

# Internet Appendix to Shrinking the Cross-Section\*

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## Internet Appendix A. Properties of the naive SDF coefficient estimator

Consider an orthogonal rotation  $P_t = Q'F_t$  with  $\Sigma_T = QD_TQ'$ ,  $Q$  is the matrix of eigenvectors of  $\Sigma_T$  and  $D_T$  is the sample diagonal matrix of eigenvalues,  $d_j$ , ordered in decreasing magnitude. If we express the SDF as  $M_t = 1 - b'_P(P_t - \mathbb{E}P_t)$  we have

$$\hat{b}_P = \left( \frac{T - N - 2}{T} \right) D_T^{-1} \bar{\mu}_P. \quad (1)$$

Consider the analytically simple case when  $D$  is known and replace  $\left( \frac{T - N - 2}{T} \right) D_T^{-1}$  with  $D^{-1}$ .<sup>1</sup> Then we have

$$\sqrt{T} (\hat{b}_P - b_P) \sim \mathcal{N} (0, D^{-1}), \quad (2)$$

which shows that estimated SDF coefficients on small-eigenvalue PCs (small  $d_i$ ) have explosive uncertainty.

The above results give exact small sample distributions, assuming returns are jointly normal. As a simple robustness exercise, consider dividing the data into  $k = 5$  sub-samples and estimating  $b_P$  separately in each.<sup>2</sup> Then we can compute the theoretical variance of these estimates is simply,

$$\text{var} (\hat{b}) = \frac{k}{T} D^{-1}, \quad (3)$$

which is larger than in Eq. (2) by a factor of  $k$  due to the shorter samples. Fig. 1 plots the sample values of  $\text{var} (\hat{b}_i)$  vs  $d_i^{-1}$  (on a log-log scale) for the PCs of the 50 anomaly portfolios we use in Section 4 of the paper. The solid line plots the relationship derived in Eq. (3). The good fit confirms that the theoretical relationship given in Eq. (2) is valid even with non-normally distributed actual return data.<sup>3</sup> Notice that the ratio of largest to smallest eigenvalue is of the order  $10^3$ . This implies that the variance of the estimated  $b$  associated with the smallest eigenvalue portfolio has 3 orders of magnitude larger sampling variance as the  $b$  associated with the largest eigenvalue portfolio.

This problem is somewhat exacerbated when  $D^{-1}$  is unknown, and thus, estimated. It is well known that the sample eigenvalues of  $D$  (equivalently,  $\Sigma$ ) are “over-dispersed” relative to true eigenvalues, especially when the number of characteristics,  $H$ , is comparable to the sample size,  $T$ . This implies that, on average, the smallest estimated eigenvalue is too small and hence the corresponding  $\hat{b}_i$  has even greater variance than shown above. In Appendix B we discuss covariance estimation uncertainty.

## Internet Appendix B. Covariance estimation uncertainty

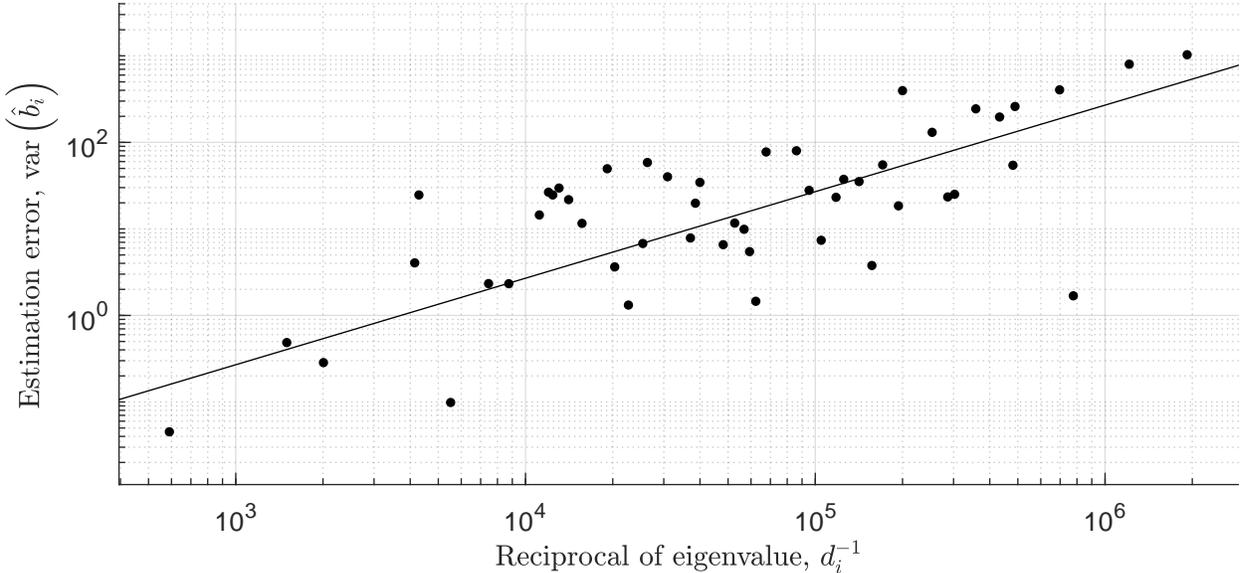
In the prior analyses, we have treated covariances ( $\Sigma$  and  $D$ ) as known. Many papers highlight the empirical difficulty in accurately estimating covariance matrices when the number of assets,  $H$ , is of the same order of magnitude as the number of time periods,  $T$ . In our main estimation with anomalies, this should not be of great concern, since  $H = 50$  and  $T \approx 11,000$ . Still, we now

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<sup>1</sup>With high-frequency data (daily) and even hundreds of factors,  $D^{-1}$  is estimated quite well as measured by the loss function  $\text{tr} (D_T^{-1}D - I)^2 / N^2$ .

<sup>2</sup>Throughout, we assume  $D$  is known. For this exercise, we estimate  $D$  from the full sample.

<sup>3</sup>This is simply an example of the central limit theorem in full effect.



**Fig. 1. Sampling variance of  $b$ .** The figure shows sample values of  $\text{var}(\hat{b}_i)$  vs reciprocal eigenvalue  $d_i^{-1}$  (on a log-log scale) for the PCs of the 50 anomaly portfolios we use in Section 4 of the paper. The solid line plots the theoretical relationship derived in Eq. (3).

analyze methods for dealing with covariance uncertainty in our empirical setting.

In a series of papers, Ledoit and Wolf (L&W) propose robust estimators of  $\Sigma$  which trade off small sample bias and variance by (asymptotically) optimally shrinking the sample covariance towards an a priori target.<sup>4</sup> They are conceptually similar but use different shrinkage targets,  $\Sigma_0$ :

$$\hat{\Sigma} = a\Sigma_0 + (1 - a)\Sigma_T$$

One choice of  $\Sigma_0$  is the diagonal matrix  $\frac{\text{tr}(\Sigma_T)}{H}I$ . The other preserves sample variances, but all correlations are set to  $\bar{\rho}$ , the average correlation coefficient extracted from  $\Sigma_T$ . The shrinkage parameter,  $a$ , is chosen to optimally balance bias and variance (to minimize estimated RMSE), given the choice of  $\Sigma_0$ . The scaled identity matrix proposed in Ledoit and Wolf (2004a) is most appropriate in our empirical setting of zero- $\beta$  anomaly portfolios. We implement their algorithm on the 50 anomaly portfolios and find  $a \approx 0.7\%$  for both methods. Ledoit and Wolf “concentrate on the covariance matrix alone without worrying about expected returns.” Hence, they set  $\hat{\mu} = \mu_T$ . The final estimator of SDF coefficients is

$$\hat{b} = (a\Sigma_0 + (1 - a)\Sigma_T)^{-1}\mu_T,$$

which appears similar to our estimator given in Eq. (22).

A fully Bayesian approach (which delivers similar results) is to specify a Wishart prior for  $\Sigma^{-1}$ , with a “flat” prior on  $\mu$ ,  $p(\mu|\Sigma) \propto 1$ , with

$$\Sigma^{-1} \sim \mathcal{W}\left(H, \frac{1}{H}\Sigma_0^{-1}\right), \quad (4)$$

<sup>4</sup>See Ledoit and Wolf (2004a), and Ledoit and Wolf (2004b).

where  $\Sigma_0 = \frac{1}{H} \text{tr}(\Sigma_T) I$ , which ensures the total expected variation under the prior matches the data, as in the L&W method. Setting the degrees of freedom to  $H$  makes the prior relatively “diffuse.” For any choice of  $\Sigma_0$ , the posterior is given by

$$\Sigma^{-1} \sim \mathcal{W}\left(H + T, [H\Sigma_0 + T\Sigma_T]^{-1}\right),$$

with expected value

$$\mathbb{E}(\Sigma^{-1}) = \left[ \left( \frac{H}{H+T} \right) \Sigma_0 + \left( \frac{T}{H+T} \right) \Sigma_T \right]^{-1}.$$

For the 50 anomaly portfolios,  $\frac{H}{H+T} \approx 0.5\%$ , similar to the shrinkage coefficient of the L&W method. We augment this with a “flat” prior on  $\mu$  so that  $\hat{\mu} = \mu_T$ . The final estimator of SDF coefficients is

$$\hat{b} = \left[ \left( \frac{H}{H+T} \right) \Sigma_0 + \left( \frac{T}{H+T} \right) \Sigma_T \right]^{-1} \mu_T,$$

which is the same as the L&W estimator except that the shrinkage constant is now deterministic.

Both the L&W method and the Bayesian approach address the known phenomenon that eigenvalues of sample covariance matrices are “over-dispersed.” That is, the largest estimated eigenvalue tends to be too large while the smallest is too small. Both methods end up shrinking all eigenvalues towards the average,  $\bar{d} = \frac{1}{H} \text{tr}(\Sigma_T)$ , while preserving the eigenvectors,  $Q$ . Since both use a flat prior for  $\mu$ , they explicitly do not address uncertainty in estimating means.

Fig. 2a shows the relative shrinkage applied to each PC portfolio of the anomalies (our main dataset) for the L&W, Wishart, and our mean-shrinkage method given by Eq. (22). We define relative shrinkage as  $\frac{\hat{b}_{P,j}}{\hat{b}_{P,j}^{\text{ols}}}$ , with  $\hat{b}_P^{\text{ols}} = Q' \Sigma_T^{-1} \bar{\mu}$ . For comparison, we include the P&S “level” shrinkage of Pástor and Stambaugh (2000), which corresponds to our  $\eta = 1$  prior.<sup>5</sup> That plot shows that this prior shrinks all coefficients uniformly towards zero.<sup>6</sup> The L&W and Wishart methods deliver very similar estimators. Importantly, these covariance shrinkage methods are characteristically different from our method (KNS) though they appear superficially similar. Whereas we shrink all coefficients, with greater shrinkage applied to smaller PCs, those methods actually slightly inflate the SDF coefficients associated with large PCs and apply much less shrinkage to small PCs. Indeed, for the smallest PC, the ratio of the L&W estimator to our estimator is approximately equal to 1,700.

### B.1. $\Sigma$ and $\mu$ both uncertain

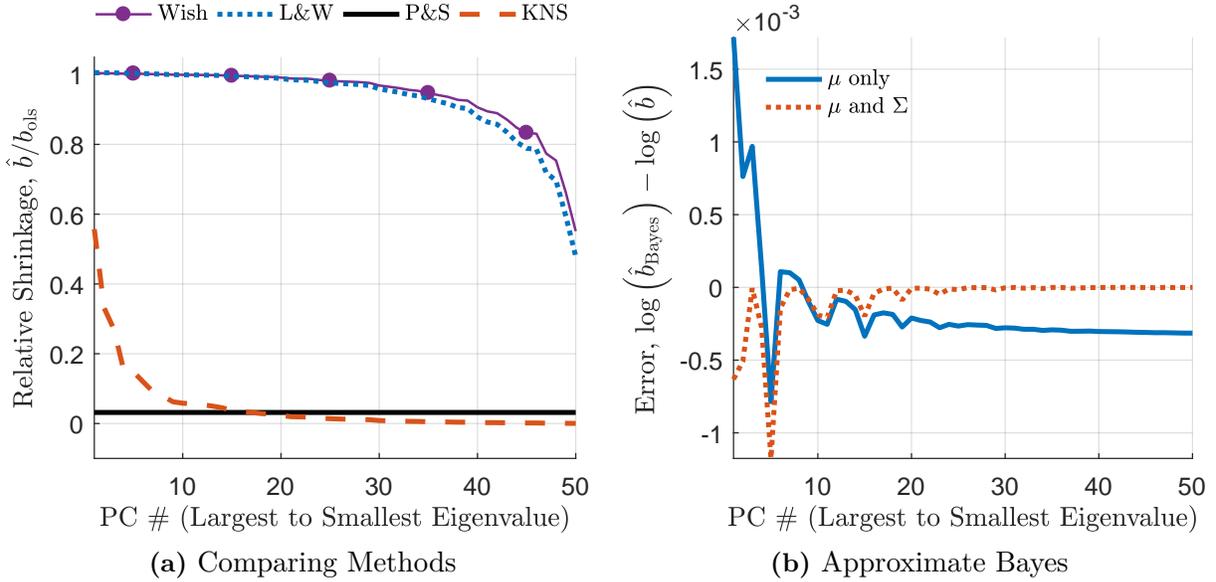
We now analyze the impact of recognizing uncertainty in both  $\mu$  and  $\Sigma$ . As in our main estimation, we specify

$$\mu | \Sigma \sim \mathcal{N}\left(0, \frac{\kappa^2}{\tau} \Sigma^2\right), \quad (5)$$

where  $\tau = \text{tr}(\Sigma_T)$ . For  $\Sigma$ , we use a similar prior to Eq. (4), with a slight modification for numerical tractability since the posterior is not fully analytic. First, we assume eigenvectors (but

<sup>5</sup>We repeat the cross-validation exercise using the prior  $\eta = 1$ , which induces the posterior estimate  $\hat{\mu} = \frac{1}{1+\gamma} \mu_T$ . For this shrinkage, the cross-validated optimum is attained at  $\frac{1}{1+\gamma} \approx 4.3\%$ .

<sup>6</sup>The degree of shrinkage is determined by cross-validation, as described in Section 3.3 of the paper.



**Fig. 2. Relative Shrinkage by Method.** Panel (a) plots the ratio of regularized estimates of PC SDF coefficients to OLS estimates for various methods. Panel (b) plots the relative difference between the fully Bayesian estimates taking into consideration uncertainty in both  $\mu$  and  $\Sigma$  and two alternative estimators. The line “ $\mu$  only” represents the estimator which treats the sample covariance matrix as the truth. The line “ $\mu$  and  $\Sigma$ ” represents the approximate Bayesian solution which first computes the posterior variance assuming sample means are the true means, then computes posterior means assuming the posterior variance is the true variance.

not eigenvalues) are known a priori, so the return covariance matrix can be orthogonalized. Let  $D$  be the covariance of PC portfolios. The marginal prior for each PC (each diagonal element of  $D^{-1}$ ) is an independent scaled inverse-chi squared priors. Let  $\sigma^2 = \text{tr}(D_T)/H$ , where  $D_T$  is the sample covariance matrix of eigen-portfolios. Under the identity Wishart prior for  $D^{-1}$  (with known  $\mu$ ), we had  $\mathbb{E}_{\text{prior}}(d_i^{-1}) = \sigma^2$ . The independent priors can be constructed by letting each diagonal element of  $D^{-1}$  have a Wishart prior with the same parameters, except to collapse the distribution to one-dimensional:

$$d_i^{-1} \sim \mathcal{W}\left(H, \frac{1}{H} \frac{1}{\sigma}\right),$$

which preserves the level of uncertainty (degrees of freedom) relative to Eq. (4). The assumption that eigenvectors are known implies that off-diagonals of  $D$  are set to identically 0 under the prior (and hence under the posterior). Along with conditional independence of  $\mu|D$ , this assumption implies that the prior, likelihood, and posterior can be factored into independent terms, one for each PC. Hence inference can be done PC-by-PC instead of jointly.<sup>7</sup>

We also consider an approximation given by the following procedure: first regularize the covariance matrix according to the Wishart prior, Eq. (4). Then, we estimate  $\hat{b}$  treating the covariance matrix as known. This method is fully analytic and closely approximates the fully Bayesian solution. Fig. 2b shows the ratio of the full Bayes estimate to the approximate Bayes estimate, and

<sup>7</sup>Since  $\mu|D$  is multivariate normal with zero correlation across PCs, the elements of  $\mu$  are conditionally independent.

to the estimator which ignores covariance uncertainty,  $\hat{b}_P = (D_T + \gamma I)^{-1} \bar{\mu}$  with  $\gamma = \frac{\tau}{\kappa^2 T}$ . As the figure shows, even the simple estimator which treats covariances as known provides a good approximation to the (numerically solved) Bayesian solution. The approximate solution is even better, delivering nearly identical estimates. Throughout our empirical work we use this approximate solution, since covariance uncertainty is potentially important when we consider thousands of portfolios in Section 4.3 of the paper.

## Internet Appendix C. Interpreting interactions

What is the economic interpretation of interactions portfolios? For simplicity, consider two binary strategies with characteristic values that can be either high or low ( $\pm 1$ ). Let  $z_s^1$  and  $z_s^2$  be the characteristic values for stock  $s$ . The pair  $\{z_s^1, z_s^2\}$  takes on four values, shown in the table below:

|                          |    |    |
|--------------------------|----|----|
| $z_s^1 \backslash z_s^2$ | -1 | +1 |
| +1                       | A  | B  |
| -1                       | C  | D  |

The letters A to D are names attached to each cell. Let  $\mu_i$ ,  $i \in \{A, B, C, D\}$  be the mean returns of stocks in each cell. For simplicity, suppose the characteristics are uncorrelated so that each cell contains the same number of firms. Further, suppose returns are cross-sectionally de-meaned (equivalent to including a time fixed-effect, or an equal-weight market portfolio factor). What is the expected return on the  $z_s^1$  mimicking portfolio? That is, what is  $\lambda_1 \equiv \mathbb{E}[z_s^1 R_s]$ ? Simply  $\frac{1}{2}(\mu_A + \mu_B - \mu_C - \mu_D)$ . Similarly,  $\lambda_2 \equiv \mathbb{E}[z_s^2 R_s] = \frac{1}{2}(-\mu_A + \mu_B - \mu_C + \mu_D)$  and  $\lambda_{12} \equiv \mathbb{E}[(z_s^1 z_s^2) R_s] = \frac{1}{2}(-\mu_A + \mu_B + \mu_C - \mu_D)$ . The fact that returns are cross-sectionally de-meaned implies  $(\mu_A + \mu_B + \mu_C + \mu_D) = 0$ , so we can easily recover  $\mu_i$  from knowledge of  $\lambda_1, \lambda_2, \lambda_{12}$  by the identity

$$\lambda \equiv \begin{bmatrix} 0 \\ \lambda_1 \\ \lambda_2 \\ \lambda_{12} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \mu_A \\ \mu_B \\ \mu_C \\ \mu_D \end{bmatrix} = G\mu \quad (6)$$

since the matrix is invertible, where the first equation imposes market clearing (all our assets are market neutral, so the total risk premium on the portfolio of all stocks in the economy is zero).

Given the three managed portfolios, how would we construct something like the “small  $\times$  value” strategy which buys small-value stocks and shorts small-growth stocks?<sup>8</sup> If  $z^1$  measures market capitalization and  $z^2$  measures BE/ME, the strategy is long D and short C. Let  $G$  be the square matrix in Eq. (6). The mean of the desired strategy is  $\mu_D - \mu_C$ , which is also equal to

$$\mu_D - \mu_C = \iota'_{DC} G^{-1} \lambda$$

where  $\iota_{DC} = [0 \ 0 \ -1 \ 1]'$ , which shows the desired strategy of long D and short C can be constructed with weights equal to  $[0 \ 0 \ 1 \ -1]$  on the four managed portfolio strategies.<sup>9</sup> Hence, combining the interaction with the base strategies allows for construction of any “mixed”

<sup>8</sup>The value anomaly is larger for small stocks, which we would like our methodology to recover.

<sup>9</sup>We include the risk-free strategy (with zero excess) return for algebraic convenience.

strategies. Conceptually, what is required is that the managed portfolios form an approximate “basis” of the potential strategies.

## Internet Appendix D. Variable definitions

### D.1. Anomaly characteristics

Our anomaly definitions and descriptions are based on the lists of characteristics compiled by Hou et al. (2015); Kogan and Tian (2015); McLean and Pontiff (2016); Novy-Marx and Velikov (2016). All accounting variables are properly lagged. For annual rebalancing, returns from July of year  $t$  to June of year  $t + 1$  are matched to variables in December of  $t - 1$ . Returns from January to June of year  $t$  are matched to variables in December of year  $t - 2$ . Financial variables with a subscript “Dec” below are computed using the same timing convention. Flow variables (like dividends or investment) are annual totals as of the measurement date, unless otherwise specified. For monthly rebalancing, returns are matched to the latest quarterly report, lagged one month. Additional lagging (if required) is reported for each variable below individually. All subindices below are measured in months. A time subscript  $t$  refers to the time at which a portfolio is formed.<sup>10</sup>

1. **Size** (*size*). Follows Fama and French (1993).  $size = ME_{Jun}$ . The CRSP end of June price times shares outstanding. Rebalanced annually.
2. **Value (annual)** (*value*). Follows Fama and French (1993).  $value = BE/ME$ . At the end of June of each year, we use book equity from the previous fiscal year and market equity from December of the previous year. Rebalanced annually.
3. **Gross Profitability** (*prof*). Follows Novy Marx (2013).  $prof = GP/AT$ , where GP is gross profits and AT is total assets. Rebalanced annually.
4. **Value-Profitability** (*valprof*). Follows Novy Marx (2013).  $valprof = rank(value) + rank(prof)$ . Sum of ranks in univariate sorts on book-to-market and profitability. Annual book-to-market and profitability values are used for the entire year. Rebalanced monthly.
5. **Piotroski’s F-score** (*F-score*). Follows Piotroski (2000).  $F\text{-score} = 1_{IB>0} + 1_{\Delta ROA>0} + 1_{CFO>0} + 1_{CFO>IB} + 1_{\Delta DTA<0|DLTT=0|DLTT_{-12}=0} + 1_{\Delta ATL>0} + 1_{EqIss\leq 0} + 1_{\Delta GM>0} + 1_{\Delta ATO>0}$ , where IB is income before extraordinary items, ROA is income before extraordinary items scaled by lagged total assets, CFO is cash flow from operations, DTA is total long-term debt scaled by total assets, DLTT is total long-term debt, ATL is total current assets scaled by total current liabilities, EqIss is the difference between sales of common stock and purchases of common stock recorded on the cash flow statement, GM equals one minus the ratio of cost of goods sold and total revenues, and ATO equals total revenues, scaled by total assets. Rebalanced annually.
6. **Debt Issuance** (*debtiss*). Follows Spiess and Affleck-Graves (1999).  $debtiss = 1_{DLTISS\leq 0}$ . Binary variable equal to one if long-term debt issuance indicated in statement of cash flow. Updated annually.
7. **Share Repurchases** (*repurch*). Follows Ikenberry et al. (1995).  $repurch = 1_{PRSTKC>0}$ . Binary variable equal to one if repurchase of common or preferred shares indicated in statement of cash flow. Updated annually.

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<sup>10</sup>We make the data available at: <https://sites.google.com/site/serhiykozak/data>

8. **Share Issuance (annual)** (*nissa*). Follows Pontiff and Woodgate (2008).  $nissa = \text{shROUT}_{\text{Jun}} / \text{shROUT}_{\text{Jun}-12}$ , where *shROUT* is the number of shares outstanding. Change in real number of shares outstanding from past June to June of the previous year. Excludes changes in shares due to stock dividends and splits, and companies with no changes in *shROUT*.
9. **Accruals** (*accruals*). Follows Sloan (1996).  $\text{accruals} = \frac{\Delta\text{ACT} - \Delta\text{CHE} - \Delta\text{LCT} + \Delta\text{DLC} + \Delta\text{TXP} - \Delta\text{DP}}{(\text{AT} + \text{AT}_{-12})/2}$ , where  $\Delta\text{ACT}$  is the annual change in total current assets,  $\Delta\text{CHE}$  is the annual change in total cash and short-term investments,  $\Delta\text{LCT}$  is the annual change in current liabilities,  $\Delta\text{DLC}$  is the annual change in debt in current liabilities,  $\Delta\text{TXP}$  is the annual change in income taxes payable,  $\Delta\text{DP}$  is the annual change in depreciation and amortization, and  $(\text{AT} + \text{AT}_{-12})/2$  is average total assets over the last two years. Rebalanced annually.
10. **Asset Growth** (*growth*). Follows Cooper et al. (2008).  $\text{growth} = \text{AT}/\text{AT}_{-12}$ . Rebalanced annually.
11. **Asset Turnover** (*aturnover*). Follows Soliman (2008).  $\text{aturnover} = \text{SALE}/\text{AT}$ . Sales to total assets. Rebalanced annually.
12. **Gross Margins** (*gmargins*). Follows Novy Marx (2013).  $\text{gmargins} = \text{GP}/\text{SALE}$ , where GP is gross profits and SALE is total revenues. Rebalanced annually.
13. **Dividend Yield** (*divp*). Follows Naranjo et al. (1998).  $\text{divp} = \text{Div}/\text{ME}_{\text{Dec}}$ . Dividend scaled by price. Both are measured in December of the year  $t - 1$  or  $t - 2$  (for returns in months prior to July). Rebalanced annually.
14. **Earnings/Price** (*ep*). Follows Basu (1977).  $\text{ep} = \text{IB}/\text{ME}_{\text{Dec}}$ . Net income scaled by market value of equity. Updated annually.
15. **Cash Flow / Market Value of Equity** (*cfp*). Follows Lakonishok et al. (1994).  $\text{cfp} = (\text{IB} + \text{DP})/\text{ME}_{\text{Dec}}$ . Net income plus depreciation and amortization, all scaled by market value of equity measured at the same date. Updated annually.
16. **Net Operating Assets** (*noa*). Follows Hirshleifer et al. (2004).  $\text{noa} = (\text{AT} - \text{CHE}) - (\text{AT} - \text{DLC} - \text{DLTT} - \text{MIB} - \text{PSTK} - \text{CEQ})$ , where AT is total assets, CHE is cash and short-term investments, DLC is debt in current liabilities, DLTT is long term debt, MIB is non-controlling interest, PSTK is preferred capital stock, and CEQ is common equity. Updated annually.
17. **Investment** (*inv*). Follows Chen et al. (2011); Lyandres et al. (2007).  $\text{inv} = (\Delta\text{PPEGT} + \Delta\text{INVT})/\text{AT}_{-12}$ , where  $\Delta\text{PPEGT}$  is the annual change in gross total property, plant, and equipment,  $\Delta\text{INVT}$  is the annual change in total inventories, and  $\text{AT}_{-12}$  is lagged total assets. Rebalanced annually, uses the full period.
18. **Investment-to-Capital** (*invcap*). Follows Xing (2008).  $\text{invcap} = \text{CAPX}/\text{PPENT}$ . Investment to capital is the ratio of capital expenditure (Compustat item CAPX) over property, plant, and equipment (Compustat item PPENT).
19. **Investment Growth** (*growth*). Follows Xing (2008).  $\text{growth} = \text{CAPX}/\text{CAPX}_{-12}$ . Investment growth is the percentage change in capital expenditure (Compustat item CAPX).
20. **Sales Growth** (*sgrowth*). Follows Lakonishok et al. (1994).  $\text{sgrowth} = \text{SALE}/\text{SALE}_{-12}$ . Sales growth is the percent change in net sales over turnover (Compustat item SALE).

21. **Leverage** (*lev*). Follows Bhandari (1988).  $lev = AT/ME_{Dec}$ . Market leverage is the ratio of total assets (Compustat item AT) over the market value of equity. Both are measured in December of the same year.
22. **Return on Assets (annual)** (*roaa*). Follows Chen et al. (2011).  $roaa = IB/AT$ . Net income scaled by total assets. Updated annually.
23. **Return on Equity (annual)** (*roea*). Follows Haugen and Baker (1996).  $roea = IB/BE$ . Net income scaled by book value of equity. Updated annually.
24. **Sales-to-Price** (*sp*). Follows Barbee Jr. et al. (1996).  $sp = SALE/ME_{Dec}$ . Total revenues divided by stock price. Updated annually.
25. **Growth in LTNOA** (*gltnoa*). Follows Fairfield et al. (2003).  $gltnoa = GRNOA - ACC$ . Growth in Net Operating Assets minus Accruals.  $NOA = (RECT + INVT + ACO + PPENT + INTAN + AO - AP - LCO - LO) / AT$ ,  $GRNOA = NOA - NOA_{-12}$ ,  $ACC = ((RECT - RECT_{-12}) + (INVT - INVT_{-12}) + (ACO - ACO_{-12}) - (AP - AP_{-12}) - (LCO - LCO_{-12}) - DP) / ((AT + AT_{-12}) / 2)$ , where RECT = Receivables, INVT = Total Inventory, ACO = Current Assets, AP = Accounts Payable, LCO = Current Liabilities (Other), DP = Depreciation and Amortization, AT = Assets, PPENT = Property, Plant, and Equipment (net), INTAN = Intangible Assets, AO = Assets (Other), LO = Liabilities (Other). Updated annually.
26. **Momentum (6m)** (*mom*). Follows Jegadeesh and Titman (1993).  $mom = \sum_{l=2}^7 r_{t-l}$ . Cumulated past performance in the previous 6 months by skipping the most recent month. Rebalanced monthly.
27. **Industry Momentum** (*indmom*). Follows Moskowitz and Grinblatt (1999).  $indmom = \text{rank}(\sum_{l=1}^6 r_{t-l}^{ind})$ . In each month, the Fama and French 49 industries are ranked on their value-weighted past 6-months performance. Rebalanced monthly.
28. **Value-Momentum** (*valmom*). Follows Novy Marx (2013).  $valmom = \text{rank}(B/M) + \text{rank}(Mom)$ . Sum of ranks in univariate sorts on book-to-market and momentum. Annual book-to-market values are used for the entire year. Rebalanced monthly.
29. **Value-Momentum-Profitability** (*valmomprof*). Follows Novy Marx (2013).  $valmomprof = \text{rank}(B/M) + \text{rank}(Prof) + \text{rank}(Mom)$ . Sum of ranks in univariate sorts on book-to-market, profitability, and momentum. Annual book-to-market and profitability values are used for the entire year. Rebalanced monthly.
30. **Short Interest** (*shortint*). Follows Dechow et al. (1998).  $shortint = \text{Shares Shorted} / \text{Shares Outstanding}$ . Updated monthly.
31. **Momentum (1 year)** (*mom12*). Follows Jegadeesh and Titman (1993).  $mom12 = \sum_{l=2}^{12} r_{t-l}$ . Cumulated past performance in the previous year by skipping the most recent month. Rebalanced monthly.
32. **Momentum-Reversal** (*momrev*). Follows Jegadeesh and Titman (1993).  $momrev = \sum_{l=14}^{19} r_{t-l}$ . Buy and hold returns from  $t - 19$  to  $t - 14$ . Updated monthly.
33. **Long-term Reversals** (*lrrev*). Follows DeBondt and Thaler (1985).  $lrrev = \sum_{l=13}^{60} r_{t-l}$ . Cumulative returns from  $t - 60$  to  $t - 13$ . Updated monthly.

34. **Value (monthly)** (*valuem*). Follows Asness and Frazzini (2013).  $valuem = BEQ_{-3}/ME_{-1}$ . Book-to-market ratio using the most up-to-date prices and book equity (appropriately lagged). Rebalanced monthly.
35. **Share Issuance (monthly)** (*nissm*). Follows Pontiff and Woodgate (2008).  $nissm = shROUT_{t-13} / shROUT_{t-1}$ , where *shROUT* is the number of shares outstanding. Change in real number of shares outstanding from  $t - 13$  to  $t - 1$ . Excludes changes in shares due to stock dividends and splits, and companies with no changes in *shROUT*.
36. **PEAD (SUE)** (*sue*). Follows Foster et al. (1984).  $sue = \frac{IBQ - IBQ_{-12}}{\sigma_{IBQ_{-24}:IBQ_{-3}}}$ , where *IBQ* is income before extraordinary items (updated quarterly), and  $\sigma_{IBQ_{-24}:IBQ_{-3}}$  is the standard deviation of *IBQ* in the past two years skipping the most recent quarter. Earnings surprises are measured by Standardized Unexpected Earnings (SUE), which is the change in the most recently announced quarterly earnings per share from its value announced four quarters ago divided by the standard deviation of this change in quarterly earnings over the prior eight quarters. Rebalanced monthly.
37. **Return on Book Equity** (*roe*). Follows Chen et al. (2011).  $roe = IBQ/BEQ_{-3}$ , where *IBQ* is income before extraordinary items (updated quarterly), and *BEQ* is book value of equity. Rebalanced monthly.
38. **Return on Market Equity** (*rome*). Follows Chen et al. (2011).  $rome = IBQ/ME_{-4}$ , where *IBQ* is income before extraordinary items (updated quarterly), and *ME* is market value of equity. Rebalanced monthly.
39. **Return on Assets** (*roa*). Follows Chen et al. (2011).  $roa = IBQ/ATQ_{-3}$ . Net income scaled by total assets. Updated quarterly.
40. **Short-term Reversal** (*strev*). Follows Jegadeesh (1990).  $strev = r_{t-1}$ . Return in the previous month. Updated monthly.
41. **Idiosyncratic Volatility** (*ivol*). Follows Ang et al. (2006).  $ivol = \text{std}(R_{i,t} - \beta_i R_{M,t} - s_i \text{SMB}_t - h_i \text{HML}_t)$ . The standard deviation of the residual from firm-level regression of daily stock returns on the daily innovations of the Fama and French three-factor model using the estimation window of three months. Lagged one month.
42. **Beta Arbitrage** (*beta*). Follows Cooper et al. (2008).  $beta = \beta_{t-60:t-1}$ . Beta with respect to the CRSP equal-weighted return index. Estimated over the past 60 months (minimum 36 months) using daily data and lagged one month. Updated monthly.
43. **Seasonality** (*season*). Follows Heston and Sadka (2008).  $season = \sum_{l=1}^5 r_{t-l \times 12}$ . Average monthly return in the same calendar month over the last 5 years. As an example, the average return from prior Octobers is used to predict returns this October. The firm needs at least one year of data to be included in the sample. Updated monthly.
44. **Industry Relative Reversals** (*indrrev*). Follows Da et al. (2013).  $indrrev = r_{-1} - r_{-1}^{\text{ind}}$ , where  $r$  is the return on a stock and  $r^{\text{ind}}$  is return on its industry. Difference between a stocks' prior month's return and the prior month's return of its industry (based on the Fama and French 49 industries). Updated monthly.

45. **Industry Relative Reversals (Low Volatility)** (*indrrevlv*). Follows Da et al. (2013).  $indrrevlv = r_{-1} - r_{-1}^{ind}$  if  $vol < NYSE$  median, where  $r$  is the return on a stock and  $r^{ind}$  is return on its industry. Difference between a stocks' prior month's return and the prior month's return of its industry (based on the Fama and French 49 industries). Only stocks with idiosyncratic volatility lower than the NYSE median for month are included in the sorts. Updated monthly.
46. **Industry Momentum-Reversal** (*indmomrev*). Follows Moskowitz and Grinblatt (1999).  $indmomrev = rank(\text{industry momentum}) + rank(\text{industry relative-reversals low-vol})$ . Sum of Fama and French 49 industries ranks on industry momentum and industry relative reversals (low vol). Rebalanced monthly.
47. **Composite Issuance** (*ciss*). Follows Daniel and Titman (2006).  $ciss = \log\left(\frac{ME_{t-13}}{ME_{t-60}}\right) - \sum_{l=13}^{60} r_{t-l}$ , where  $r$  is the log return on the stock and ME is total market equity. Updated monthly.
48. **Price** (*price*). Follows Blume and Husic (1973).  $price = \log(ME/shrout)$ , where ME is market equity and shrout is the number of shares outstanding. Log of stock price. Updated monthly.
49. **Firm Age** (*age*). Follows Barry and Brown (1984).  $age = \log(1 + \text{number of months since listing})$ . The number of months that a firm has been listed in the CRSP database.
50. **Share Volume** (*shvol*). Follows Datar et al. (1998).  $shvol = \frac{1}{3} \sum_{i=1}^3 \text{volume}_{t-i}/shrout_t$ . Average number of shares traded over the previous three months scaled by shares outstanding. Updated monthly.

## D.2. WRDS financial ratios

WRDS Industry Financial Ratio is a collection of most commonly used financial ratios by academic researchers. The data are provided by the Financial Ratios Suite by WRDS. There are in total over 70 financial ratios grouped into the following seven categories: Capitalization, Efficiency, Financial Soundness/Solvency, Liquidity, Profitability, Valuation and Others.

1. **P/E (Diluted, Excl. EI)** (*pe\_exi*) – Valuation. Price-to-Earnings, excl. Extraordinary Items (diluted).
2. **P/E (Diluted, Incl. EI)** (*pe\_inc*) – Valuation. Price-to-Earnings, incl. Extraordinary Items (diluted).
3. **Price/Sales** (*ps*) – Valuation. Multiple of Market Value of Equity to Sales.
4. **Price/Cash flow** (*pcf*) – Valuation. Multiple of Market Value of Equity to Net Cash Flow from Operating Activities.
5. **Enterprise Value Multiple** (*evm*) – Valuation. Multiple of Enterprise Value to EBITDA.
6. **Book/Market** (*bm*) – Valuation. Book Value of Equity as a fraction of Market Value of Equity.
7. **Shiller's Cyclically Adjusted P/E Ratio** (*capei*) – Valuation. Multiple of Market Value of Equity to 5-year moving average of Net Income.

8. **Dividend Payout Ratio** (*dpr*) – Valuation. Dividends as a fraction of Income Before Extra-Items.
9. **Net Profit Margin** (*npm*) – Profitability. Net Income as a fraction of Sales.
10. **Operating Profit Margin Before Depreciation** (*opmbd*) – Profitability. Operating Income Before Depreciation as a fraction of Sales.
11. **Operating Profit Margin After Depreciation** (*opmad*) – Profitability. Operating Income After Depreciation as a fraction of Sales.
12. **Gross Profit Margin** (*gpm*) – Profitability. Gross Profit as a fraction of Sales.
13. **Pre-tax Profit Margin** (*ptpm*) – Profitability. Pretax Income as a fraction of Sales.
14. **Cash Flow Margin** (*cfm*) – Financial Soundness. Income before Extraordinary Items and Depreciation as a fraction of Sales.
15. **Return on Assets** (*roa*) – Profitability. Operating Income Before Depreciation as a fraction of average Total Assets based on most recent two periods.
16. **Return on Equity** (*roe*) – Profitability. Net Income as a fraction of average Book Equity based on most recent two periods, where Book Equity is defined as the sum of Total Parent Stockholders' Equity and Deferred Taxes and Investment Tax Credit.
17. **Return on Capital Employed** (*roce*) – Profitability. Earnings Before Interest and Taxes as a fraction of average Capital Employed based on most recent two periods, where Capital Employed is the sum of Debt in Long-term and Current Liabilities and Common/Ordinary Equity.
18. **After-tax Return on Average Common Equity** (*aftret\_eq*) – Profitability. Net Income as a fraction of average of Common Equity based on most recent two periods.
19. **After-tax Return on Invested Capital** (*aftret\_invcapx*) – Profitability. Net Income plus Interest Expenses as a fraction of Invested Capital.
20. **After-tax Return on Total Stockholders' Equity** (*aftret\_equity*) – Profitability. Net Income as a fraction of average of Total Shareholders' Equity based on most recent two periods.
21. **Pre-tax return on Net Operating Assets** (*pretret\_noa*) – Profitability. Operating Income After Depreciation as a fraction of average Net Operating Assets (NOA) based on most recent two periods, where NOA is defined as the sum of Property Plant and Equipment and Current Assets minus Current Liabilities.
22. **Pre-tax Return on Total Earning Assets** (*pretret\_earnat*) – Profitability. Operating Income After Depreciation as a fraction of average Total Earnings Assets (TEA) based on most recent two periods, where TEA is defined as the sum of Property Plant and Equipment and Current Assets.
23. **Common Equity/Invested Capital** (*equity\_invcap*) – Capitalization. Common Equity as a fraction of Invested Capital.

24. **Long-term Debt/Invested Capital** (*debt\_invcap*) – Capitalization. Long-term Debt as a fraction of Invested Capital.
25. **Total Debt/Invested Capital** (*totdebt\_invcap*) – Capitalization. Total Debt (Long-term and Current) as a fraction of Invested Capital.
26. **Interest/Average Long-term Debt** (*int\_debt*) – Financial Soundness. Interest as a fraction of average Long-term debt based on most recent two periods.
27. **Interest/Average Total Debt** (*int\_totdebt*) – Financial Soundness. Interest as a fraction of average Total Debt based on most recent two periods.
28. **Cash Balance/Total Liabilities** (*cash\_lt*) – Financial Soundness. Cash Balance as a fraction of Total Liabilities.
29. **Inventory/Current Assets** (*invnt\_act*) – Financial Soundness. Inventories as a fraction of Current Assets.
30. **Receivables/Current Assets** (*rect\_act*) – Financial Soundness. Accounts Receivables as a fraction of Current Assets.
31. **Total Debt/Total Assets** (*debt\_at*) – Solvency. Total Liabilities as a fraction of Total Assets.
32. **Short-Term Debt/Total Debt** (*short\_debt*) – Financial Soundness. Short-term Debt as a fraction of Total Debt.
33. **Current Liabilities/Total Liabilities** (*curr\_debt*) – Financial Soundness. Current Liabilities as a fraction of Total Liabilities.
34. **Long-term Debt/Total Liabilities** (*lt\_debt*) – Financial Soundness. Long-term Debt as a fraction of Total Liabilities.
35. **Free Cash Flow/Operating Cash Flow** (*fcf\_ocf*) – Financial Soundness. Free Cash Flow as a fraction of Operating Cash Flow, where Free Cash Flow is defined as the difference between Operating Cash Flow and Capital Expenditures.
36. **Advertising Expenses/Sales** (*adv\_sale*) – Other. Advertising Expenses as a fraction of Sales.
37. **Profit Before Depreciation/Current Liabilities** (*profit\_lct*) – Financial Soundness. Operating Income before D&A as a fraction of Current Liabilities.
38. **Total Debt/EBITDA** (*debt\_ebitda*) – Financial Soundness. Gross Debt as a fraction of EBITDA.
39. **Operating CF/Current Liabilities** (*ocf\_lct*) – Financial Soundness. Operating Cash Flow as a fraction of Current Liabilities.
40. **Total Liabilities/Total Tangible Assets** (*lt\_ppent*) – Financial Soundness. Total Liabilities to Total Tangible Assets.
41. **Long-term Debt/Book Equity** (*dltt\_be*) – Financial Soundness. Long-term Debt to Book Equity.

42. **Total Debt/Total Assets** (*debt\_assets*) – Solvency. Total Debt as a fraction of Total Assets.
43. **Total Debt/Capital** (*debt\_capital*) – Solvency. Total Debt as a fraction of Total Capital, where Total Debt is defined as the sum of Accounts Payable and Total Debt in Current and Long-term Liabilities, and Total Capital is defined as the sum of Total Debt and Total Equity (common and preferred).
44. **Total Debt/Equity** (*de\_ratio*) – Solvency. Total Liabilities to Shareholders' Equity (common and preferred).
45. **After-tax Interest Coverage** (*intcov*) – Solvency. Multiple of After-tax Income to Interest and Related Expenses.
46. **Cash Ratio** (*cash\_ratio*) – Liquidity. Cash and Short-term Investments as a fraction of Current Liabilities.
47. **Quick Ratio (Acid Test)** (*quick\_ratio*) – Liquidity. Quick Ratio: Current Assets net of Inventories as a fraction of Current Liabilities.
48. **Current Ratio** (*curr\_ratio*) – Liquidity. Current Assets as a fraction of Current Liabilities.
49. **Capitalization Ratio** (*capital\_ratio*) – Capitalization. Total Long-term Debt as a fraction of the sum of Total Long-term Debt, Common/Ordinary Equity and Preferred Stock.
50. **Cash Flow/Total Debt** (*cash\_debt*) – Financial Soundness. Operating Cash Flow as a fraction of Total Debt.
51. **Inventory Turnover** (*inv\_turn*) – Efficiency. COGS as a fraction of the average Inventories based on the most recent two periods.
52. **Asset Turnover** (*at\_turn*) – Efficiency. Sales as a fraction of the average Total Assets based on the most recent two periods.
53. **Receivables Turnover** (*rect\_turn*) – Efficiency. Sales as a fraction of the average of Accounts Receivables based on the most recent two periods.
54. **Payables Turnover** (*pay\_turn*) – Efficiency. COGS and change in Inventories as a fraction of the average of Accounts Payable based on the most recent two periods.
55. **Sales/Invested Capital** (*sale\_invcap*) – Efficiency. Sales per dollar of Invested Capital.
56. **Sales/Stockholders Equity** (*sale\_equity*) – Efficiency. Sales per dollar of total Stockholders' Equity.
57. **Sales/Working Capital** (*sale\_nwc*) – Efficiency. Sales per dollar of Working Capital, defined as difference between Current Assets and Current Liabilities.
58. **Research and Development/Sales** (*RD\_SALE*) – Other. R&D expenses as a fraction of Sales.
59. **Accruals/Average Assets** (*Accrual*) – Other. Accruals as a fraction of average Total Assets based on most recent two periods.

60. **Gross Profit/Total Assets** (*GProf*) – Profitability. Gross Profitability as a fraction of Total Assets.
61. **Book Equity** (*be*) – Other. Firm size as measured by total book equity.
62. **Cash Conversion Cycle (Days)** (*cash\_conversion*) – Liquidity. Inventories per daily COGS plus Account Receivables per daily Sales minus Account Payables per daily COGS.
63. **Effective Tax Rate** (*efftax*) – Profitability. Income Tax as a fraction of Pretax Income.
64. **Interest Coverage Ratio** (*intcov\_ratio*) – Solvency. Multiple of Earnings Before Interest and Taxes to Interest and Related Expenses.
65. **Labor Expenses/Sales** (*staff\_sale*) – Other. Labor Expenses as a fraction of Sales.
66. **Dividend Yield** (*divyield*) – Valuation. Indicated Dividend Rate as a fraction of Price.
67. **Price/Book** (*ptb*) – Valuation. Multiple of Market Value of Equity to Book Value of Equity.
68. **Trailing P/E to Growth (PEG) ratio** (*PEG\_trailing*) – Valuation. Price-to-Earnings, excl. Extraordinary Items (diluted) to 3-Year past EPS Growth.
- 69-80. **Return in Month  $t - i$**  (*ret\_lag<sub>i</sub>*) – Other. Past one-month returns in months  $t - i$  for  $i = \{1, \dots, 12\}$ .

**Table 1. Part I:** Mean annualized returns on anomaly portfolios, %

The table lists all basic “anomaly” characteristics used in our analysis and shows annualized mean returns on managed portfolios which are linear in characteristics. Columns (1)-(3) show mean annualized returns (in %) for managed portfolios corresponding to all characteristics in the full sample, pre-2005 sample, and post-2005 sample, respectively. All managed portfolios’ returns are based on a monthly-rebalanced buy-and-hold strategy and are further rescaled to have standard deviations equal to the in-sample standard deviation of excess returns on the aggregate market index. The sample is daily from November 1973 to December 2017.

|                               | (1)         | (2)      | (3)       |
|-------------------------------|-------------|----------|-----------|
|                               | Full Sample | Pre 2005 | Post 2005 |
| 1. Size                       | -2.3        | -2.8     | -1.0      |
| 2. Value (A)                  | 6.2         | 9.2      | -0.9      |
| 3. Gross profitability        | 3.6         | 2.4      | 6.7       |
| 4. Value-profitablity         | 13.2        | 17.8     | 2.3       |
| 5. F-score                    | 8.1         | 10.0     | 3.8       |
| 6. Debt issuance              | 1.8         | 1.0      | 3.6       |
| 7. Share repurchases          | 6.9         | 7.5      | 5.5       |
| 8. Net issuance (A)           | -9.5        | -11.5    | -4.7      |
| 9. Accruals                   | -5.6        | -7.7     | -0.5      |
| 10. Asset growth              | -8.6        | -11.1    | -2.5      |
| 11. Asset turnover            | 5.3         | 3.9      | 8.6       |
| 12. Gross margins             | -1.2        | 0.1      | -4.1      |
| 13. Dividend/Price            | 3.6         | 5.5      | -1.1      |
| 14. Earnings/Price            | 8.3         | 10.6     | 2.6       |
| 15. Cash Flows/Price          | 7.9         | 10.4     | 1.9       |
| 16. Net operating assets      | 1.9         | 3.6      | -2.2      |
| 17. Investment/Assets         | -10.0       | -12.5    | -3.9      |
| 18. Investment/Capital        | -4.1        | -4.9     | -2.1      |
| 19. Investment growth         | -9.0        | -10.9    | -4.4      |
| 20. Sales growth              | -5.7        | -5.9     | -5.3      |
| 21. Leverage                  | 4.9         | 7.7      | -2.1      |
| 22. Return on assets (A)      | 2.4         | 0.5      | 7.1       |
| 23. Return on book equity (A) | 4.7         | 4.8      | 4.6       |
| 24. Sales/Price               | 9.4         | 11.6     | 4.1       |
| 25. Growth in LTNOA           | -2.5        | -1.7     | -4.4      |
| 26. Momentum (6m)             | 2.1         | 4.1      | -2.7      |
| 27. Industry momentum         | 5.6         | 8.1      | -0.2      |
| 28. Value-momentum            | 5.1         | 7.8      | -1.3      |
| 29. Value-momentum-prof.      | 6.5         | 9.6      | -0.8      |
| 30. Short interest            | 0.3         | 1.6      | -2.8      |

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**Table 1. Part II:** Mean annualized returns on anomaly portfolios, %

|                               | (1)   | (2)   | (3)  |
|-------------------------------|-------|-------|------|
| 31. Momentum (12m)            | 9.0   | 12.7  | 0.1  |
| 32. Momentum-reversals        | -5.7  | -7.8  | -0.7 |
| 33. Long-run reversals        | -5.4  | -7.9  | 0.4  |
| 34. Value (M)                 | 5.5   | 7.9   | -0.3 |
| 35. Net issuance (M)          | -8.7  | -9.9  | -5.9 |
| 36. Earnings surprises        | 12.0  | 15.2  | 4.5  |
| 37. Return on equity          | 10.5  | 12.2  | 6.5  |
| 38. Return on market equity   | 12.2  | 15.3  | 5.0  |
| 39. Return on assets          | 7.1   | 7.1   | 7.0  |
| 40. Short-term reversals      | -8.0  | -11.9 | 1.3  |
| 41. Idiosyncratic volatility  | -3.1  | -3.7  | -1.6 |
| 42. Beta arbitrage            | -0.7  | -0.3  | -1.5 |
| 43. Seasonality               | 11.5  | 18.6  | -5.6 |
| 44. Industry rel. reversals   | -17.8 | -25.6 | 0.8  |
| 45. Industry rel. rev. (L.V.) | -34.9 | -47.3 | -5.1 |
| 46. Ind. mom-reversals        | 20.1  | 29.1  | -1.7 |
| 47. Composite issuance        | -8.4  | -10.2 | -4.1 |
| 48. Price                     | -1.1  | -1.1  | -1.2 |
| 49. Age                       | 3.5   | 4.7   | 0.4  |
| 50. Share volume              | -1.2  | -1.3  | -1.1 |

**Table 2. Part I:** Mean annualized returns on WFR portfolios, %

The table lists all basic WFR characteristics used in our analysis and shows annualized mean returns on managed portfolios which are linear in characteristics. Columns (1)-(3) show mean annualized returns (in %) for managed portfolios corresponding to all characteristics in the full sample, pre-2005 sample, and post-2005 sample, respectively. All managed portfolios' returns are based on a monthly-rebalanced buy-and-hold strategy and are further rescaled to have standard deviations equal to the in-sample standard deviation of excess returns on the aggregate market index. The sample is daily from September 1964 to December 2017.

|   | (1)            | (2)      | (3)       |
|---|----------------|----------|-----------|
|   | Full<br>Sample | Pre 2005 | Post 2005 |
| 1. P/E (diluted, excl. EI)                        | -10.3          | -11.8    | -5.6      |
| 2. P/E (diluted, incl. EI)                        | -13.1          | -15.5    | -5.9      |
| 3. Price/Sales                                    | -7.9           | -9.1     | -4.2      |
| 4. Price/Cash flow                                | -4.6           | -5.0     | -3.4      |
| 5. Enterprise value multiple                      | -10.2          | -11.5    | -6.3      |
| 6. Book/Market                                    | 4.0            | 5.7      | -1.0      |
| 7. Shillers cyclically adjusted P/E Ratio         | -5.6           | -7.7     | 1.0       |
| 8. Dividend payout ratio                          | -1.8           | -2.0     | -1.3      |
| 9. Net profit margin                              | 2.1            | 2.8      | -0.2      |
| 10. Operating profit margin before depreciation   | 2.1            | 3.8      | -2.9      |
| 11. Operating profit margin after depreciation    | 2.6            | 4.0      | -1.6      |
| 12. Gross profit margin                           | 1.0            | 2.4      | -3.4      |
| 13. Pre-tax profit margin                         | 2.8            | 3.6      | 0.4       |
| 14. Cash flow margin                              | 0.9            | 1.8      | -1.7      |
| 15. Return on assets                              | 7.0            | 6.9      | 7.6       |
| 16. Return on equity                              | 7.3            | 7.6      | 6.1       |
| 17. Return on capital employed                    | 8.9            | 8.6      | 9.9       |
| 18. After-tax return on average common equity     | 8.0            | 8.8      | 5.6       |
| 19. After-tax return on invested capital          | 6.1            | 6.1      | 6.2       |
| 20. After-tax return on total stockholders equity | 7.9            | 8.6      | 5.6       |
| 21. Pre-tax return on net operating assets        | 7.1            | 8.2      | 3.7       |
| 22. Pre-tax return on total earning assets        | 6.7            | 7.8      | 3.3       |
| 23. Common equity/Invested capital                | 1.2            | 0.9      | 2.1       |
| 24. Long-term debt/Invested capital               | -0.4           | -0.1     | -1.6      |
| 25. Total debt/Invested capital                   | -0.6           | 0.0      | -2.4      |
| 26. Interest/Average long-term debt               | 3.1            | 4.5      | -1.3      |
| 27. Interest/Average total debt                   | 3.2            | 4.5      | -0.4      |
| 28. Cash balance/Total liabilities                | 1.2            | 1.1      | 1.6       |
| 29. Inventory/Current assets                      | 0.1            | -0.7     | 2.6       |
| 30. Receivables/Current assets                    | 0.2            | 0.3      | -0.1      |

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**Table 2. Part II:** Mean annualized returns on WFR portfolios, %

|  | (1)   | (2)   | (3)  |
|--|-------|-------|------|
| 31. Total debt/Total assets                        | -2.7  | -2.7  | -2.7 |
| 32. Short-term debt/Total debt                     | -0.3  | 0.7   | -3.3 |
| 33. Current liabilities/Total liabilities          | 2.6   | 3.0   | 1.4  |
| 34. Long-term debt/Total liabilities               | -5.1  | -5.9  | -2.4 |
| 35. Free cash flow/Operating cash flow             | 17.0  | 20.9  | 5.1  |
| 36. Advertising expenses/Sales                     | 2.1   | 2.2   | 1.7  |
| 37. Profit before depreciation/Current liabilities | 3.3   | 4.3   | 0.1  |
| 38. Total debt/EBITDA                              | -1.8  | -1.1  | -4.0 |
| 39. Operating CF/Current liabilities               | 11.2  | 14.7  | 0.3  |
| 40. Total liabilities/Total tangible assets        | 3.0   | 4.2   | -0.8 |
| 41. Long-term debt/Book equity                     | -1.2  | -1.0  | -1.9 |
| 42. Total debt/Total assets                        | 2.2   | 2.8   | 0.6  |
| 43. Total debt/capital                             | 1.0   | 1.8   | -1.1 |
| 44. Total debt/Equity                              | 1.9   | 2.4   | 0.3  |
| 45. After-tax interest coverage                    | 5.1   | 5.0   | 5.4  |
| 46. Cash ratio                                     | 0.8   | 1.2   | -0.4 |
| 47. Quick ratio (acid test)                        | -1.3  | -1.4  | -0.9 |
| 48. Current ratio                                  | -1.6  | -2.0  | -0.3 |
| 49. Capitalization ratio                           | -0.4  | -0.1  | -1.2 |
| 50. Cash flow/Total debt                           | 11.4  | 13.0  | 6.5  |
| 51. Inventory turnover                             | 2.7   | 3.8   | -0.5 |
| 52. Asset turnover                                 | 6.5   | 5.8   | 8.5  |
| 53. Receivables turnover                           | 3.7   | 3.1   | 5.2  |
| 54. Payables turnover                              | -1.2  | -4.2  | 8.0  |
| 55. Sales/Invested capital                         | 8.9   | 8.4   | 10.4 |
| 56. Sales/Stockholders equity                      | 7.9   | 7.9   | 7.8  |
| 57. Sales/Working capital                          | 2.8   | 3.7   | 0.2  |
| 58. Research and development/Sales                 | 3.5   | 3.6   | 3.2  |
| 59. Accruals/Average assets                        | 12.0  | 13.5  | 7.7  |
| 60. Gross profit/Total assets                      | 6.7   | 6.4   | 7.8  |
| 61. Book equity                                    | 0.7   | 1.3   | -1.0 |
| 62. Cash conversion cycle (days)                   | -3.5  | -4.2  | -1.1 |
| 63. Effective tax rate                             | 3.6   | 4.1   | 2.3  |
| 64. Interest coverage ratio                        | 6.8   | 6.5   | 7.4  |
| 65. Labor expenses/Sales                           | 0.7   | 1.7   | -2.5 |
| 66. Dividend yield                                 | 3.4   | 4.7   | -0.6 |
| 67. Price/Book                                     | -5.0  | -6.7  | 0.4  |
| 68. Trailing P/E to growth (PEG) ratio             | -10.5 | -12.0 | -5.9 |
| 69. Month $t - 1$                                  | -8.6  | -11.8 | 1.1  |
| 70. Month $t - 2$                                  | 0.4   | 0.7   | -0.6 |

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**Table 2. Part III:** Mean annualized returns on WFR portfolios, %

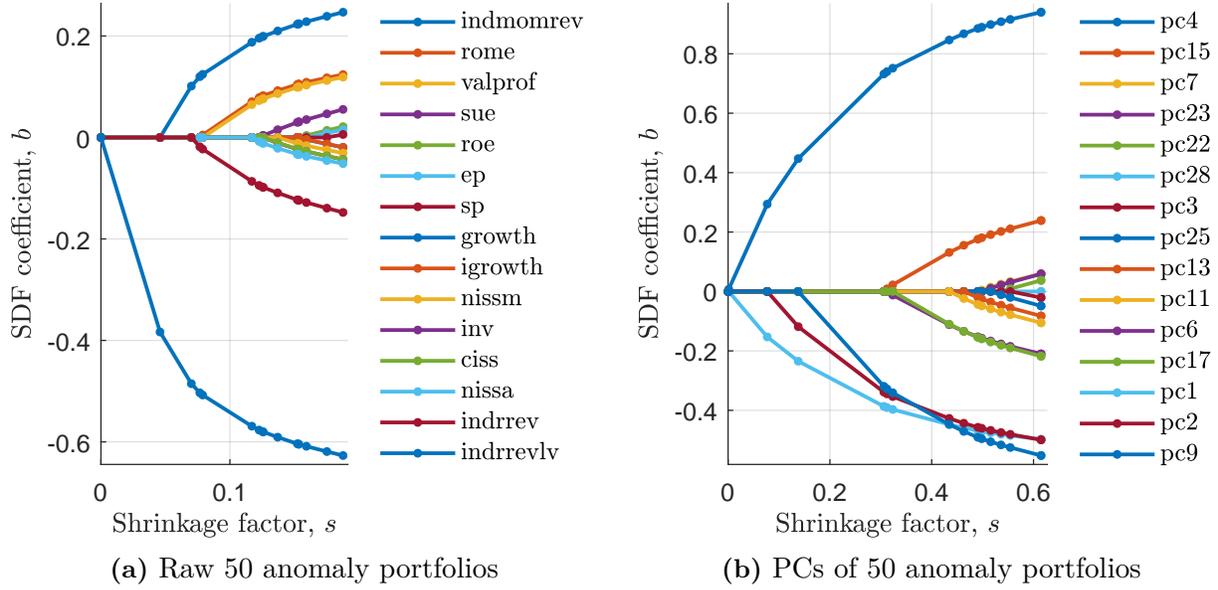
|                    | (1) | (2)  | (3)  |
|--------------------|-----|------|------|
| 71. Month $t - 3$  | 3.4 | 4.5  | 0.0  |
| 72. Month $t - 4$  | 3.1 | 3.0  | 3.3  |
| 73. Month $t - 5$  | 2.7 | 3.4  | 0.5  |
| 74. Month $t - 6$  | 5.9 | 8.6  | -2.5 |
| 75. Month $t - 7$  | 3.2 | 4.6  | -1.1 |
| 76. Month $t - 8$  | 3.2 | 3.6  | 2.0  |
| 77. Month $t - 9$  | 9.6 | 12.2 | 1.8  |
| 78. Month $t - 10$ | 5.3 | 8.0  | -3.2 |
| 79. Month $t - 11$ | 9.0 | 8.4  | 10.9 |
| 80. Month $t - 12$ | 7.2 | 10.0 | -1.3 |

## Internet Appendix E. Supplementary plots and tables

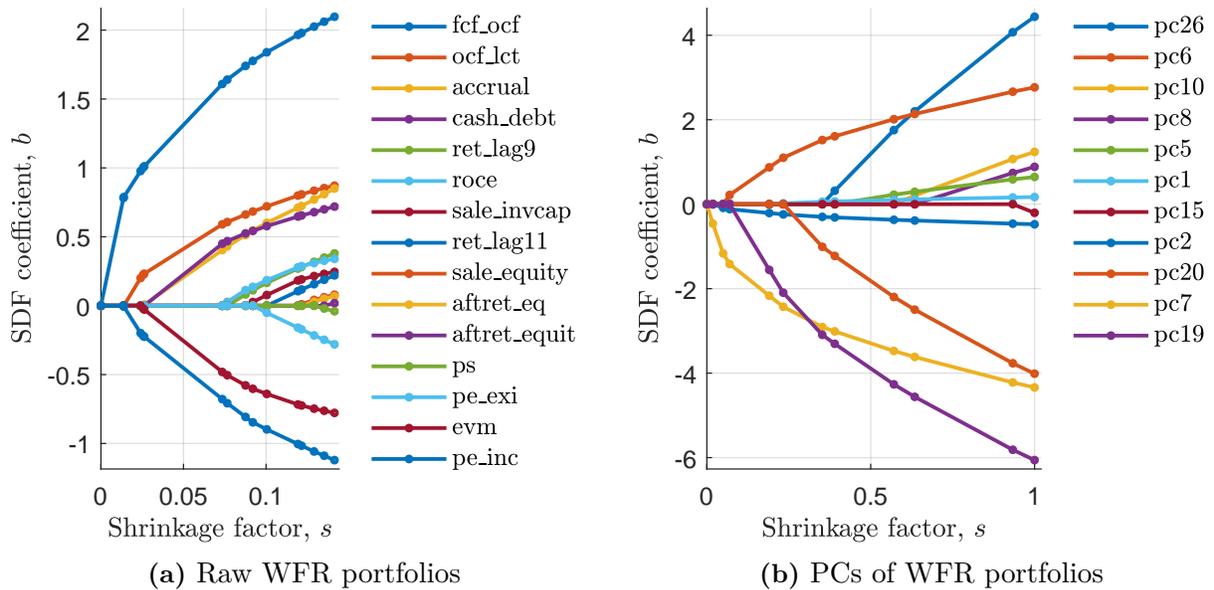
**Table 3.** Percentage of variance explained by PCs, %

The table shows the percentage of variance explained by  $PC_j$  of portfolio returns in each dataset (“% of var.” rows) and cumulative variance explained by  $PC_1$  through  $PC_j$  (“Cumulative”). All portfolios are orthogonalized with respect to the aggregate market.

|  | PC <sub>1</sub> | PC <sub>2</sub> | PC <sub>5</sub> | PC <sub>10</sub> | PC <sub>20</sub> | PC <sub>50</sub> | PC <sub>100</sub> | PC <sub>200</sub> | PC <sub>500</sub> | PC <sub>1000</sub> |
|--|-----------------|-----------------|-----------------|------------------|------------------|------------------|-------------------|-------------------|-------------------|--------------------|
| PCs of Fama and French 25 ME/BM portfolios |                 |                 |                 |                  |                  |                  |                   |                   |                   |                    |
| % of var.                                  | 31.6            | 12.4            | 3.8             | 2.5              | 1.8              | -                | -                 | -                 | -                 | -                  |
| Cumulative                                 | 31.6            | 44.1            | 57.0            | 71.8             | 92.3             | -                | -                 | -                 | -                 | -                  |
| PCs of 50 anomaly portfolios               |                 |                 |                 |                  |                  |                  |                   |                   |                   |                    |
| % of var.                                  | 34.2            | 15.0            | 4.8             | 1.8              | 0.6              | 0.0              | -                 | -                 | -                 | -                  |
| Cumulative                                 | 34.2            | 49.2            | 70.4            | 83.0             | 93.9             | 100.0            | -                 | -                 | -                 | -                  |
| PCs of 80 WFR portfolios                   |                 |                 |                 |                  |                  |                  |                   |                   |                   |                    |
| % of var.                                  | 29.8            | 18.8            | 4.6             | 1.4              | 0.9              | 0.1              | -                 | -                 | -                 | -                  |
| Cumulative                                 | 29.8            | 48.6            | 68.3            | 77.7             | 89.1             | 99.1             | -                 | -                 | -                 | -                  |
| PCs of interactions of anomalies           |                 |                 |                 |                  |                  |                  |                   |                   |                   |                    |
| % of var.                                  | 13.4            | 5.9             | 2.1             | 1.2              | 0.7              | 0.3              | 0.1               | 0.1               | 0.0               | 0.0                |
| Cumulative                                 | 13.4            | 19.3            | 28.6            | 36.1             | 44.6             | 57.2             | 67.3              | 77.4              | 89.4              | 96.7               |
| PCs of interactions of WFR portfolios      |                 |                 |                 |                  |                  |                  |                   |                   |                   |                    |
| % of var.                                  | 14.2            | 5.4             | 2.1             | 0.9              | 0.5              | 0.3              | 0.1               | 0.1               | 0.0               | 0.0                |
| Cumulative                                 | 14.2            | 19.6            | 28.4            | 34.3             | 40.8             | 51.9             | 60.3              | 67.7              | 76.7              | 83.4               |



**Fig. 3.**  $L^1$  coefficient paths for the optimal model (50 anomaly portfolios). Paths of coefficients based on the optimal (dual-penalty) sparse model that uses 50 anomaly portfolios sorted portfolios (Panel a) and 50 PCs based on anomaly portfolios (Panel b). Labels are ordered according to the vertical ordering of estimates at the right edge of the plot. In Panel b coefficient paths are truncated at the first 15 variables.



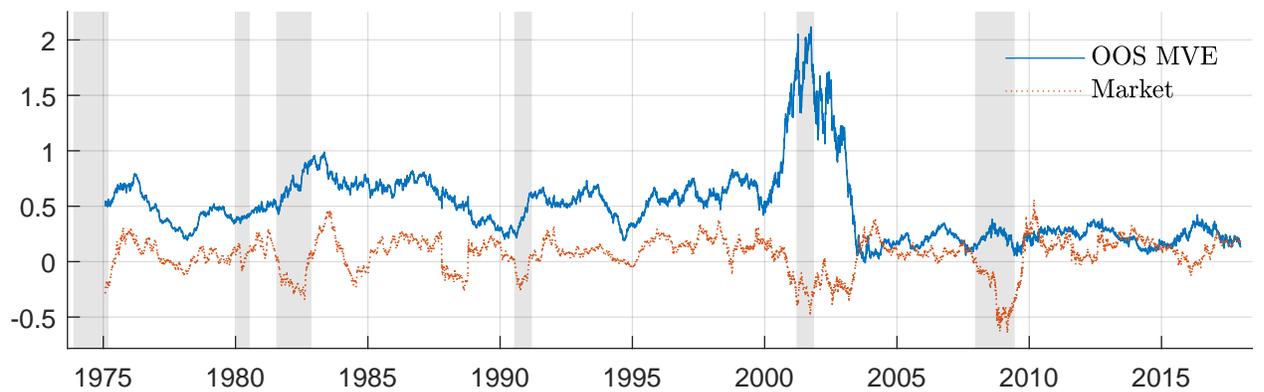
**Fig. 4.**  $L^1$  coefficient paths for the optimal model (WFR portfolios). Paths of coefficients based on the optimal (dual-penalty) sparse model that uses 80 WFR portfolios sorted portfolios (Panel a) and 80 PCs based on WFR portfolios (Panel b). Labels are ordered according to the vertical ordering of estimates at the right edge of the plot.



(a) Returns on MVE portfolio based on 50 anomalies in withheld sample



(b) Returns on MVE portfolio based on interactions of 50 anomalies in withheld sample



(c) Returns on MVE portfolio based on interactions of 50 anomalies in full sample

**Fig. 5. Time-series of returns on the MVE portfolio.** The figure plots the time-series of one-year overlapping returns on the regularized market-neutral MVE portfolio implied by our SDF (blue solid line) and returns on the market (for comparison only; red dashed line). Panel (a) plots MVE portfolio returns in the withheld sample (2005-present) implied by the SDF that was constructed using 50 anomaly portfolios. Panel (b) plots MVE returns in the withheld sample using a model based on interactions of 50 anomalies. Panel (c) plots MVE returns in full sample implied by the model with interactions.

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