that is subject

to some kind of ordered sequence represents limited knowledge – the absolute must comprehend all relative objectifications simultaneously, as an overarching unity, but remain static and complete. It can never be represented as an object to a subject; it can only reveal itself to itself as itself.

Conclusion:

The Jain notion of omniscience is traditionally defined by Umāsvāti as knowledge of all substances in all of their modes. If we take the *Tattvārtha* as a comprehensive description of the constitution of substances, we can imagine a set of perspectival truths that, if known *simultaneously*, is equivalent to the Jain conception of omniscience. Nevertheless, we have argued with the insights of set theory that this conception of omniscience is flawed *a priori*. In order to ease this tension, we defended Kundakunda's 'gnostic' take on omniscience, which has a characteristically Upaniṣadic emphasis on the ineffable transcendence of self-knowledge. We claim that the 'gestalt' nature of this knowledge – its 'inaccessibility' – lends itself to a more consistent and desirable interpretation of omniscience within the Jain system.

Before closing, we wish to emphasize that the traditional Jain teachings about substances and the relative *nayas* are decidedly *not* invalidated on Kundakunda's reading. Rather, we must simply grant that the verbal definition of omniscience as 'knowledge of all substances in all their modes' is *distinct* from the actual, phenomenal emancipation of self-realization. This self-realization would entail a gestalt state that escapes any *prima facie* conception of omniscience as a terminal sum of all distinct truths from all relative perspectives. Perhaps one could argue that this move reduces the superpower of the Jain notion of literal omniscience to a more esoteric, Vedāntic conception of the absolute transcendence of self-knowledge. And, indeed, one could view Kundakunda as having endorsed precisely this mitigated interpretation.

But one of the upshots of Kundakunda's 'gnosticism' is that it supports later Jain thinkers like Haribhadra, who wished to advertise the *anekāntavāda* doctrine as having uniquely grasped what is contemporarily described (in the wake of Aldous Huxley) as the spiritual philosophy of 'perennialism': the pluralistic tolerance for diverse traditions grounded on the idea that 'paths are many, truth is One.' In the *Yogadr̥ṣṭisamuccaya*, for instance, Haribhadra writes: "The multiplicity of the teachings is suited according to the qualities of the disciples...The greatest truth of going beyond *samsāra* is known as *nirvāṇa*. And this truth [achieved] from spiritual discipline is singular, although distinct in regards to name. 'Eternal Shiva, Greatest Brahman, Self-Realization, and Suchness': It is spoken of with these terms, but they conform to only this one meaning."²⁸ (v.129-130, 134) Haribhadra believed that the Jina's omniscience represented the same absolute understanding as professed in other traditions, ones that do not necessarily describe omniscience as 'knowledge of all substances in all their modes'.

Certainly, if we stop to consider this ecumenical interpretation of Jainism, wouldn't the *literal, prima facie* idea of omniscience include this classically *upāyic* construal *anyway*? After all, as the Buddhists understood well, knowledge of *everything* must by definition embrace knowledge that pertains to the expediency of spiritual transmission. Perhaps, in this sense, the notion of omniscience presupposes that when a liberated being preaches about omniscience with the intent to help others achieve liberation, they tailor its meaning to suit their disciples' understanding. For Umāsvāti, then, the best way to describe this state for the Jain community is 'knowledge of all substances in all of their modes.' But the actual experience of omniscience must be such that no conceivable description could ever be given, because it ultimately transcends any representational

²⁸ *citrā tu deśasanaiteṣāṃ syād vineyānugūnyataḥ...samsārātītātattvaṃ tu param nirvāṇasamjñitam | taddhy ekam eva niyamāc chabdabhede'pi tattvataḥ. sadāśivaḥ parambrahma siddhātmā tathateti ca | śabdais tad ucyate'nvarthād ekam evaivamādibhiḥ*

content. In sum, Kundakunda's mystical understanding of omniscience actually helps validate the scope of the *anekāntavāda* doctrine, which, according to later thinkers like Haribhadra, should promote a pluralistic vision of the validity of diverse traditions that seek the singular goal of liberation.

To end on a more speculative note, I would like to say something about how this whole topic broadly relates to process thought, the specter of which has been haunting this conversation. Consider that a well-known innovation of process theology is the rejection of God's foreknowledge. (Hartshorne, p.284) In a pragmatic spirit, Whitehead and Hartshorne saw any form of static absolutism in an epistemic sense as an affront to the metaphysical flow of experience and the irreducible reality of change. One's experience of learning a 'new' fact about the world is the subjective validation of the primordial procession of novelty. According to the *prima facie* understanding of omniscience, then, time itself would cease to mean anything for the Jina, insofar as we experience time as *necessarily* linked to the disclosure of novelty, and thus our own being *qua* unknowing. As discussed, the distinction between simultaneity and sequential knowledge is critical for the Jains; sequential knowledge means that we are forced to employ a *naya*, and thus it represents ignorance. Presumably, then, for the Jina the *kramatic* flow of time that we capture in the snapshots of the *nayas* must be experienced as a static *yugapad* state, in which all future facts present themselves in a static panorama, wherein no new growth or conceptual development is possible. A completed, standing absolute with no new developmental or additive capabilities is also Cantor's vision of the cardinality of Ω .

For a Whiteheadian, this idea commits a gross fallacy of misplaced concreteness; we incorrectly imagine concrete reality as the reified object of a collection of discrete facts, fully explicable with abstract universals. Thus when we think in these wrongheaded terms, we fancy

that some perspective can possibly exist where objective knowledge ends and becoming itself halts. Cantor and mainstream Jainism both believed in the comprehensive and complete perspective of this transcendent absolute. But, for process thought, *existence entails some degree of ignorance* - even God's - insofar as genuine novelty emerges as a function of creative existence itself.

Thus, in closing, we would like to present an overview of the case that Cantor interpreted his results incorrectly, and unwittingly supplied grist for the process-philosophy mill. To do justice to the argument would require another paper. But, briefly, it would go something like this: We noted that Cantor believed that, to account for the sequence of the natural numbers, we required a static domain like \aleph_0 as the first actual infinite cardinality:

Whereas the potential infinite means nothing more than an indeterminate, always finite, variable magnitude taking values which become either as small as we please or larger than any arbitrary finite bound, the actual infinite refers to a fixed in itself, constant quantum which is larger than any finite magnitude of the same kind. Thus, for example, a variable magnitude x successively taking the different finite whole number values 1, 2, 3, ..., v , ... represents a potential infinite, while the set (v) of all whole finite numbers, conceptually determined in full by a conceptual law, offers the simplest example of an actual-infinite quantum. (Cantor 1887-88, p. 409)

Cantor reasons on the basis of this insight that, in the same way that we need a completed, statically-infinite domain like \aleph_0 to make sense of the sequence of the rational numbers, so too do we need to posit the completed domain of Ω to make sense of the ordinal rank of the transfinite cardinals. According to Cantor, the transfinite ordinal structure thus testified to a fully independent, absolute being at the end of the rainbow. This was a being that encompassed the transfinite and physical universe, but also transcended them as a static, self-determinate unity.

At this point, a process thinker would say that Cantor's analogy reflects a category mistake of hypostatization: the 'gestalt' quality of \aleph_0 and Ω are ultimately incommensurable, as *one has determinate abstract properties in a relational sequence and the other has no relative determinations whatsoever*. Since the transcendent absolute cannot be a determinate object at all

– and any attempt to represent its reality inevitably fails – we should consider it an ontologically ambiguous *process*. Cantor’s results thus make perfect sense from a Whiteheadian perspective, which would insist that the transfinite universe in which Cantor conceptually maneuvers is an abstraction of the truly ‘unrepresentable’ – i.e. the concrete reality of becoming. This fallacy of misplaced concreteness arguably does not obtain for Kundakunda, because the nature of the Self, by definition, is not an objectified determinacy. But I do not think that Cantor, as a traditional theist, can reconcile the objectified notion of a completed, static absolute with the endless incompleteness of determinate representations demonstrated by the formal results of set theory.²⁹

²⁹ What I find particularly fascinating is the sense in which the potentially infinite and the absolute infinite become reflections of each other through their mutually entailed *incompletion*; all conventional reality - which is *potentially* infinite - has its source in an absolute infinite - which is infinite *potential*. Process thinkers hold that true reality can’t be represented as any determinate ‘thing’ - as soon as you represent any determinate sort of coherence, it is in the nature of becoming to go beyond it. Thus, in the case of Cantor, the utter deficiency of actual infinity for the potential infinite, and the sheer surfeit of actual infinity in the absolute infinity, both demonstrate that unrepresentability, indeterminateness, and incompleteness are the hallmarks of the truly infinite. In the end, then, the ‘incomplete’ infinity that Cantor feared in the potential infinite was not vanquished with the presumption of \aleph_0 , but rather eternally deferred.

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